This week
Feedback, revisited

I. A short review of feedback basics (Chapter 9)

II. Measurement and simulation of loop gains and other closed-loop quantities (Sections 9.6, 9.5.4, and B.2.2)

III. Analysis of closed-loop circuits using null double injection (supplementary notes)

IV. Examples (supplementary notes)
I. Short review of feedback basics

Regulator system small-signal block diagram
Solution of block diagram

Manipulate block diagram to solve for $\hat{v}(s)$. Result is

$$
\hat{v} = \hat{v}_{ref} \frac{G_c G_{vd} / V_M}{1 + HG_c G_{vd} / V_M} + \hat{v}_g \frac{G_{vg}}{1 + HG_c G_{vd} / V_M} - \hat{i}_{load} \frac{Z_{out}}{1 + HG_c G_{vd} / V_M}
$$

which is of the form

$$
\hat{v} = \hat{v}_{ref} \frac{1}{H} \frac{T}{1 + T} + \hat{v}_g \frac{G_{vg}}{1 + T} - \hat{i}_{load} \frac{Z_{out}}{1 + T}
$$

with $T(s) = H(s) G_c(s) G_{vd}(s) / V_M = "loop gain"

Loop gain $T(s)$ = products of the gains around the negative feedback loop.
9.2.2. Feedback causes the transfer function from the reference input to the output to be insensitive to variations in the gains in the forward path of the loop

Closed-loop transfer function from \( \hat{v}_{ref} \) to \( \hat{v}(s) \) is:

\[
\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \bigg|_{v_g = 0, i_{load} = 0} = \frac{1}{H(s)} \frac{T(s)}{1 + T(s)}
\]

If the loop gain is large in magnitude, i.e., \( \|T\| > 1 \), then \((1 + T) \approx T\) and \(T/(1+T) \approx T/T = 1\). The transfer function then becomes

\[
\frac{\hat{v}(s)}{\hat{v}_{ref}(s)} \approx \frac{1}{H(s)}
\]

which is independent of the gains in the forward path of the loop.

This result applies equally well to dc values:

\[
\frac{V}{V_{ref}} = \frac{1}{H(0)} \frac{T(0)}{1 + T(0)} \approx \frac{1}{H(0)}
\]
Example: construction of $T/(1+T)$

\[
\frac{T}{1+T} \approx \begin{cases} 
\frac{1}{T} & \text{for } \|T\| \gg 1 \\
T & \text{for } \|T\| \ll 1
\end{cases}
\]
9.2.1. Feedback reduces the transfer functions from disturbances to the output

Original (open-loop) line-to-output transfer function:

\[
G_{vg}(s) = \frac{\hat{v}(s)}{\hat{v}_g(s)} \bigg|_{\hat{v}_d = 0, i_{load} = 0}
\]

With addition of negative feedback, the line-to-output transfer function becomes:

\[
\frac{\hat{v}(s)}{\hat{v}_g(s)} \bigg|_{\hat{v}_ref = 0, i_{load} = 0} = \frac{G_{vg}(s)}{1 + T(s)}
\]

Feedback reduces the line-to-output transfer function by a factor of

\[
\frac{1}{1 + T(s)}
\]

If \( T(s) \) is large in magnitude, then the line-to-output transfer function becomes small.
Closed-loop output impedance

Original (open-loop) output impedance:

\[ Z_{out}(s) = -\frac{\hat{v}(s)}{\hat{i}_{load}(s)} \bigg|_{\hat{d} = 0, \hat{v}_g = 0} \]

With addition of negative feedback, the output impedance becomes:

\[ \frac{\hat{v}(s)}{-\hat{i}_{load}(s)} \bigg|_{\hat{v}_{ref} = 0, \hat{v}_g = 0} = \frac{Z_{out}(s)}{1 + T(s)} \]

Feedback reduces the output impedance by a factor of

\[ \frac{1}{1 + T(s)} \]

If \( T(s) \) is large in magnitude, then the output impedance is greatly reduced in magnitude.
Same example: construction of $1/(1+T)$

\[
\frac{1}{1+T(s)} \approx \begin{cases} 
\frac{1}{T(s)} & \text{for } ||T|| \gg 1 \\
1 & \text{for } ||T|| \ll 1 
\end{cases}
\]
Phase margin test for stability

Closed-loop system is stable if phase margin is positive.

Phase margin affects Q-factor of poles at $f_c$ in closed-loop transfer functions.

\[ \angle T(j2\pi f_c) = -112^\circ \]

\[ \varphi_m = 180^\circ - 112^\circ = +68^\circ \]
$Q$ vs. $\phi_m$

- $Q = 1 \Rightarrow 0 \text{dB}$, $\phi_m = 52^\circ$
- $Q = 0.5 \Rightarrow -6 \text{dB}$, $\phi_m = 76^\circ$
Transient response vs. damping factor
II. Measurement and simulation of loop gains and other closed-loop quantities

Objective: experimentally determine loop gain $T(s)$, by making measurements at point $A$

Correct result is

$$T(s) = G_1(s) \left( \frac{Z_2(s)}{Z_1(s) + Z_2(s)} \right) G_2(s) H(s)$$
Conventional approach: break loop, measure $T(s)$ as conventional transfer function

measured gain is

$$T_m(s) = \frac{\hat{v}_y(s)}{\hat{v}_x(s)} \bigg|_{v_{ref} = 0, v_g = 0}$$

$$T_m(s) = G_1(s) G_2(s) H(s)$$

$$T_m(s) \approx T(s) \quad \text{provided that} \quad \|Z_2\| >> \|Z_1\|$$
9.6.1. Voltage injection

Ac injection source $v_z$ is connected between blocks 1 and 2

Dc bias is determined by biasing circuits of the system itself

Injection source does modify loading of block 2 on block 1

$T_v(s) \approx T(s)$ provided

(i) $\|Z_1(s)\| << \|Z_2(s)\|$, and

(ii) $\|T(s)\| >> \left\| \frac{Z_1(s)}{Z_2(s)} \right\|$
9.6.2. Current injection

\[ T_i(s) = \left. \frac{{\dot{i}_y(s)}}{{\dot{i}_x(s)}} \right|_{\varphi_{\text{ref}} = 0, \varphi_x = 0} \]

Now require:

(i) \[ \| Z_2(s) \| \ll \| Z_1(s) \|, \quad \text{and} \]

(ii) \[ \| T(s) \| \gg \left| \frac{Z_2(s)}{Z_1(s)} \right| \]
Simulation of loop gain via injection

*(design example of Section 9.5.4, simulated in B.2.2)*

**Example:** voltage injection after error amplifier

**PSPICE issues**

**Divergence!**

1. Use `.nodeset` to improve convergence

2. Limit maximum and minimum duty cycles

![Circuit Diagram](image)
Buck voltage regulator example
Sections B.2.2 and 9.5.4

Buck voltage regulator, closed-loop

```
.param  R=10
.param  L=50uH fs=100KHz
.step PARAM R LIST 3,10,25
.ac dec 101 5 50KHz
.nodeset v(3)=15 v(5)=5 v(6)=4.144 v(8)=0.536
.lib switch.lib
.lib nom.lib
Vg 1 0 28V
Xswitch 1 2 2 0 8 CCM-DCM1 PARAMS: L={L} fs={fs}
L1 2 3 {L}
C1 3 0 500uF
Rload 3 0 {R}
Vcc p 0 12V
Vref ref 0 5V
Xopamp ref 5 p 0 6 LM324
R1 3 4 11K
R2 4 5 85K
C2 4 5 1.1nF
R4 5 0 47K
R3 6 6x 120K
C3 6x 5 2.7nF
Vz 6 7 dc 2 ac 1
Epwm 8 0 value={LIMIT(V(7)*0.25,0.1,0.9)}
.probe
.end
```

Plotting the loop gain via injection $v_z$

Use PROBE to plot $v_y/v_x$

Loop gain is plotted at three values of load resistance

File included on website also plots transient response

If .nodeset is omitted, PSPICE diverges
9.5.4. Design example: buck converter
With $G_c = 1$, the loop gain is

$$T_u(s) = T_{u0} \frac{1}{1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$T_{u0} = \frac{HV}{DV_M} = 2.33 \Rightarrow 7.4\text{dB}$$

$$f_c = 1.8\text{kHz}, \phi_m = 5^\circ$$
Lead (PD) compensator

- Obtain a crossover frequency of 5kHz, with phase margin of 52°
- $T_u$ has phase of approximately -180° at 5kHz, hence lead (PD) compensator is needed to increase phase margin

- Lead compensator should have phase of +52° at 5kHz
- $T_u$ has magnitude of -20.6dB at 5kHz
- Lead compensator gain should have magnitude of +20.6dB at 5kHz
Loop gain, with lead compensator

\[ T(s) = T_{u0} \cdot G_{c0} \frac{(1 + \frac{s}{\omega_c})}{(1 + \frac{s}{\omega_p})(1 + \frac{s}{Q_0\omega_0} + \left(\frac{s}{\omega_0}\right)^2)} \]

- Good phase margin
- Wide bandwidth
- Low-frequency loop gain is low — hence not much rejection of disturbances
1/(1+T), with lead compensator

- need more low-frequency loop gain
- hence, add inverted zero (PID controller)
Improved compensator (PID)

$$G_c(s) = G_{cm} \frac{1 + \frac{s}{\omega_L}}{1 + \frac{s}{\omega_p}} \frac{1 + \frac{\omega_L}{s}}{\left(1 + \frac{s}{\omega_p}\right)}$$

- add inverted zero to PD compensator, without changing dc gain or corner frequencies
- choose $f_L$ to be $f_c/10$, so that phase margin is unchanged
$T(s)$ and $1/(1+T(s))$, with PID compensator
Feedback, revisited

Line-to-output transfer function

\[ G_{vg}(0) = D \]

\[ \frac{D}{T_{u0}G_{cm}} \]

20dB/dec

-40dB/dec

open-loop \( \| G_{vg} \| \)

closed-loop \( \left\| \frac{G_{vg}}{1 + T} \right\| \)

\( f_c \)

\( f_z \)

\( f_L \)

\( f_0 \)

\( Q_0 \)

Fundamentals of Power Electronics
Simulation results
Loop gain, bandwidth, phase margin

- Results at full load (nominal design point, $R = 3 \, \Omega$) are close to design goals
- Very different results at light load (in DCM at $R = 25 \, \Omega$)!
- As load current is reduced: $Q$ at first increases because of reduced damping. Then $Q$ decreases in DCM.
Simulation results
Audiosusceptibility

Open loop, $d(t) = \text{constant}$

Closed loop

$R = 3 \, \Omega$

$R = 25 \, \Omega$
Effect of feedback on transient response

PSPICE-generated transient response, same closed-loop buck example