Summary

Feedback analysis via null double injection
ECEN 5807

$$\frac{u_o}{u_i} = G(s) = G_\infty(s)\,\frac{T(s)}{1+T(s)} + G_0(s)\,\frac{1}{1+T(s)}$$

Quantity	**Current injection**	**Voltage injection**
T(s) Loop gain	$T(s) = \left.\dfrac{i_y(s)}{i_x(s)}\right\vert_{u_i=0}$	$T(s) = \left.\dfrac{v_y(s)}{v_x(s)}\right\vert_{u_i=0}$
$T_n(s)$ Null loop gain	$T_n(s) = \left.\dfrac{i_y(s)}{i_x(s)}\right\vert_{u_o=0}$	$T_n(s) = \left.\dfrac{v_y(s)}{v_x(s)}\right\vert_{u_o=0}$
$G_\infty(s)$ Ideal forward gain	$G_\infty(s) = \left.\dfrac{u_o(s)}{u_i(s)}\right\vert_{i_y=0}$	$G_\infty(s) = \left.\dfrac{u_o(s)}{u_i(s)}\right\vert_{v_y=0}$
$G_0(s)$ Direct forward transmission through feedback path, or open-loop disturbance transfer function	$G_0(s) = \left.\dfrac{u_o(s)}{u_i(s)}\right\vert_{i_x=0}$	$G_0(s) = \left.\dfrac{u_o(s)}{u_i(s)}\right\vert_{v_x=0}$

To find *T(s)*, the input $u_i(s)$ is set to zero. In the presence of the injection source $v_z(s)$ or $i_z(s)$ (see Sections 9.6.1 and 9.6.2 of the text), the signal $v_x(s)$ or $i_x(s)$ is followed around the loop, to find $v_y(s)$ or $i_y(s)$.

To find $T_n(s)$, $G_\infty(s)$, or $G_0(s)$, the signal $v_z(s)$ or $i_z(s)$ is injected in the presence of the input $u_i(s)$. The injection signal is varied as necessary to null the specified quantity to zero. Note that nulling a signal is not the same as shorting it to zero, and the null condition can be used to calculate the given quantity in a relatively simple way.

For example, finding $G_\infty(s)$ is similar to using the principle of virtual ground to find the ideal gain of an op amp circuit. Nulling $v_y(s)$ is equivalent to nulling the error signal. The ratio of the system output to input is found under the conditions that the error signal is nulled.

The following reciprocity identity relates the four quantities:

$$\frac{T_n(s)}{T(s)} = \frac{G_\infty(s)}{G_0(s)}$$