A closer look at the correction factor

RWE

$$\frac{\left(1+\frac{z}{z_N}\right)}{\left(1+\frac{z}{z_D}\right)}$$

has a numerator term $\left(1+\frac{z}{z_N}\right)$ and a denominator term $\frac{1}{(1+z/z_D)}$. Both terms can be handled in a similar manner.

There are usually three cases of interest:

1. $1 \gg \left\|\frac{z}{z_N}\right\|$ $\left(\text{or } 1 \gg \left\|\frac{z}{z_D}\right\|\right)$

   then $1+\frac{z}{z_N} \approx 1$ $\left(\text{or } \frac{1}{1+\frac{z}{z_D}} \approx 1\right)$

2. $\left\|\frac{z}{z_N}\right\| \sim 1$ $\left(\text{or } \left\|\frac{z}{z_D}\right\| \sim 1\right)$

   we get poles or zeroes in this case

3. $1 \ll \left\|\frac{z}{z_N}\right\|$ $\left(\text{or } 1 \ll \left\|\frac{z}{z_D}\right\|\right)$

   - then $1+\frac{z}{z_N} \approx \frac{z}{z_N}$

     $$\frac{1}{1+\frac{z}{z_D}} \approx \frac{1}{\left(\frac{z}{z_D}\right)}$$

   - when $1 \ll \left\|\frac{z}{z_N}\right\|$ <u>and</u> $1 \ll \left\|\frac{z}{z_D}\right\|$

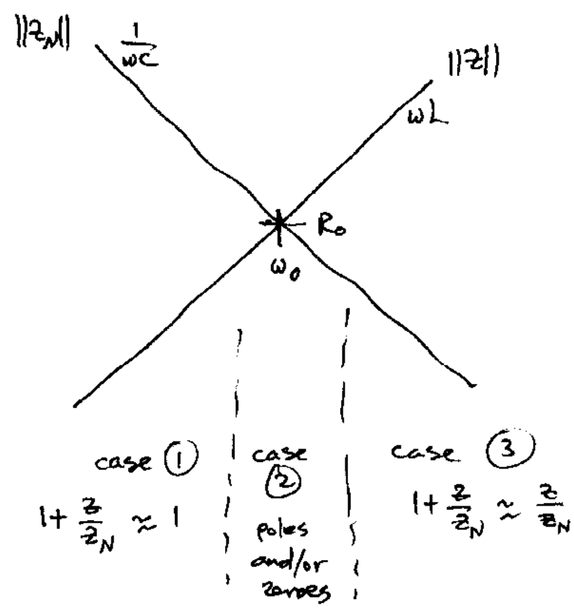
     then $$\frac{1+\frac{z}{z_N}}{1+\frac{z}{z_D}} \approx \frac{z_D}{z_N}$$

Let's look at some examples

A. $Z_N(s) = \frac{1}{sC}$, $Z(s) = sL$

$\|Z_N\| = \frac{1}{\omega C}$, $\angle Z_N = -90°$

$\|Z\| = \omega L$, $\angle Z = +90°$

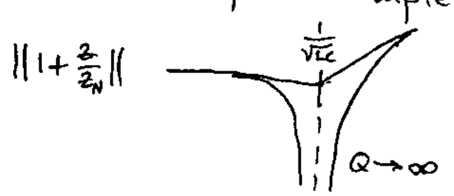


At $\omega = \omega_0$, Z and Z_N have equal magnitude but opposite phase.
Hence, $\frac{Z}{Z_N} = -1$ at $\omega = \omega_0$ $\left(\text{try it: } \frac{Z(j\omega)}{Z_N(j\omega)} = \frac{j\omega_0 L}{\left(\frac{1}{j\omega_0 C}\right)} = \frac{jR_0}{\left(\frac{R_0}{j}\right)} = j^2 = -1\right)$

So $1 + \frac{Z}{Z_N} \to 0 \Rightarrow -\infty$ dB

actual result: $1 + \frac{Z}{Z_N} = 1 + \frac{sL}{\left(\frac{1}{sC}\right)} = 1 + s^2LC$

undamped complex zeroes

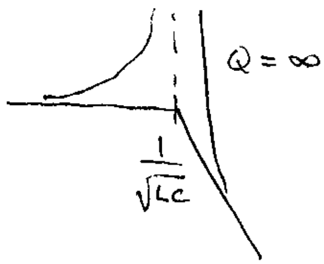


When $\angle \frac{Z}{Z_N} = \angle Z - \angle Z_N = \pm 180°$, then $\|1 + \frac{Z}{Z_N}\| \to 0$ at $\|Z\| = \|Z_N\|$

Denominator is similar, but poles are obtained instead of zeroes:

if $Z_D = \frac{1}{sC}$ and $Z = sL$ then $\frac{1}{1+\frac{Z}{Z_D}} = \frac{1}{1+s^2LC}$

undamped resonant poles



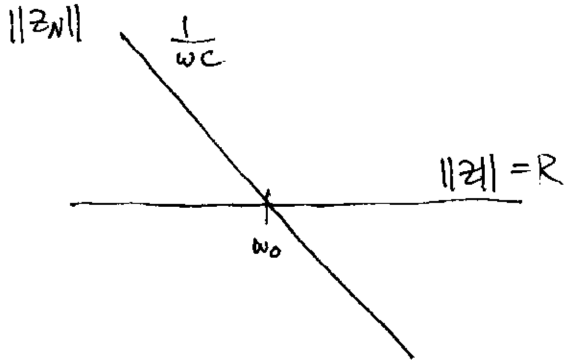
when $\angle \frac{Z}{Z_D} = \angle Z - \angle Z_D = \pm 180°$

then $\left\| \frac{1}{1+\frac{Z}{Z_D}} \right\| \to \infty$ at $\|Z\| = \|Z_D\|$

B. $Z_N(s) = \frac{1}{sC}$, $Z = R$

$\|Z_N\| = \frac{1}{\omega C}$, $\angle Z_N = -90°$

$\|Z\| = R$, $\angle Z = 0°$



At $\omega = \omega_0$, Z and Z_N have equal magnitudes.
Their phase differs by 90°.
Hence, $\frac{Z}{Z_N} = j$ at $\omega = \omega_0$ $\left(\frac{Z}{Z_N} = \frac{R}{\left(\frac{1}{j\omega_0 C}\right)} \right.$ with $\frac{1}{\omega_0 C} = R$

$\left. \Rightarrow \frac{Z}{Z_N} = \frac{R}{\left(\frac{R}{j}\right)} = j \text{ at } \omega = \omega_0 \right)$

So $1 + \frac{Z}{Z_N} \to 1 + j = \sqrt{2}\,\angle 45°$

$\left\| 1 + \frac{Z}{Z_N} \right\| = \sqrt{2} \Rightarrow +3\,dB$

We obtain a real zero at $\omega = \omega_0$

$$1 + \frac{Z}{Z_N} = 1 + \frac{R}{\left(\frac{1}{sC}\right)} = 1 + sRC$$

Denominator behavior is similar, but poles are obtained instead of zeroes.

The general case: How the correction factor terms $\left(1 + \frac{Z}{Z_N}\right)$ and $\left(1 + \frac{Z}{Z_D}\right)^{-1}$ depend on $\frac{Z}{Z_N}$ and $\frac{Z}{Z_D}$.

A related question: if we don't want to substantially change a given transfer function, how should Z be chosen? Sure, we want $\|Z\| \ll \|Z_N\|$ and $\|Z\| \ll \|Z_D\|$, but how small is small enough?

At a given frequency,

Let $\frac{Z}{Z_N} = R e^{j\theta}$ so $\frac{\|Z\|}{\|Z_N\|} = R$, $\angle Z - \angle Z_N = \theta$

Then $\left(1 + \frac{Z}{Z_N}\right) = 1 + R e^{j\theta} = 1 + R\cos\theta + jR\sin\theta$

$$\left\| 1 + \frac{Z}{Z_N} \right\| = \sqrt{(1 + R\cos\theta)^2 + (R\sin\theta)^2} = \sqrt{1 + R^2 + 2R\cos\theta}$$

$$\angle\left(1+\frac{Z}{Z_N}\right) = \tan^{-1}\left(\frac{R\sin\theta}{1+R\cos\theta}\right)$$

See Figs. C.6 and C.7 of text for plots of these functions: How the magnitude and phase of the correction factor term $\left(1+\frac{Z}{Z_N}\right)$ depends on $\frac{\|Z\|}{\|Z_N\|}$ and $\angle Z - \angle Z_N$.

If $\|Z\| < \frac{1}{10}\|Z_N\|$ $\quad$ (so $\|Z\|_{dB} < \|Z_N\|_{dB} - 20\,dB$)

then $$-1\,dB \le \left\|1+\frac{Z}{Z_N}\right\| \le +1\,dB$$

and $$-7^\circ \le \angle\left(1+\frac{Z}{Z_N}\right) \le +7^\circ$$

If $\|Z\| < \frac{1}{3}\|Z_N\|$ $\quad$ (so $\|Z\|_{dB} < \|Z_N\|_{dB} - 10\,dB$)

then $$-3\tfrac{1}{2}\,dB \le \left\|1+\frac{Z}{Z_N}\right\| \le +3\tfrac{1}{2}\,dB$$

$$-20^\circ \le \angle\left(1+\frac{Z}{Z_N}\right) \le +20^\circ$$

Results for $\frac{1}{\left(1+\frac{Z}{Z_D}\right)}$ term are similar since, if $\frac{Z}{Z_D} = Re^{j\theta}$ then

$$\left\|\frac{1}{1+\frac{Z}{Z_D}}\right\| = \frac{1}{\sqrt{1+R^2+2R\cos\theta}}, \quad \left\|\frac{1}{1+\frac{Z}{Z_D}}\right\|_{dB} = -\left\|\sqrt{1+R^2+2R\cos\theta}\right\|_{dB}$$

$$\angle\frac{1}{1+\frac{Z}{Z_D}} = -\tan^{-1}\left(\frac{R\sin\theta}{1+R\cos\theta}\right).$$ See Figs. C.8 and C.9

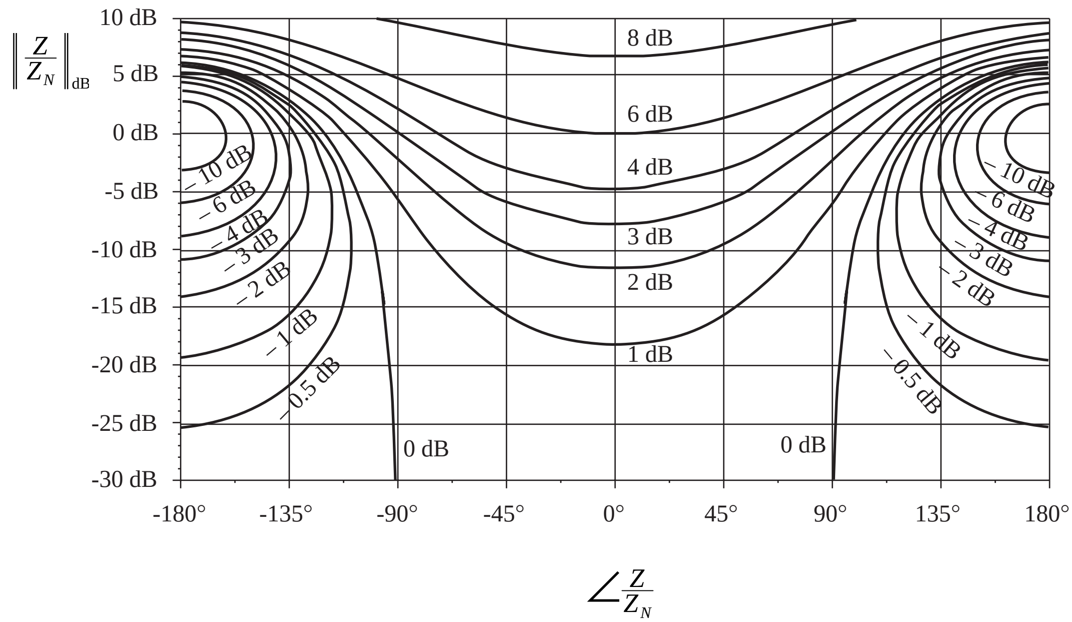


Fig. C.6 Contours of constant $\| 1 + Z/Z_N \|$, as a function of the magnitude and phase of Z/Z_N.

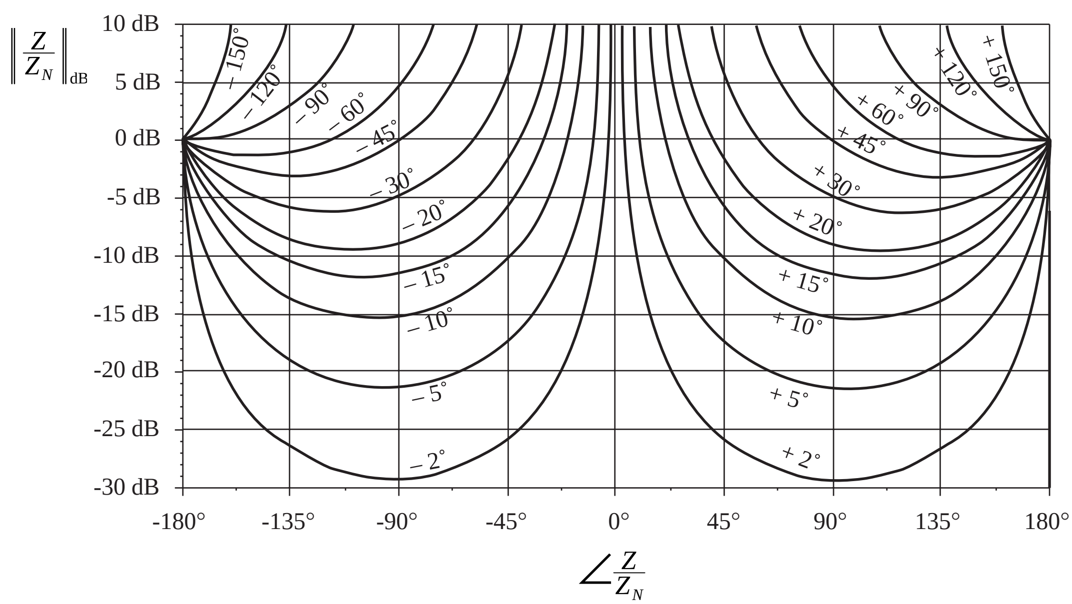


Fig. C.7 Contours of constant $\angle(1 + Z/Z_N)$, as a function of the magnitude and phase of Z/Z_N.

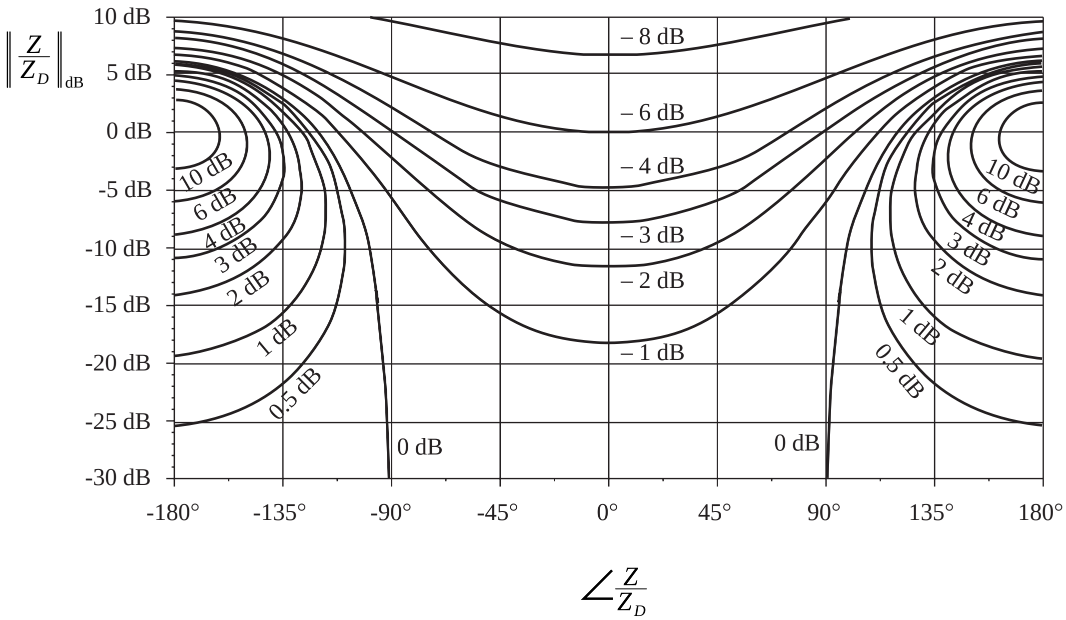


Fig. C.8 Contours of constant $\| 1/(1 + Z/Z_D) \|$, as a function of the magnitude and phase of Z/Z_D

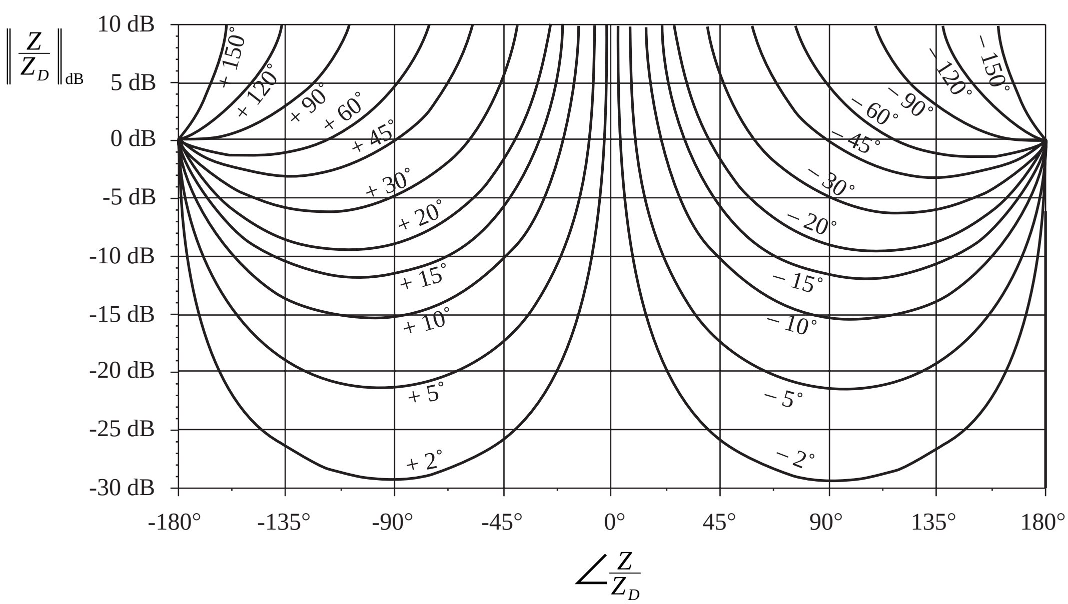


Fig. C.9 Contours of constant $\angle 1/(1 + Z/Z_D)$, as a function of the magnitude and phase of Z/Z_D.