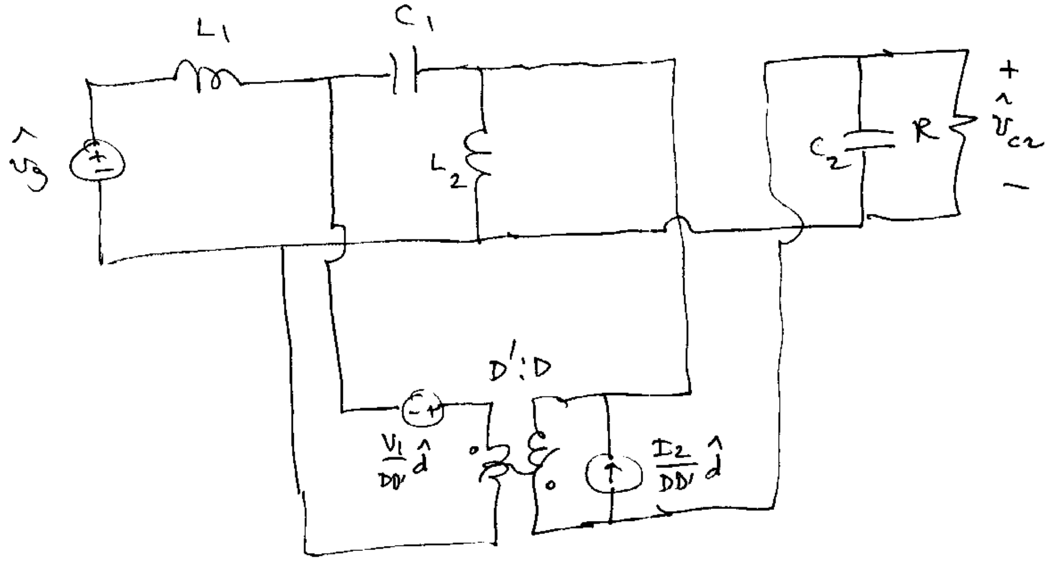
RWE

Using the Extra Element Theorem to gain insight into small signal behavior of SEPIC

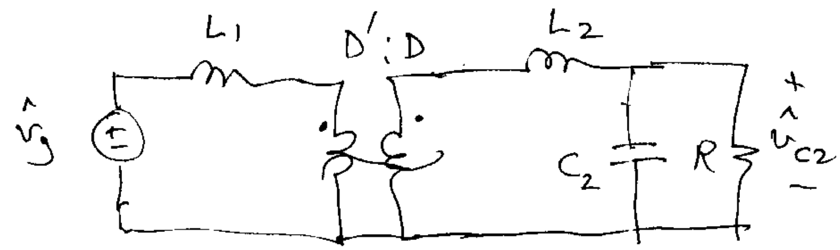
Averaged switch model



Find $G_{vg}(s) = \left.\dfrac{\hat{v}_{C2}}{\hat{v}_g}\right|_{\hat{d}=0}$

would be a lot easier if C_1 weren't present!

if $C_1 \to$ open ckt, model (with $\hat{d}=0$) becomes



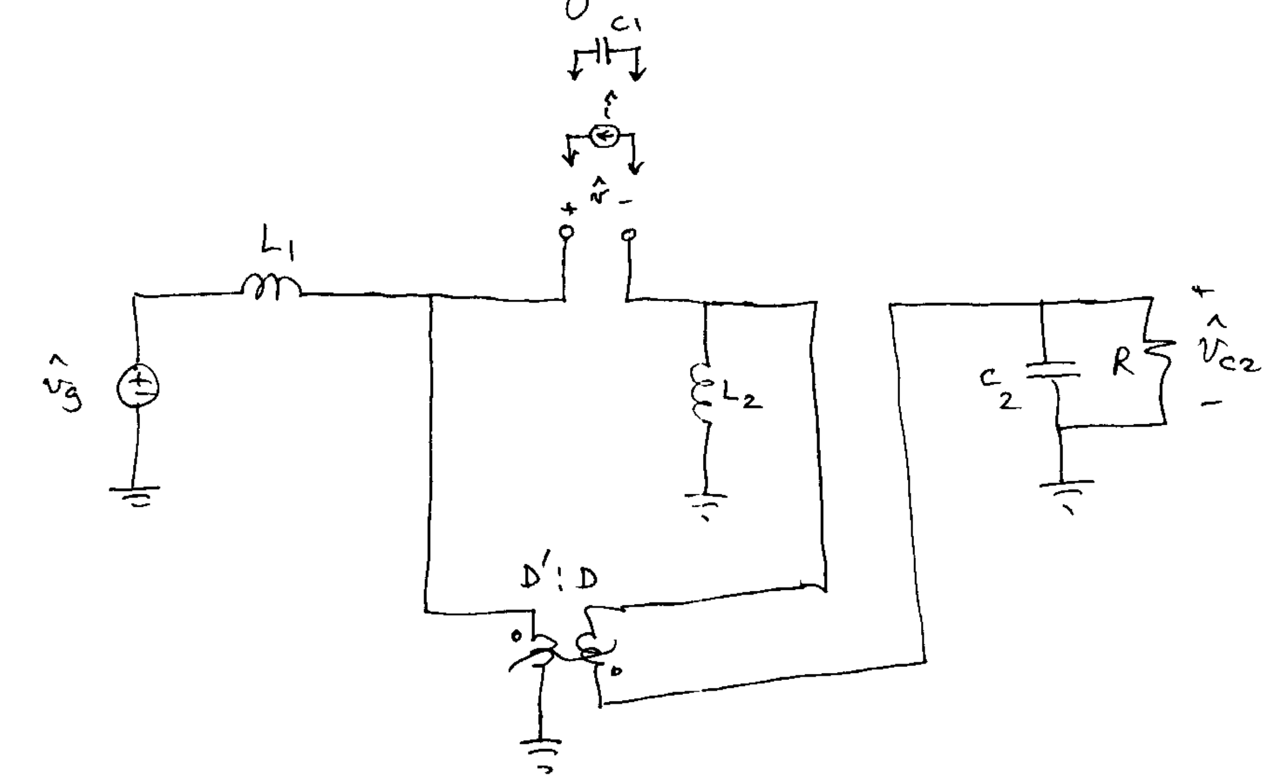
$$R \| \frac{1}{sC_2} = \frac{R}{1+sRC_2}$$

$$G_{vg\text{-}old}(s) = \frac{D}{D'} \frac{R\|\frac{1}{sC_2}}{R\|\frac{1}{sC_2} + s\left(L_2 + \left(\frac{D}{D'}\right)^2 L_1\right)}$$

$$= \frac{D}{D'} \frac{1}{1 + s\dfrac{L_2 + \left(\frac{D}{D'}\right)^2 L_1}{R} + s^2 C_2 \left(L_2 + \left(\frac{D}{D'}\right)^2 L_1\right)}$$

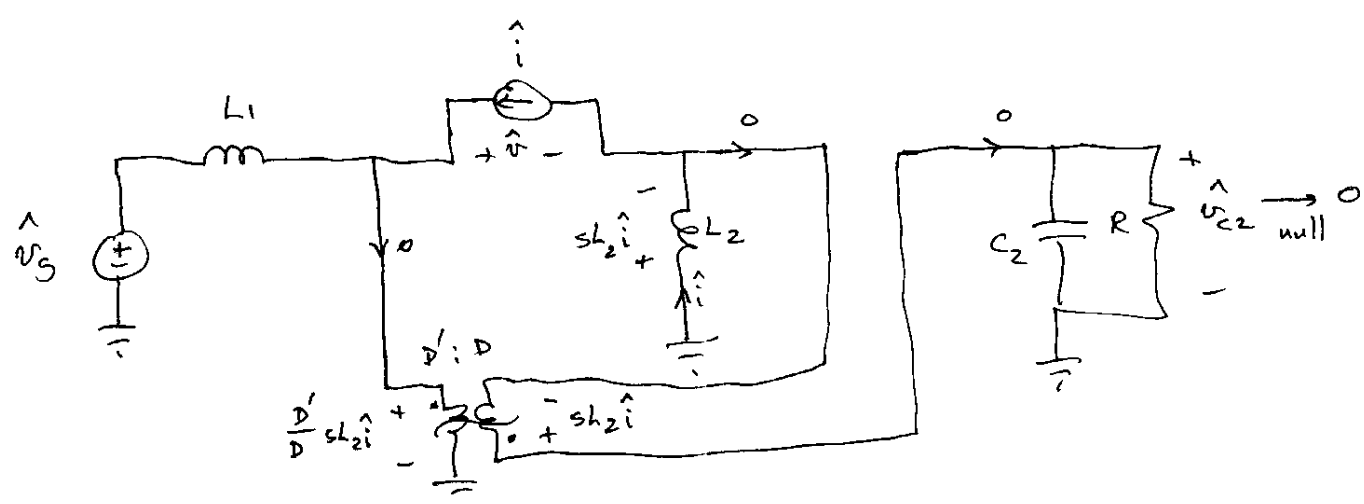
2 pole response $= \dfrac{D}{D'} \dfrac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$

Now add C_1 using extra element theorem



$$G_{vg}(s) = G_{vg\text{-old}}(s) \cdot \frac{1 + \frac{Z_N}{Z}}{1 + \frac{Z_D}{Z}} \qquad \text{with } Z = \frac{1}{sC_1}$$

<u>$Z_N(s)$</u> in the presence of $\hat{v}_g$, adjust $\hat{i}$ to null $\hat{v}_{c2}$



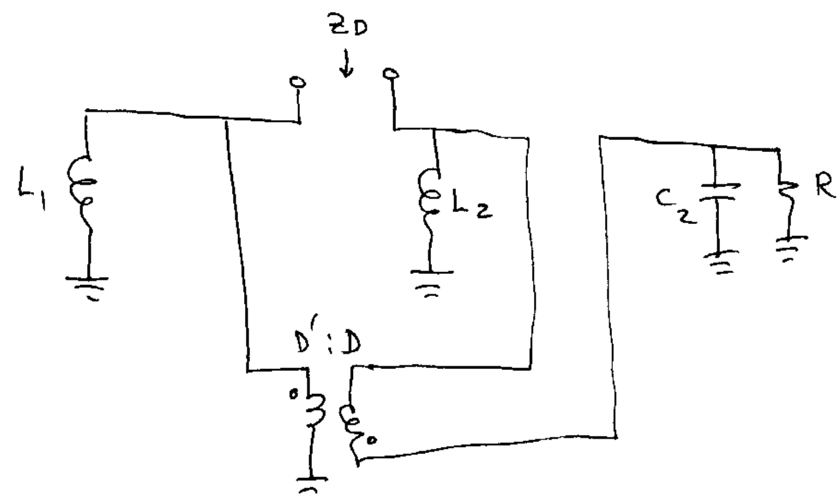
$$\hat{v} = \frac{D'}{D} sL_2 \hat{i} + sL_2 \hat{i}$$

$$= \hat{i} \; sL_2 \left(\frac{D'}{D} + 1 \right)$$

$$= \hat{i} \; \frac{sL_2}{D}$$

so $\quad Z_N(s) = \dfrac{sL_2}{D}$

$$Z_D(s) = \left. \frac{\hat{v}}{\hat{i}} \right|_{\hat{v}_g = 0}$$



After some analysis, one can show that

$$Z_D(s) = s(L_1 + L_2) \; \frac{1 + s \dfrac{L_1 \| L_2}{D'^2 R} + s^2 \dfrac{L_1 \| L_2 \, C_2}{D'^2}}{1 + s \dfrac{L_2 + \left(\frac{D}{D'}\right)^2 L_1}{R} + s^2 C_2 \left(L_2 + \left(\frac{D}{D'}\right)^2 L_1 \right)}$$

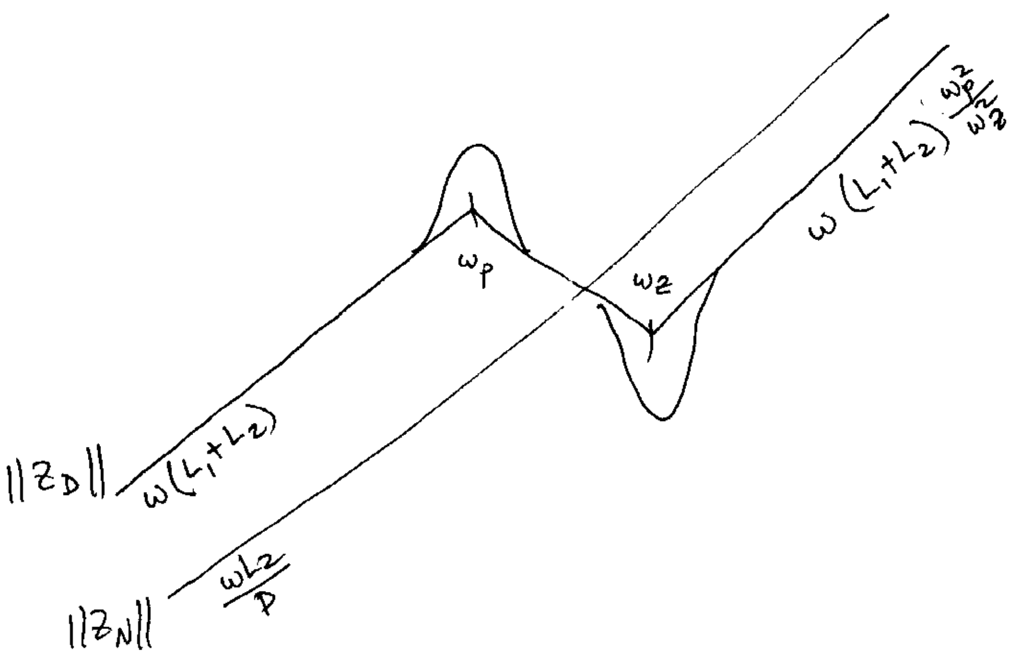
which is of the form

$$Z_D(s) = s(L_1 + L_2) \; \frac{1 + \dfrac{s}{Q_z \omega_z} + \left(\dfrac{s}{\omega_z}\right)^2}{1 + \dfrac{s}{Q_p \omega_p} + \left(\dfrac{s}{\omega_p}\right)^2}$$

with
$$\omega_z = \frac{D'}{\sqrt{L_1 \| L_2 \ C_2}}$$

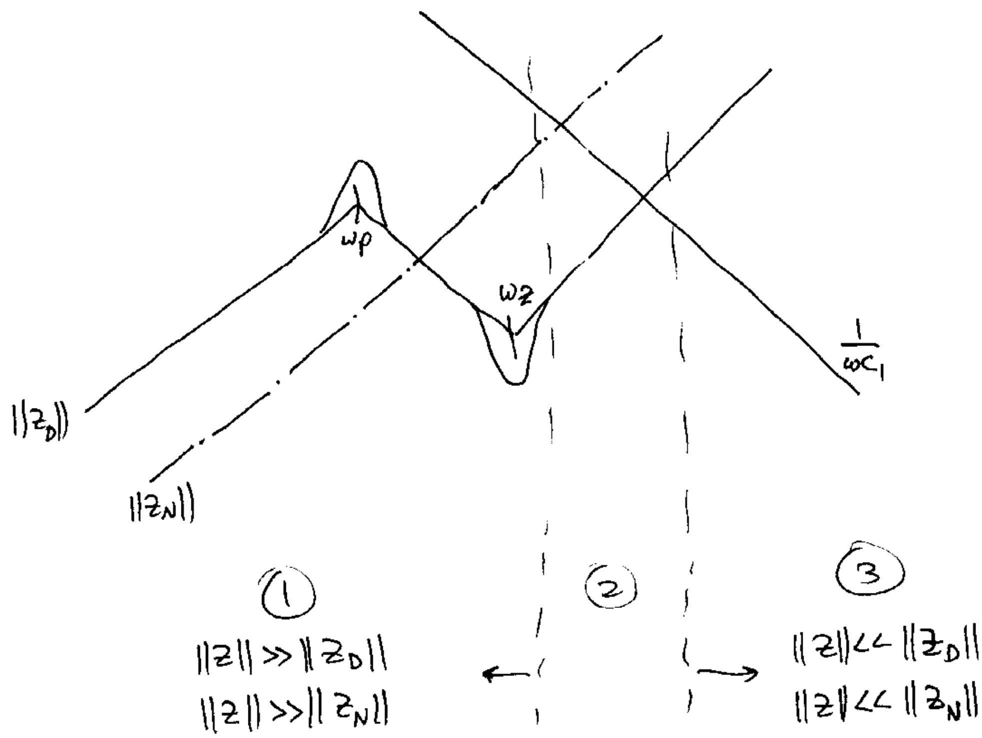
$$\omega_p = \frac{1}{\sqrt{C_2\left(L_2 + \left(\frac{D}{D'}\right)^2 L_1\right)}} = \omega_0$$

Asymptotes



Analysis of correction factor

Suppose $\|Z\| = \frac{1}{\omega C_1}$ is as follows:



Region 1

$$\text{correction factor} = \frac{1+\frac{Z_N}{Z}}{1+\frac{Z_D}{Z}} \approx 1$$

Region 2

$Z(s) = \frac{1}{sC_1}$ is comparable in magnitude to Z_N and/or Z_D. Correction factor deviates substantially from 1

Region 3

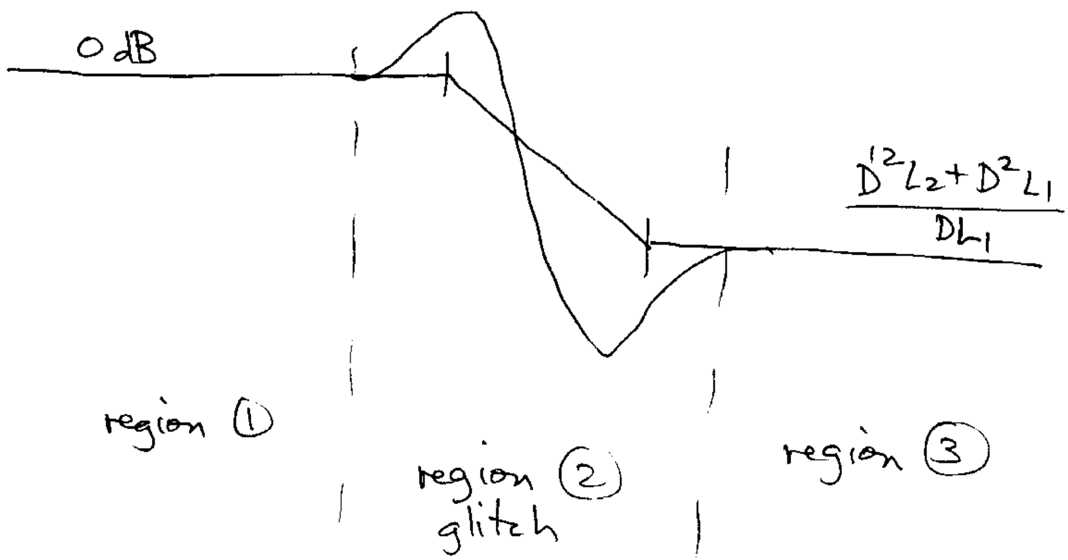
$\|Z\| \ll \|Z_D\|$ and $\|Z\| \ll \|Z_N\|$. Correction factor becomes

$$\frac{1+\frac{Z_N}{Z}}{1+\frac{Z_D}{Z}} \approx \frac{\left(\frac{Z_N}{Z}\right)}{\left(\frac{Z_D}{Z}\right)} = \frac{Z_N}{Z_D}.$$

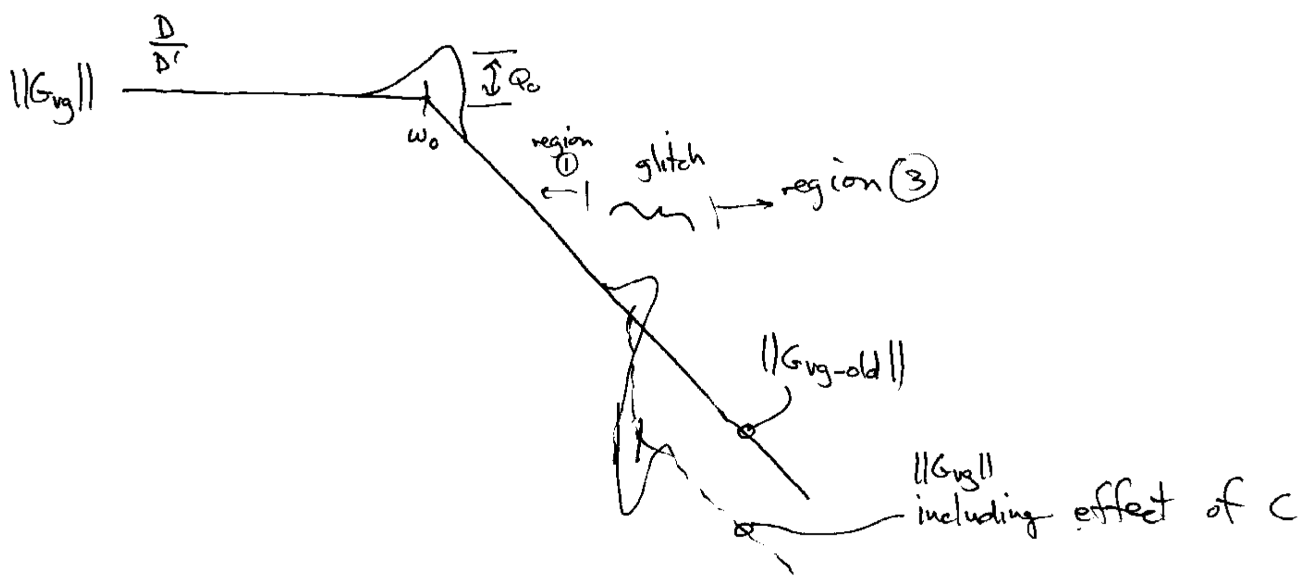
$$\frac{Z_N}{Z_D} = \frac{\left(\frac{sL_2}{D}\right)}{s(L_1+L_2)\left(\frac{\omega_p^2}{\omega_z^2}\right)} = \frac{L_2}{L_1+L_2}\ \frac{1}{D}\ \frac{D'^2}{L_1 \| L_2 \cdot C}\ C\left(L_2+\left(\frac{D}{D'}\right)^2 L_1\right)$$

$$= \frac{D'^2}{D\,L_1}\left(L_2+\frac{D^2}{D'^2}L_1\right) = \frac{D'^2 L_2 + D^2 L_1}{D\,L_1}$$

||correction factor||



<u>Composite transfer function</u>



Physical interpretation

Converter operates as second-order system ($G_{vg\text{-old}}$) plus a pair of resonant poles and resonant zeroes caused by C_1 interactions. These pairs of poles & zeroes cause an additional "glitch" in the transfer functions, near the frequencies where $\frac{1}{\omega C_1}$ intersects the $\|Z_N\|$ and $\|Z_D\|$ asymptotes.

From Figs. C.6 to C.9:

$Z(j\omega) = \frac{1}{j\omega C_1}$ has phase $-90°$

$Z_N(j\omega) = \frac{j\omega L_2}{D}$ has phase $+90°$

$Z_D(j\omega)$ has phase close to $+90°$ for $\omega \ll \omega_p$ or $\omega \gg \omega_z$

So when $\|Z\| = \|Z_N\|$, $1 + \frac{Z_N}{Z} = 0$ because Z_N and Z have opposite phase

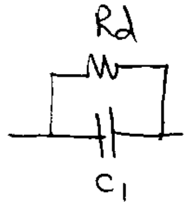
$\Rightarrow$ undamped complex zeroes at the frequency where $\|Z\|$ crosses $\|Z_N\|$

also, when $\|Z\| = \|Z_D\|$, $\dfrac{1}{1 + \frac{Z_D}{Z}}$ becomes large in magnitude because Z_D and Z have approximately opposite phase

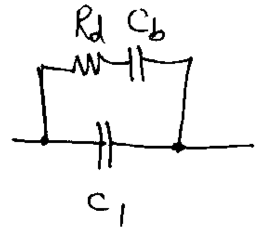
$\Rightarrow$ resonant poles near the frequency where $\|Z\| = \|Z_D\|$

Extensions

(A) Damping the glitch: add damping resistor in parallel with C_1

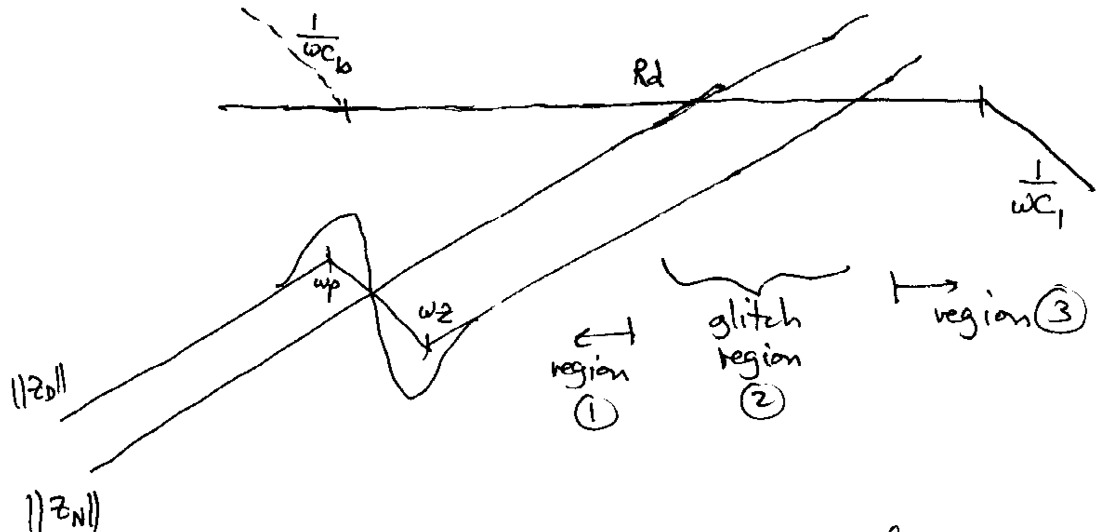


or



$C_b >> C_1$

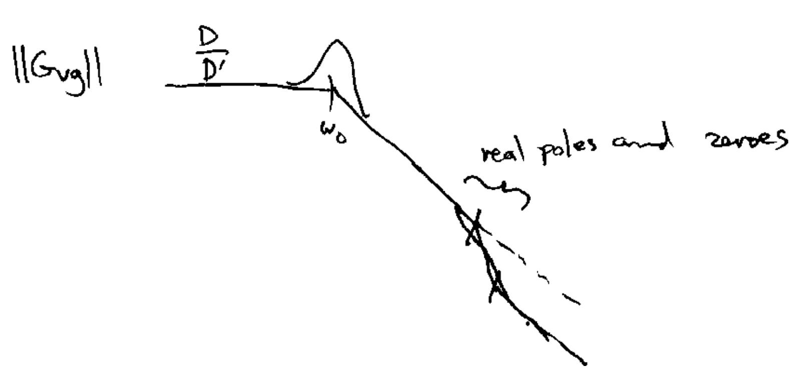
then $Z(s) = R_d \| \frac{1}{sC_1}$



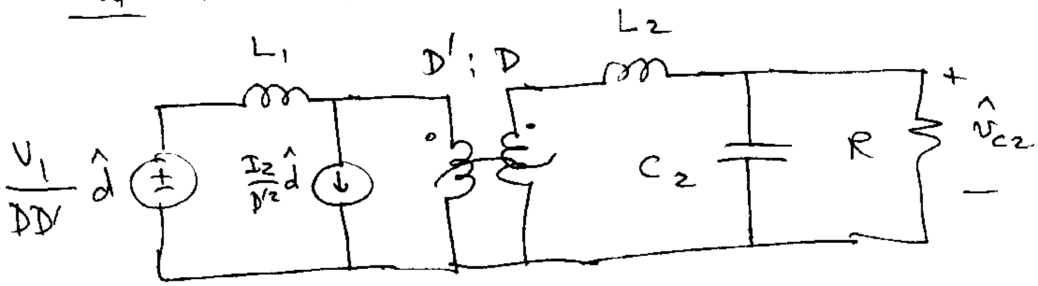
This reduces the Q-factors of the poles and zeroes associated with the glitch. In the vicinity of region ②, $Z(s) \approx R_d$ has phase $0°$

Z_N and Z_D have phase close to $90°$

$\Rightarrow$ $1+\frac{Z_N}{Z}$ and $\frac{1}{1+\frac{Z_D}{Z}}$ terms have real poles and zeroes



$G_{vd\text{-}old}$ $\hat{v}_g = 0$

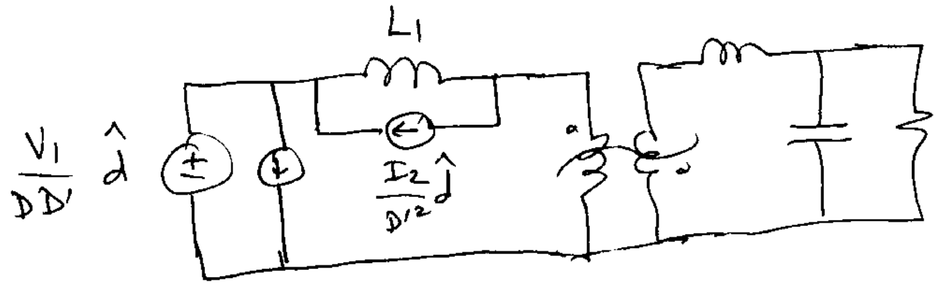


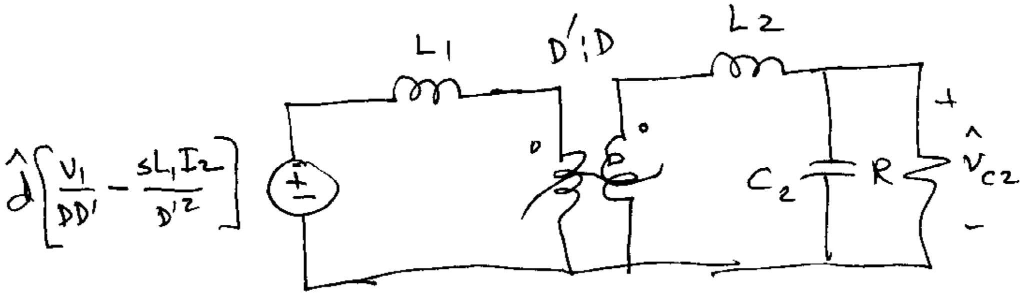
$V_1 = V_g$

$I_2 = \frac{V_{c2}}{R}$

$= \left(\frac{D}{D'}\right) \frac{V_g}{R}$

Push current source through L_1:





$$G_{vd\text{-}old}(s) = \left.\frac{\hat{v}_{c2}}{\hat{d}}\right|_{\substack{\frac{1}{\omega C_1} \to \text{open} \\ \hat{v}_g = 0}} = \frac{D}{D'} \frac{\left(\frac{V_1}{DD'} - \frac{sL_1 I_2}{D'^2}\right)}{1 + \frac{s\left(L_2 + \left(\frac{D}{D'}\right)^2 L_1\right)}{R} + s^2 C_2\left(L_2 + \left(\frac{D}{D'}\right)^2 L_1\right)}$$

$$= \frac{D}{D'} \frac{V_1}{DD'} \frac{\left(1 - \frac{sL_1 I_2 DD'}{V_1 D'^2}\right)}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

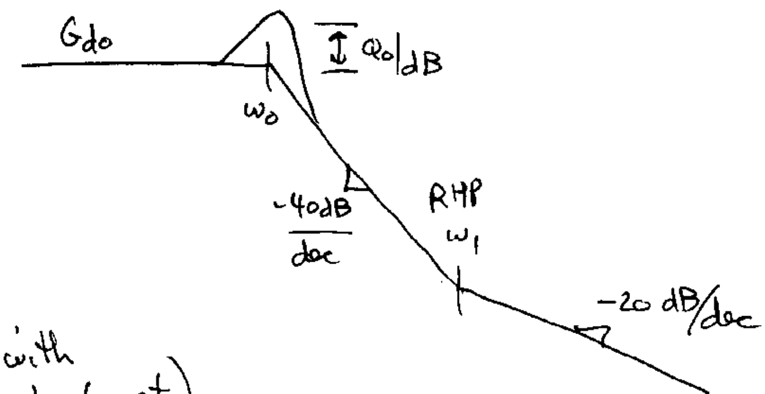
Q_0 & ω_0 as on p. 1

$$= G_{d0} \frac{1 - \frac{s}{\omega_1}}{1 + \frac{s}{Q_0 \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$G_{d0} = \frac{V_g}{D'^2}$

RHP zero at $\omega_1 = \frac{R}{L_1}\left(\frac{D'}{D}\right)^2$

$\|G_{vd\text{-}old}\|$



(compare with CCM buck-boost)

Now add C_1 in again using EET

$$G_{vd}(s) = \left(G_{vd\text{-}old}\right)\frac{1+\frac{Z_N}{Z}}{1+\frac{Z_D}{Z}} \qquad \text{with } Z(s)=\frac{1}{sC_1}$$

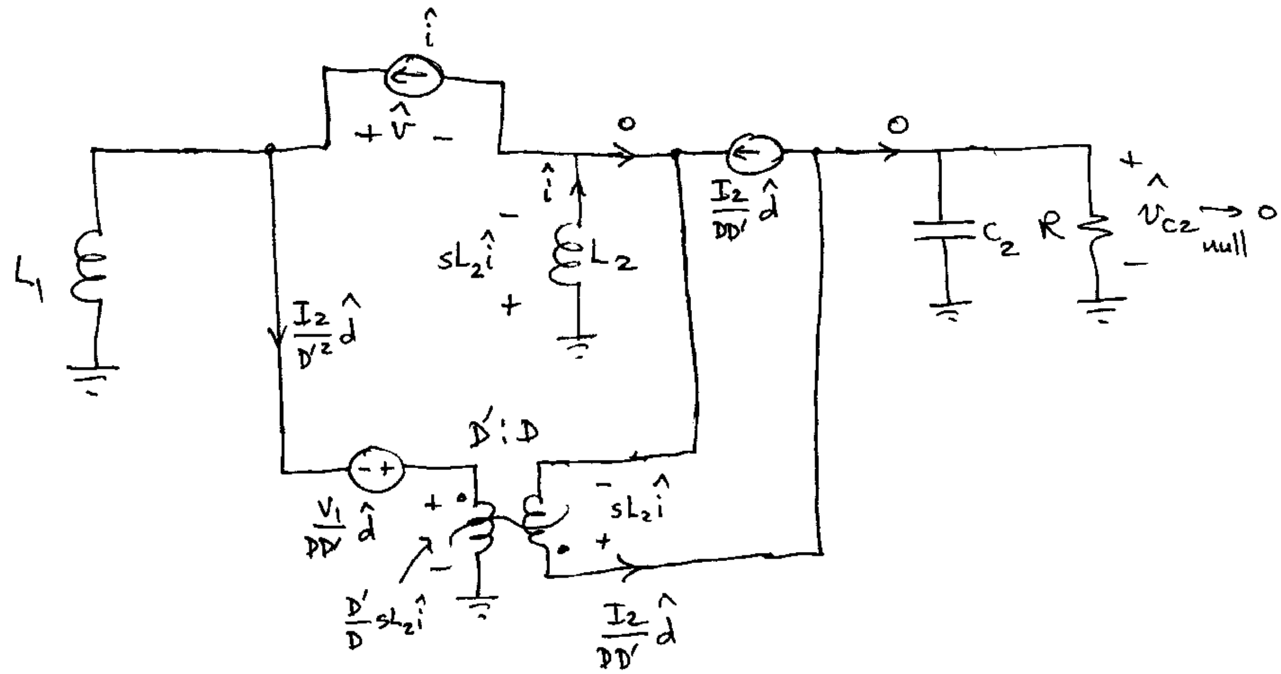
$$Z_N(s) = \left.\frac{\hat{v}}{\hat{i}}\right|_{\hat{v}_{c2}\xrightarrow[\text{null}]{}0}$$

In presence of $\hat{d}$, adjust $\hat{i}$ to null $\hat{v}_{c2}$

$$Z_D(s) = \left.\frac{\hat{v}}{\hat{i}}\right|_{\hat{d}=0}$$

– same $Z_D(s)$ as for $G_{vg}(s)$ analysis

$Z_N(s)$ for $G_{vd}(s)$ analysis



$$\hat{v} = \frac{D'}{D} sL_2 \hat{i} - \frac{V_1}{DD'}\hat{d} + sL_2\hat{i} = \frac{sL_2}{D}\hat{i} - \frac{V_1}{DD'}\hat{d}$$

with $\hat{i} = \frac{I_2}{D'^2}\hat{d} + \frac{1}{sL_1}\left(\frac{D'}{D} sL_2\hat{i} - \frac{V_1}{DD'}\hat{d}\right)$

$$\Rightarrow \hat{d}\left(\frac{I_2}{D'^2} - \frac{V_1}{DD' sL_1}\right) = \hat{i}\left(1 - \frac{D'}{D}\frac{L_2}{L_1}\right)$$

so $\hat{d} = \hat{i} \dfrac{\left(1 - \frac{D'}{D}\frac{L_2}{L_1}\right)}{\left(1 - \frac{V_1 D'}{I_2 D s L_1}\right)} \dfrac{D'^2}{I_2}$

$$\Rightarrow \hat{v} = \frac{sL_2}{D}\hat{i} - \frac{V_1}{DD'}\hat{i}\frac{D'^2}{I_2}\frac{\left(1 - \frac{D'}{D}\frac{L_2}{L_1}\right)}{\left(1 - \frac{V_1 D'}{s D L_1 I_2}\right)}$$

$$\frac{\hat{v}}{\hat{i}} = Z_N = \frac{sL_2}{D} - \frac{D'}{D}\frac{V_1}{I_2}\frac{1 - \frac{D'L_2}{DL_1}}{1 - \frac{D'V_1}{s D L_1 I_2}}$$

note $\frac{V_1}{I_2} = \frac{D'}{D} R$

$$Z_N(s) = \frac{sL_2}{D} - \left(\frac{D'}{D}\right)^2 R \; \frac{1 - \frac{D'L_2}{DL_1}}{1 - \frac{sD^2L_1}{D'^2R}} \left(-\frac{sD^2L_1}{D'^2R}\right)$$

$$= \frac{sL_2}{D} + \frac{sL_1\left(1 - \frac{D'L_2}{DL_1}\right)}{1 - s\frac{D^2L_1}{D'^2R}}$$

$$= \frac{sL_2}{D} + \frac{s\left(L_1 - \frac{D'}{D}L_2\right)}{1 - s\frac{D^2L_1}{D'^2R}} = \frac{s\frac{L_2}{D} - s^2\frac{L_1L_2D}{D'^2R} + sL_1 - s\frac{D'}{D}L_2}{\left(1 - s\frac{D^2L_1}{D'^2R}\right)}$$

$$= \frac{s\left(L_1 + L_2\right) - s^2\left(\frac{L_1L_2D}{D'^2R}\right)}{1 - s\frac{D^2L_1}{D'^2R}}$$

$$= s\left(L_1 + L_2\right)\frac{\left(1 - s\frac{L_1 \| L_2\, D}{D'^2R}\right)}{\left(1 - s\frac{D^2L_1}{D'^2R}\right)}$$ which contains RHP pole and RHP zero