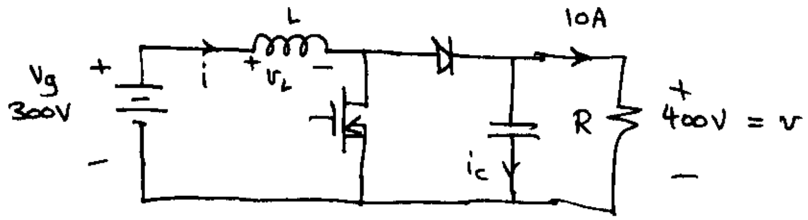
Tutorial

Solution to Problem 3.9

Note paragraph above Problem 3.8

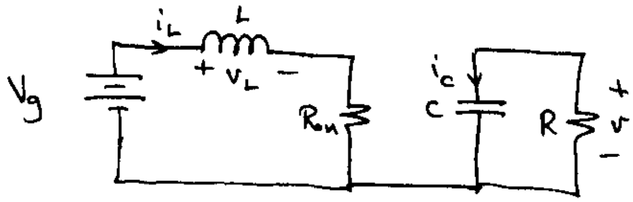
Interface a 300V battery to a 400V, 10A load.
Investigate boost and buck-boost converters: which has better efficiency?
MOSFET has 0.5 Ω on-resistance. All other losses can be ignored.

Boost converter



$$R = \frac{400V}{10A} = 40\Omega$$

transistor on   diode off

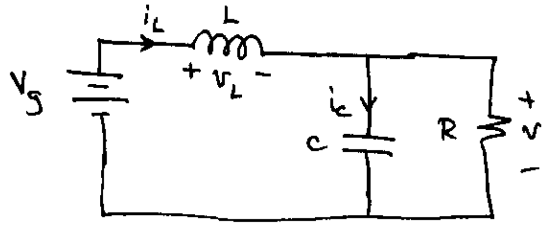


sm. ripple approximation

$$v_L(t) = V_g - i_L(t)R_{on} \approx V_g - I_L R_{on}$$

$$i_c(t) = -v(t)/R \approx -\frac{V}{R}$$

transistor off   diode on



$$v_L(t) = V_g - v(t) \approx V_g - V$$

$$i_c(t) = i_L(t) - \frac{v(t)}{R} \approx I_L - \frac{V}{R}$$

Volt-second balance

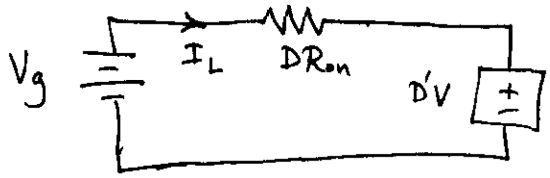
$$\langle v_L(t)\rangle = 0 = D(V_g - I_L R_{on}) + D'(V_g - V) = V_g - DR_{on}I_L - D'V$$

charge balance

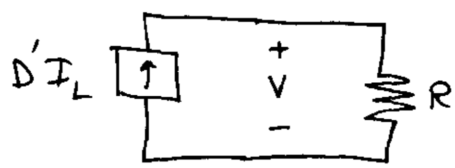
$$\langle i_c(t)\rangle = 0 = D\left(-\frac{V}{R}\right) + D'\left(I_L - \frac{V}{R}\right) = D'I_L - \frac{V}{R}$$

Construct equivalent circuit

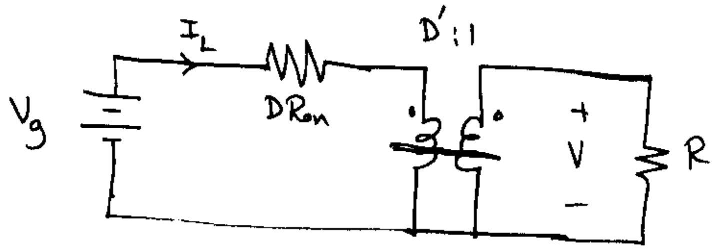
inductor – loop equation $\langle v_L \rangle = 0 = V_g - D R_{on} I_L - D' V$



capacitor – node equation



Combine circuits, replace dependent sources with $D':1$ transformer



Solution :

$$V = \frac{1}{D'} V_g \frac{1}{1 + \frac{D}{(D')^2} \frac{R_{on}}{R}} \qquad I_L = \frac{V}{D' R}$$

$$\eta = \frac{1}{1 + \frac{D}{(D')^2} \frac{R_{on}}{R}} \qquad P_{loss} = I_L^2 D R_{on}$$

For the values $V_g = 300$, $V = 400$, $R = 40$, $R_{on} = 0.5$

find D and η (and also I_L and P_{loss})

Solve quadratic equation for D $\left(\text{arises from } \frac{V}{V_g} = \frac{1}{1-D} \frac{1}{1 + \frac{D}{(1-D)^2} \frac{R_{on}}{R}} \right)$

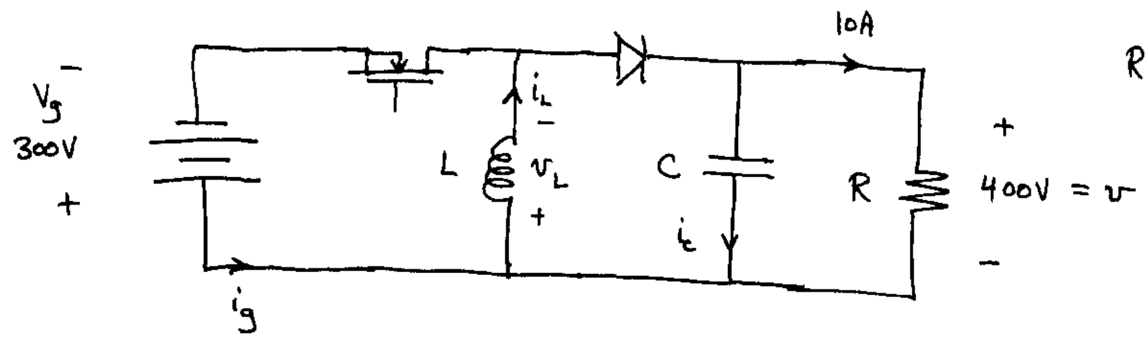
$\Rightarrow D = 0.2543$ (note $D \to 0.2500$ for $R_{on} \to 0$)

Then $\eta = 0.9943$ 99.43%

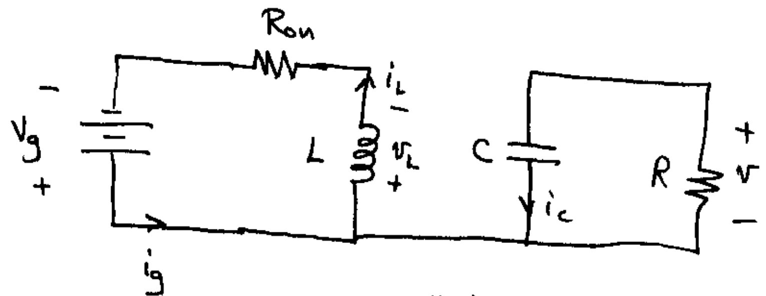
$I_L = 13.4A$

$P_{loss} = 22.9$ W

Buck-boost converter



transistor on $0 < t < DT_s$ diode off



[Since the converter input current $i_g(t)$ is pulsating, we must write equations for $i_g(t)$ during each subinterval, then average to find the dc component]

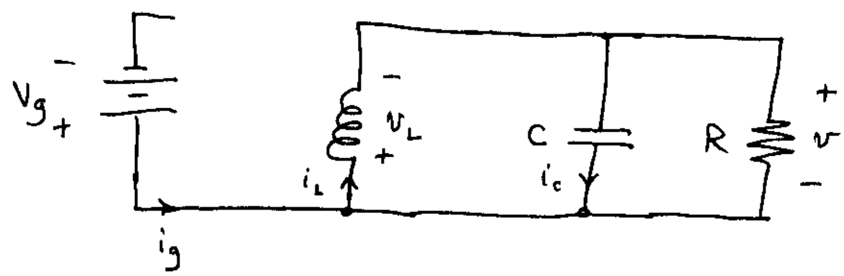
small ripple approximation

$$v_L(t) = V_g - i_L R_{on} \approx V_g - I_L R_{on}$$

$$i_c(t) = -v(t)/R \approx -V/R$$

$$i_g(t) = i_L(t) \approx I_L$$

transistor off $\quad DT_s < t < T_s \quad$ diode on



$$v_L(t) = -v(t) \approx -V$$

$$i_c(t) = i_L(t) - v(t)/R \approx I_L - V/R$$

$$i_g(t) = 0$$

Volt-second balance

$$\langle v_L(t)\rangle = 0 = D\left(V_g - I_L R_{on}\right) + D'\left(-V\right)$$

charge balance

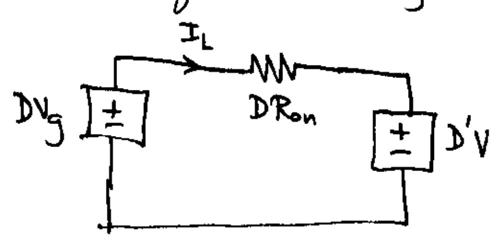
$$\langle i_c(t)\rangle = 0 = D\left(-V/R\right) + D'\left(I_L - V/R\right)$$

average input current

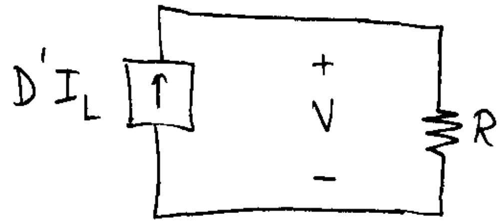
$$\langle i_g\rangle = I_g = D\left(I_L\right) + D'\left(0\right)$$

Construct equivalent circuit

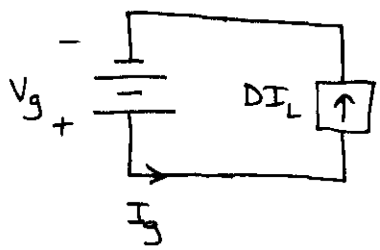
inductor loop equation: $DV_g - I_L D R_{on} - D'V = \langle v_L\rangle = 0$



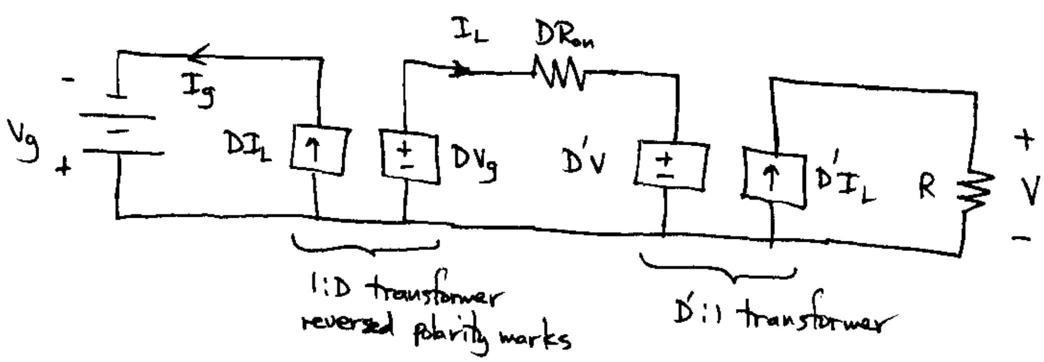
Capacitor node equation: $D'I_L - \frac{V}{R} = \langle i_c \rangle = 0$



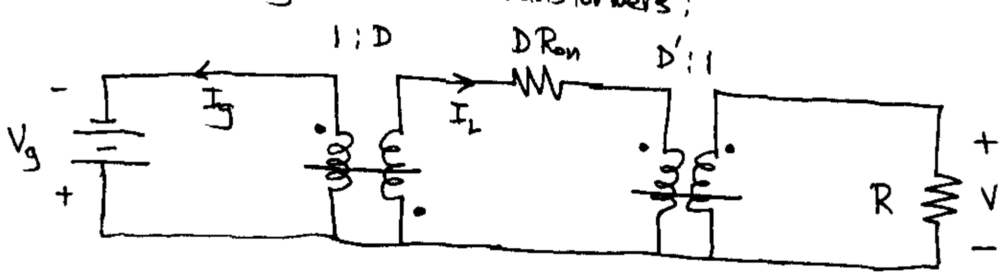
Input current (node) equation: $I_g = DI_L$



Draw circuit models together:



Model including ideal dc transformers:



Solution of model:

$$V = \frac{D}{D'} V_g \frac{R}{R + \frac{D}{(D')^2} R_{on}} \Rightarrow \frac{V}{V_g} = \frac{D}{D'} \frac{1}{1 + \frac{D}{(D')^2} \frac{R_{on}}{R}}$$

$$\eta = \frac{1}{1 + \frac{D}{(D')^2} \frac{R_{on}}{R}}$$

$$I_L = \frac{V}{D' R}$$

$$P_{loss} = I_L^2 D R_{on}$$

Note that, with the exception of the extra factor of D in the $\frac{V}{V_g}$ equation, all of the above equations are identical to the respective boost converter equations on page 2. Because of the extra factor of D, the buck-boost converter must operate at a larger duty cycle. This leads to increased inductor current, increased transistor conduction time, and increased power loss.

Solution

$D = 0.5813$

$\eta = 0.9602$ 96,02%

$I_L = 23.89$ A

$P_{loss} = 165.8$ W — more than 7 times larger than boost

heatsink must be 7 times larger

boost is much better than buck-boost in this application

End of problem