Solution to
Problem Set 8
Power Electronics 3
Spring 2000

Dc and ac analysis of a quasi-resonant converter. The converter illustrated below is a transformer-isolated quasi-resonant converter. It includes a controller that regulates the converter output voltage by variation of the switching frequency. You may assume that the controller is well-designed, such that it tightly regulates the dc output voltage.

Fig. 1 Closed-loop quasi-resonant converter:

(a) What is the parent dc–dc converter? What kind of resonant switch is employed?

(b) Determine the steady-state numerical values of: the output voltage $V$, the switch conversion ratio $\mu$, and the dc current flowing through the 20 $\mu$H inductor. Estimate the quiescent value of the switching frequency (you may do this graphically if you wish).

(c) Write an expression for the control-to-output transfer function (from switching frequency variations $f_s$ to output voltage $V$), and give expressions for the salient features (dc gain, corner frequencies, and $Q$-factor). You may neglect the high-frequency dynamics caused by the tank elements. Does the transfer function contain a right half-plane zero?

(d) Construct the Bode plot of the magnitude and phase of your transfer function derived in part (c).
a) Resonant elements (swell) are the 0.36 \mu H inductor and the 0.64 \mu F capacitor.

20 \mu H inductor and 220 \mu F capacitor are relatively large, and constitute the PWM filter elements.

Set resonant elements to zero to obtain the parent dc–dc converter:

![Flyback converter diagram]

Set PWM filter elements and \( v_g \) to their high-frequency ac states (\( C_F \) & \( v_g \rightarrow \text{short}, L_m \rightarrow \text{open} \)) to obtain the resonant switch network:

![Resonant switch network diagram]
Push secondary elements through transformer and rearrange:

\[ V = \frac{V_s}{n} \frac{M}{1-M} \]

with \( \mu \) = switch conversion ratio

Solve for \( \mu \):

\[ (5V) = \frac{(15V)}{3} \frac{M}{1-M} \Rightarrow \mu = 0.5 \]
Also, we already know the solution for the dc magnetizing current \( I_m \) of the CCM flyback converter:

\[
I_m = \frac{V}{nR} \frac{1}{1-M}
\]

\[
= \frac{(5V)}{3(0.5 \Omega)} \frac{1}{1-0.5} = 6.67 \text{A}
\]

To estimate the switching frequency, we first need to find \( J \), then use \( \mu = \frac{P_2}{P_1} (J) \) to solve for \( F \).

Referred to the transformer primary, the base normalizing quantities are:

\[ V_{\text{base}} = V_g + nV \] (same as off-state voltage of MOSFET in parent PWM converter)

\[ I_{\text{base}} = \frac{V_{\text{base}}}{R_0} \]

\[ R_0 = \sqrt{\frac{L}{C \cdot n^2}} \]

The normalized current \( J \) is

\[ J = \frac{I_m}{I_{\text{base}}} \] (normalize the on-state current of MOSFET in parent PWM converter)

Plug in numbers:

\[ R_0 = \sqrt{\frac{0.36 \mu H}{0.64 \mu F}} \cdot 3^2 = 0.25 \text{\Omega} \]

\[ V_{\text{base}} = 15 + 3.5 = 30 \text{V} \]
\[ I_{\text{base}} = \frac{30V}{2.25\Omega} = 13.33A \]

\[ J = \frac{6.67A}{13.33A} = 0.5 \]

Now solve for the normalized switching frequency \( F \):

\[ \mu = F \frac{P_\frac{1}{2}(J)}{F} \Rightarrow F = \frac{P_\frac{1}{2}(J)}{\mu} \]

with \( P_\frac{1}{2}(J) = \frac{1}{2\pi} \left[ \frac{1}{2}J + \pi + \sin^{-1}(J) + \frac{1}{J} \left( 1 + \sqrt{1 - J^2} \right) \right] \)

\[ P_\frac{1}{2}(0.5) = 1.22 \]

so \( F = \frac{0.5}{1.22} = 0.411 = \frac{f_s}{f_0} \)

The tank resonant frequency is

\[ f_0 = \frac{1}{2\pi \sqrt{L \cdot \frac{C}{\mu^2}}} = \frac{1}{2\pi \sqrt{(0.36\mu H)(0.64\mu F/3^2)}} \]

\[ = 995 \text{ kHz} \quad (\text{i.e., } 1 \text{ MHz}) \]

So \( f_s = Ff_0 = (0.411)(995 \text{ kHz}) = 409 \text{ kHz} \)
c) Derive control-to-output transfer function

\[ G_{vc}(s) = \frac{\hat{V}}{\hat{e}_s} \bigg|_{\Delta = 0} \]

We have

\[ J = \frac{i_M}{i_{base}} = \frac{i_M R_0}{v_g + n v} \]

and

\[ \mu = \frac{f_s}{f_0} \frac{P_2}{2} \left( \frac{i_M R_0}{v_g + n v} \right) \]

Perturb and linearize:

\[ \hat{\mu} = K_c \hat{f}_s + K_v \hat{v} + K_i \hat{i}_M + K_g \hat{v}_g \]

Since we weren't asked to find \( G_{vg}(s) = \frac{\hat{v}}{\hat{v}_g} \), we can set \( \hat{v}_g \) to zero and ignore the \( K_g \hat{v}_g \) term.

\[ K_c = \frac{1}{f_0} \frac{P_2(r)}{2} = \frac{1.22}{995 kHz} = 1.22 \times 10^{-6} \text{ Hz}^{-1} \]

\[ K_v = F \frac{dP_2(r)}{dJ} \frac{dJ}{dv} \]

See table from notes on web, "Extension of State Space Averaging..." page 13:

\[ \frac{dP_2(r)}{dJ} = \frac{1}{2\pi} \left[ \frac{1}{2} - \frac{1}{J^2} \right] = -1.108 \]
\[ \frac{\partial J}{\partial V} = -\frac{I_m R_o n}{(V_g+nV)^2} = -0.05 \]

so \( K_V = (0.411)(-1.108)(-0.05) = 0.0228 \)

\[ K_i = F \frac{dP_i(s)}{dJ} \frac{\partial J}{\partial i_m} \]

with \( \frac{\partial J}{\partial i_m} = \frac{R_o}{V_g+nV} = \frac{2.255}{15V+3.3V} = \frac{1}{I_{base}} = \frac{1}{13.33A} \)

so \( K_i = (0.411)(-1.108)\left(\frac{1}{13.33A}\right) = -0.034 \) note that \( K_i \) is negative

Converter model - block diagram
$G_{vd}(s)$ and $G_{id}(s)$ are transfer functions of the parent PWM flyback converter.

Small-signal model of PWM flyback converter

(see Fig. 7.27 of textbook, p. 28)
(set $R_{on} \rightarrow 0$ and $D \rightarrow D_0$) (replace $n$ with $\frac{1}{n}$)

The transfer function $G_{vd}(s)$ is

$$G_{vd}(s) = \frac{V}{\frac{V_0}{n_0}} \frac{1 - s \frac{n_0 L_1 R}{n^3 \mu^3}}{\text{den}(s)}$$

with $\text{den}(s) = 1 + s \frac{L_n}{n^2 \mu^2 R} + s^2 \frac{L_n C F}{n^2 \mu_0^2}$

(found by setting $\hat{v}_g \rightarrow 0$ and solving for $\hat{v}$)

The transfer function $G_{id}(s)$ is also found by setting $\hat{v}_g \rightarrow 0$, then solving for $\hat{i}_M$:

push to primary side
Solve:

\[ \hat{i}_m = \frac{d}{dt} \left[ (V_0 + nV) + \frac{I_M}{\mu_0} \cdot n^2 \mu_0^2 \left( RL || \frac{1}{sC_F} \right) \right] \left[ \frac{sL_M + n^2 \mu_0^2 \left( RL || \frac{1}{sC_F} \right)}{1+sRC_F} \right] \]

so

\[ G_{id}(s) = \frac{\hat{i}_m}{V} \left|_{V_0=0} \right. = \left[ V_0 + nV + n^2 \mu_0^2 I_M \frac{R}{1+sRC_F} \right] \left[ \frac{sL_M + n^2 \mu_0^2 \left( RL || \frac{1}{sC_F} \right)}{1+sRC_F} \right] \]

note

\[ V = \frac{V_0}{n} \quad \frac{\mu_0}{\mu_0'} \quad \Rightarrow V_0 = nV \cdot \frac{\mu_0'}{\mu_0} \]

\[ I_M = \frac{V}{R} \cdot \frac{1}{n\mu_0} \]

so

\[ G_{id}(s) = \frac{nV \left( \frac{\mu_0'}{\mu_0} + 1 \right) + n^2 \mu_0' \frac{V}{R} \cdot \frac{1}{n\mu_0} \cdot \frac{R}{1+sRC_F}}{sL_M + n^2 \mu_0^2 \left( RL || \frac{1}{sC_F} \right)} \]
\[ G_{id}(s) = \frac{(1+sRC_F) \frac{nV}{\mu_0} + nV}{sLM + s^2 RLM C_{12} + n^2 \mu_0^2 R} \]

\[ = \frac{V}{n\mu_0^2 R} \frac{1 + \frac{1}{\mu_0} + s \frac{RC_F}{\mu_0}}{1 + s \frac{L}{n^2 \mu_0^2 R} + s^2 \frac{LM C_F}{n^2 \mu_0^2}} \]

\[ = \frac{V (1+\mu_0)}{n\mu_0^2 \mu_0 R} \frac{(1 + s \frac{RC_F}{1+\mu_0})}{(den's)} \]

with den's as defined previously.

Now solve the block diagram.

Could: use algebra, or manipulate block diagram.

\[ \hat{f}_s \] 
\[ \rightarrow \] 
\[ K_F \] 
\[ \rightarrow \] 
\[ + \] 
\[ \rightarrow \] 
\[ \hat{\mu} \] 
\[ \rightarrow \] 
\[ G_{ud} \] 
\[ \rightarrow \] 
\[ \hat{c}_v \] 

\[ K_F \] 
\[ \rightarrow \] 
\[ + \] 
\[ \rightarrow \] 
\[ K_i \] 
\[ \rightarrow \] 
\[ G_{id} \] 

Use feedback rule:

\[ \begin{array}{c}
A \\
B
\end{array} \rightarrow \] 
\[ \begin{array}{c}
A \\
1+AB
\end{array} \]
\[ G_{vc}(s) = \left. \frac{\hat{v}_g}{\hat{f}_s} \right|_{\hat{u}_g = 0} = \frac{K_c G_{ud}(s)}{1 + |K_i| G_{id}(s) - K_v G_{vd}(s)} \]

Next, plug in expressions for \( G_{ud}(s) \) and \( G_{id}(s) \).
\[ G_{vc}(s) = \frac{V}{K_c \frac{1}{\mu_0 \mu_0'} \left( 1 - s \frac{\mu_0 L M}{n^2 \mu_0^2 R} \right)}{(dun'(s))} \]

\[ 1 + |K_i| \frac{V(1+\mu_0)}{n^2 \mu_0^2 \mu_0 R} + \frac{(1 + s \frac{RCF}{1+\mu_0})}{(dun'(s))} - KV \frac{V}{\mu_0 \mu_0'} \left( 1 - s \frac{\mu_0 L M}{n^2 \mu_0^2 R} \right) \]

\[ \times \frac{V}{(dun'(s))} \]

\[ \text{multiply through by } dun'(s) \text{ and collect terms:} \]

\[ = \frac{V}{K_c \mu_0 \mu_0'} \left( 1 - s \frac{\mu_0 L M}{n^2 \mu_0^2 R} \right) \]

\[ + s \left( \frac{LM}{n^2 \mu_0^2 R} + |K_i| \frac{VCF}{n^2 \mu_0^2 \mu_0 R} + KV \frac{VL}{n^2 \mu_0^2 R} \right) \]

\[ + s^2 \left( \frac{LmCF}{n^2 \mu_0^2} \right) \]

\[ G_{vc}(s) = \frac{V}{\mu_0 \mu_0'} \left( 1 + |K_i| \frac{V(1+\mu_0)}{n^2 \mu_0^2 \mu_0 R} - KV \frac{V}{\mu_0 \mu_0'} \right) \cdot \frac{1 - s \frac{1}{\omega_2}}{(dun'(s))} \]

\[ \text{new poles} \]

with

\[ dun'(s) = 1 + s \left( \frac{LM}{n^2 \mu_0^2 R} + |K_i| \frac{VCF}{n^2 \mu_0^2 \mu_0 R} + KV \frac{VL}{n^2 \mu_0^2 R} \right) + s^2 \left( \frac{LmCF}{n^2 \mu_0^2} \right) \]
\[ G_{vc}(s) = G_{co} \left( \frac{1 - \frac{s}{\omega_2}}{1 + \frac{s}{Q \omega_0} + \left( \frac{s}{\omega_0} \right)^2} \right) \]

with:

\[ G_{co} = \frac{K_c V}{\mu_0 \mu'_0 \left( 1 + |k_i| \frac{V(1+\mu_0)}{\eta \mu_0^2 \mu_0 R} - \frac{V}{\mu_0 \mu'_0} \right)} \]

\[ \omega_2 = \frac{\eta^2 \mu_0^2 R}{\mu_0 L_M} \quad (RHP) \]

\[ \omega_0 = \eta \mu'_0 \sqrt{1 + |k_i| \frac{V(1+\mu_0)}{\eta \mu_0^2 \mu_0 R} - \frac{V}{\mu_0 \mu'_0}} \]

\[ Q = \sqrt{1 + |k_i| \frac{V(1+\mu_0)}{\eta \mu_0^2 \mu_0 R} - \frac{V}{\mu_0 \mu'_0}} \frac{1}{\eta \mu'_0 R \sqrt{\frac{L_M}{c_F}} \left( 1 + \frac{K_i V}{\mu'_0} \right) + \frac{|k_i| V}{\mu_0 \mu'_0} \sqrt{\frac{c_F}{L_M}}} \]
d) Plug in values

\[ G_{ce} = 1.28 \cdot 10^{-5} \text{ volts/Hz} \]

\[ f_2 = 17.9 \text{ kHz (RHP)} \]

\[ f_0 = 4.97 \text{ kHz} \]

\[ Q = 0.501 \Rightarrow \text{almost two real poles at 4.97 kHz} \]