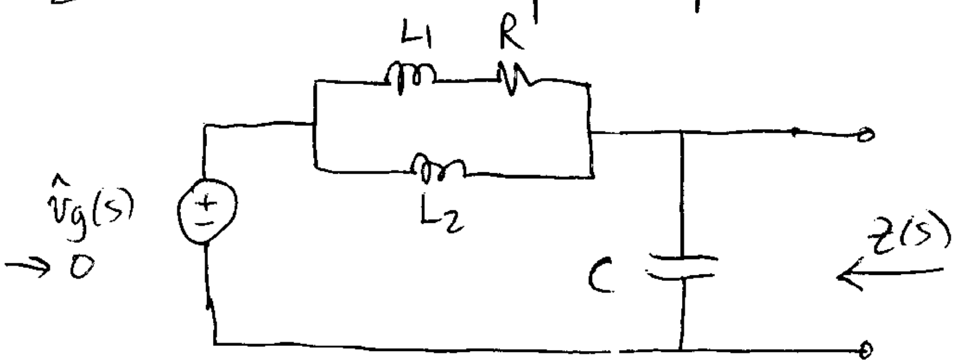
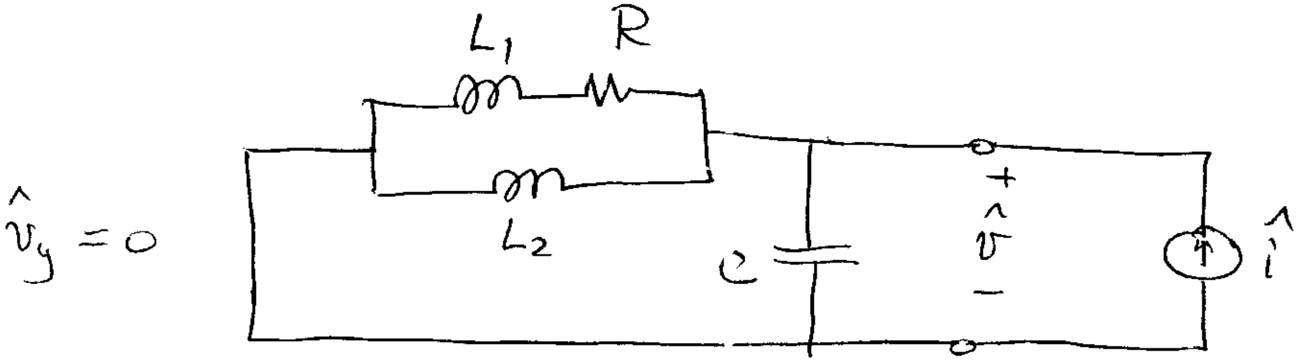
Employing the n-extra element theorem when the dc value of the asymptote is zero

Sometimes, the dc value of the transfer function is not a nonzero, finite value. this happens whenever the low-frequency asymptote has a nonzero slope. In the power electronics field, this is nearly always the case for impedances (as long as parasitic loss elements are not modeled), because we do not want the dc or 60 HZ current to flow through a lossy resistive element.

For example, consider the output impedance $Z(s)$ of the damped input filter shown below:



To measure this impedance, we would set $\hat{v}_g \to 0$, then inject a current $\hat{i}$ and measure the induced voltage $\hat{v}$:



$$Z(s) = \left.\frac{\hat{v}}{\hat{i}}\right|_{\hat{v}_g = 0}$$

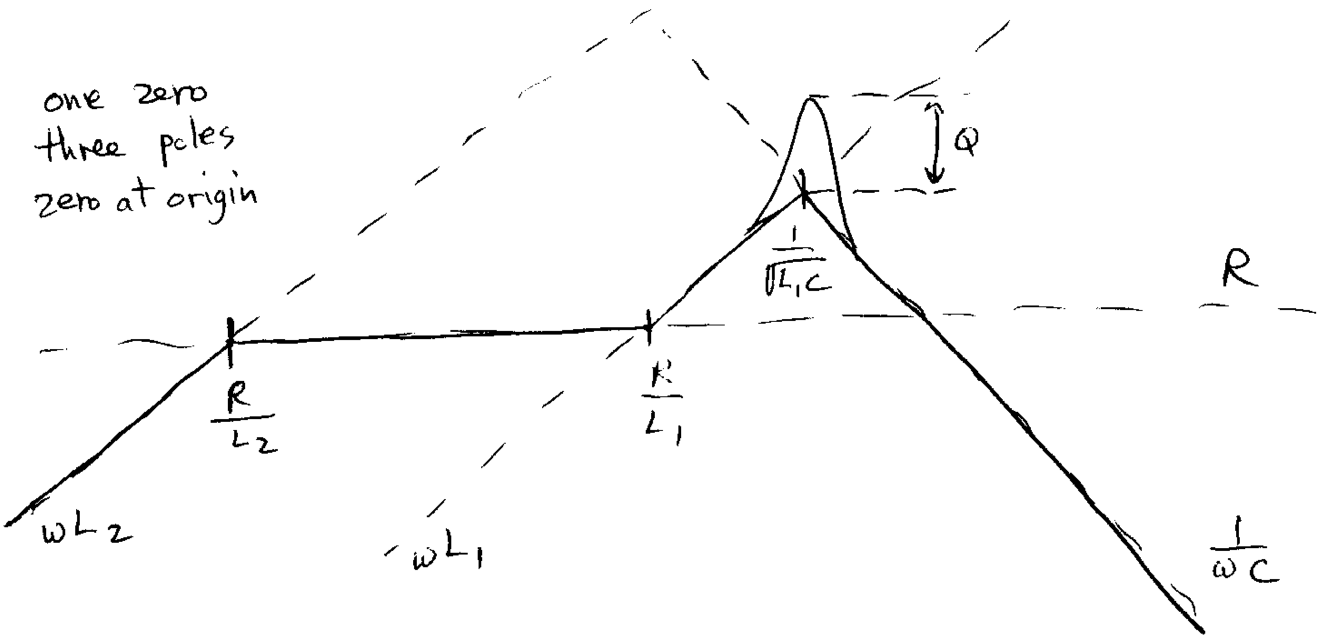
So $Z(s)$ is the transfer function from $\hat{i}$ to $\hat{v}$.

If we try to use the n-extra element theorem in its basic form, we obtain a dc value of zero, so that

$$Z(s) = 0 \cdot \frac{\text{zeroes}}{\text{poles}}$$

This isn't going to work. We need to use a form contain one or more inverted poles or zeroes, so that $Z(s)$ is referred to a non-zero midband gain.

As an aside, let's construct $Z(s)$ using the algebra on the graph technique, for the case where $L_2 >> L_1$:



It looks like we could express $Z(s)$ using an inverted pole at $\frac{R}{L_2}$, a midband asymptote of R, a regular zero at $\frac{R}{L_1}$, and regular complex poles at $\frac{1}{\sqrt{L_1 C}}$, as follows:

$$Z(s) \approx R \frac{\left(1 + \frac{sL_1}{R}\right)}{\left(1 + \frac{R}{sL_2}\right)\left(1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2\right)}$$

with $\omega_0 = \frac{1}{\sqrt{L_1 C}}$

Q is perhaps less obvious, although it can be approximated, using the $\frac{R}{L_1}$ zero, as $Q \approx \frac{1}{R}\sqrt{\frac{L_1}{C}}$

If the approximate analysis of the previous page is good enough, then our work is done. But if we want to optimize the design of the input filter (making L_2 no larger than necessary), then we can't make the approximation $L_2 >> L_1$. So we need to work out the actual $Z(s)$. In higher-order input filters having several damped L-C sections, the need for developing an accurate expression for $Z(s)$ becomes even more acute.

So we need to generalize the n-extra-element theorem to handle inverted poles and zeroes. This can be done by generalizing the ideas of "low frequency state" and "high frequency state" of a reactive element, as follows;

Replace the terms

"low-frequency state" with "normal state"

"high-frequency state" with "abnormal state"

The normal state of each inductor or capacitor can be chosen to be either a short circuit or an open circuit. The "reference gain" of the transfer function (previously called the "dc gain") is equal to the value of the transfer function when all inductors and capacitors are set to their normal states. The transfer function is then written in the form

$$G(s) = (\text{reference gain}) \underbrace{\frac{(\text{regular and/or inverted zeroes})}{(\text{regular and/or inverted poles})}}_{\text{"numerator and denominator polynomials"}}$$

The numerator and denominator polynomials may now contain terms with negative powers of s.
We use the usual method to find the polynomial coefficients, except that "low-frequency state" is replaced by "normal state", "high-frequency state" is replaced by "abnormal state", and inverted terms are used whenever the normal state coincides with the high-frequency state, as follows:

$$\frac{sL}{R} \longrightarrow \frac{R}{sL}, \qquad sRC \longrightarrow \frac{1}{sRC}$$

Impedance example

The low-frequency asymptote of $Z(s)$ tends to zero because of L_2. So let's invert the sL_2 terms. If we let

$$sL_2 \rightarrow \text{open circuit}$$
$$sL_1 \rightarrow \text{short circuit}$$
$$\frac{1}{sC} \rightarrow \text{open circuit}$$

then we obtain $Z \rightarrow R$

Therefore, let's define the normal and abnormal states as follows:

Element	normal state	abnormal state
L_1	short (dc)	open (HF)
L_2	open (HF)	short (dc)
C	open (dc)	short (HF)

We can write $Z(s)$ in the following form:

$$Z(s) = R \frac{1 + \left(\frac{sL_1}{R_h} + sCR_i + \frac{R_j}{sL_2} \right)}{1 + \left(\frac{sL_1}{R_a} + sCR_b + \frac{R_c}{sL_2} \right) + \left(sCR_b \frac{sL_1}{R_d} + sCR_b \frac{R_e}{sL_2} + \frac{R_c}{sL_2} \frac{sL_1}{R_f} \right)}$$

(Recall from p. 3 that $Z(s)$ contains three poles, one zero, and a zero at the origin)

The above form for $Z(s)$ is written in the usual (almost) manner, with three poles and one zero, except that the $\frac{sL_2}{R_x}$ terms are inverted, and $Z(s)$ is referenced to the midband gain R. The denominator polynomial is 3^{rd} order, but contains s^{-1}, s^0, s^1, and s^2 terms.

The coefficients are found in the usual manner:

<u>Denominator terms</u>

set all independent sources to zero:

$\underbrace{\hat{v}_g = 0}_{\text{short}}$, $\underbrace{\hat{i} = 0}_{\text{open}}$

R_a: resistance seen by L_1 when L_2 and C are set to their normal states

$L_2 \rightarrow$ open

$C \rightarrow$ open

$R_a = \infty$

R_b: resistance seen by C when L_1 and L_2 are set to their normal states

$L_1 \rightarrow$ short

$L_2 \rightarrow$ open

$R_b = R$

R_c: resistance seen by L_2 when L_1 and C are set to their normal states

$L_1 \rightarrow$ short

$C \rightarrow$ open

$R_c = R$

R_d: resistance seen by L_1 when C = abnormal state (short) and L_2 = normal state (open)

$R_d = R$

R_e: resistance seen by L_2 when C = abnormal state (short) and L_1 = normal state (short)

$R_e = 0$

R_f: resistance seen by L_1 when L_2 = abnormal state (short) and C = normal state (open)

$R_f = R$

Zeroes: In the presence of $\hat{i}$, inject at the L_1, L_2, or C port such that $\hat{v} \xrightarrow[\text{null}]{} 0$

R_h: resistance seen by L_1 when C and L_2 are set to their normal states and $\hat{v} \xrightarrow[\text{null}]{} 0$

$C \rightarrow$ open $\qquad L_2 \rightarrow$ open

$R_h = R$

R_i : resistance seen by C when L_1 and L_2 are set to their normal states and $\hat{v} \xrightarrow[\text{null}]{} 0$

$L_1 \rightarrow$ short $\quad L_2 \rightarrow$ open

$R_i = 0$

R_j : resistance seen by L_2 when L_1 and C are set to their normal states and $\hat{v} \xrightarrow[\text{null}]{} 0$

$L_1 \rightarrow$ short $\quad C \rightarrow$ open

$R_j = 0$

So

$$Z(s) = R \frac{1 + \frac{sL_1}{R}}{1 + sRC + \frac{R}{sL_2} + s^2 L_1 C + \frac{sL_1}{sL_2}}$$

To write in the usual form, multiply by $\dfrac{\left(\frac{sL_2}{R}\right)}{\left(\frac{sL_2}{R}\right)}$

$$Z(s) = sL_2 \frac{\left(1 + \frac{sL_1}{R}\right)}{\left(1 + \frac{s(L_1+L_2)}{R} + s^2 L_2 C + s^3 \frac{L_1 L_2 C}{R}\right)}$$