Solution

Homework Assignment 5
Resonant and Soft Switching Techniques in Power Electronics
ECEN 5817
Spring 2006

Resonant Inverter Dynamic Modeling Problems

1. The parallel resonant inverter of the previous homework assignment is modified to operate at a constant switching frequency, with control via variation of the switch duty cycle \( d(t) \) as illustrated in Fig. 9(c) of the packet 2 dynamics course notes. The object of this problem is to develop a model for analytical prediction of the control to output transfer function.

   (a) Construct a phasor equivalent circuit that models the sinusoidal steady-state output voltage phasor \( V \) as well as the small-signal output voltage phasor \( \hat{V} \), as a function of control input variations in the duty cycle \( d \). Your model should include analytical expressions for the steady-state and small-signal voltage phasors produced by the switch network.

   (b) Solve your model to find expressions for the sinusoidal steady-state output voltage phasor \( V \) and the small-signal output voltage phasor \( \hat{V} \).

   (c) Find an expression for the transfer function \( G(s) = \hat{v}_{out}(s)/\hat{d}(s) \), where \( \hat{v}_{out} \) is the scalar small-signal variation in the envelope of the output voltage \( v(t) \). It is not necessary to evaluate or simplify your expression.

2. A certain resonant inverter is illustrated in the figure below. The controller operates the full-bridge switch network so that the switch output voltage \( v(t) \) is both frequency-modulated (through variation of the switching frequency) and amplitude modulated (through variation of the switch duty cycle as in Fig. 12 of the packet 2 dynamics notes). There are four tank elements: capacitor \( C_1 \) is a dc blocking capacitor that contributes to the tank dynamics, \( L_1 \) models the magnetizing inductance of a transformer, \( L_2 \) models the leakage inductance of a transformer, and \( C_2 \) models output-side capacitance.

   Construct an equivalent circuit model that is suitable for SPICE simulation of the transfer functions from the FM and AM control inputs to the output voltage envelope:

   (a) Derive the small-signal model (real and imaginary components) for the ac side of the duty-cycle controlled switch network.

   (b) Sketch the complete circuit for simulation. It is not necessary to solve or simulate your result.
1. Parallel Resonant Inverter

Duty cycle modulation of the full bridge switch network: the ac output voltage of the switch network is

\[ V_S(t) = \frac{4V_g}{\pi} \sin \left( \frac{\pi d(t)}{2} \right) \cos (\omega_S t) \]

express as a phasor: \( V_{S1}(t) = \frac{4V_g}{\pi} \sin \left( \frac{\pi d(t)}{2} \right) \]

which is nonlinear because of the sine function.

a) First find the steady-state and small-signal voltage phasors produced by the switch network.

The steady-state voltage phasor is

\[ V_{S1} = \frac{4V_g}{\pi} \sin \left( \frac{\pi D}{2} \right) \]

where \( D \) is the steady-state duty cycle (uses peak values for phasors)

or, if we use rms values, we should divide by \( \sqrt{2} \).

To find the small-signal ac phasor:

if we let \( d(t) = D + \hat{d}(t) \) with \( |\hat{d}(t)| << D \) then

\[ V_{S1}(t) = V_{S1} + \hat{V}_{S1}(t) \] with \( \hat{V}_{S1}(t) = (\text{complex phasor gain}) \cdot \hat{d}(t) \)
The gain is the slope of $v_{s1}(t)$ at the quiescent operating point $d = D$:

$$\text{gain} = \left. \frac{d}{d(t)} \left( \frac{v_{s1}}{D} \right) \right|_{d = D} = \frac{d}{d(t)} \left[ \frac{4Vg}{\pi} \sin \left( \frac{\pi t}{2} \right) \right]_{d(t) = D}$$

$$= \frac{4Vg}{\pi} \cdot \frac{\pi}{2} \cdot \cos \left( \frac{\pi d(t)}{2} \right) \bigg|_{d(t) = D}$$

$$= 2Vg \cos \left( \frac{\pi D}{2} \right)$$

So

$$\hat{v}_{s1}(t) = 2Vg \cos \left( \frac{\pi D}{2} \right) \cdot d(t)$$

Now model the remaining part of the parallel resonant inverter using the inductor and capacitor models of the course notes packet 2 (Figs. 4 and 6)
It works to combine the large-signal (dc) model and the small-signal (ac) model because they lead to similar models that can be combined. Specifically, substitution of Eq. (15) into Eq. (12) of the packed notes leads to

\[ V + \hat{v} = L \frac{d(I + \hat{i})}{dt} + j\omega_s (I + \hat{i}) + j\omega L I + j\omega L \hat{i} \]

which can be modeled as

![Inductor Model Diagram]

So the same inductor phasor-transformed model works both for steady-state and small-signal ac.

To find the steady-state solution of the phasor-transformed equivalent circuit at the bottom of the previous page: (a) set small-signal (hat) quantities to zero: \( \hat{i} \rightarrow 0, \hat{w} \rightarrow 0 \); (b) set dynamic elements to zero: inductor \( \rightarrow \) short, capacitor \( \rightarrow \) open. The model becomes

![Steady-State Circuit Diagram]
b) The solution of the steady-state model is:

\[ V = \frac{V_{s1} \cdot RL \cdot \frac{1}{j\omega_0 C}}{j\omega_0 L + RL \cdot \frac{1}{j\omega_0 C}} = H(j\omega_0) V_{s1} \]

The small-signal ac model is obtained by setting steady-state sources to zero, the resulting model is:

The problem specifies that the converter operates at constant switching frequency. Hence \( \hat{\omega}_0 = 0 \) and the above model becomes

\[ V = \frac{V_{s1} \cdot \frac{1}{\omega_0 C}}{\left[ \frac{1}{(s+j\omega_0)C} \right] + (s+j\omega_0)L} \]

The small-signal output voltage phasor is therefore

\[ \hat{V} = \hat{V}_{s1} \cdot \frac{RL \cdot \frac{1}{(s+j\omega_0)C}}{[RL \cdot \frac{1}{(s+j\omega_0)C}] + (s+j\omega_0)L} \]

with \( \hat{V}_{s1} = (2V_s \cos(\frac{\pi N}{2}) \cdot \hat{d}) \)
c) Find an expression for the envelope transfer function \( G_{env}(s) = \frac{\hat{V}_{envelope}}{\hat{I}(s)} \) where \( \hat{V}_{envelope}(s) \) is the scalar small-signal variation in the envelope of the output voltage. It is not necessary to simplify the result.

From the packet 2 notes, pp. 22-23:

\[
\hat{V}_{envelope} = \frac{V_{1} \hat{v}^* + V_{2} \hat{v}^*}{2 ||V||}
\]

with \( \hat{v} = (2 V_{s1} \cos(\frac{\pi t}{2})) \hat{v} \)

\[
\frac{R_l \frac{1}{(s+j\omega_0)C}}{[R_l \frac{1}{(s-j\omega_0)C} + (s-j\omega_0)L]}
\]

from previous page.

Then \( \hat{v}^* = (2 V_{s1} \cos(\frac{\pi t}{2})) \hat{v} \)

\[
\frac{R_l \frac{1}{(s-j\omega_0)C}}{[R_l \frac{1}{(s-j\omega_0)C} + (s-j\omega_0)L]}
\]

and \( V = V_{s1} \frac{H(j\omega_0)}{j\omega_0} \), \( V_{1}^* = V_{s1}^* \frac{H(-j\omega_0)}{-j\omega_0} \), \( ||V|| = ||V_{s1}|| \cdot ||H(j\omega_0)|| \)

Substitute these equations into the top equation for the answer.

It is not necessary to further simplify. But it can be seen that if \( \hat{V}^{(s)} \) contains poles at the roots of the denominator polynomial \( D(s+j\omega_0) \), then the conjugate \( \hat{V}^{* (s)} \) contains poles at the roots of \( D^*(s+j\omega_0) = D(s-j\omega_0) \). Hence, the transfer function \( \frac{\hat{V}_{envelope}}{\hat{I}} \) contains poles at the roots of \( D(s+j\omega_0)D^*(s-j\omega_0) \).
which is identical to the poles of the frequency-modulated transfer function \( \hat{v}_{en}(s) / \hat{\omega}(s) \). In general, these transfer functions have the same poles but different zeroes and dc gains.

Problem 2  
LCLC inverter circuit with both AM and FM control

```
line voltage variation (not modeled in this problem: \( v_s = 0 \)) AM FM
```

Derive model suitable for SPICE simulation.

a) The switch network

As in the previous problem, we can write

\[
v_{s1}(t) = \frac{4V_3}{T} \sin\left(\frac{\pi d}{2}\right) \cos(\phi(t))
\]

with \( \phi(t) = \int \omega_s(t) \, dt \)

which can be expressed using the phasor

\[
v_{s1}(t) = \frac{4V_3}{T} \sin\left(\frac{\pi d}{2}\right)
\]
If the control inputs $V_g(t)$ and $d(t)$ and $w_s(t)$ follow quiescent values plus small ac perturbations:

$v_g(t) = V_g + \hat{v}_g(t)$  \hspace{1cm} |\hat{v}_g(t)| \ll V_g
\[
d(t) = D + \hat{d}(t) \hspace{1cm} |\hat{d}(t)| \ll D
\]
\[
\omega_s(t) = \omega_{so} + \hat{\omega}_s(t) \hspace{1cm} |\hat{\omega}_s(t)| \ll \omega_{so}
\]

then the resulting phasor representing the switch output voltage can be expressed as

\[
v_{s1}(t) = V_{s1} + \langle A \rangle \cdot \hat{v}_g(t) + \langle B \rangle \cdot \hat{d}(t) + \langle \omega_s(t) \rangle
\]

we first need to find these:

Perturb and linearize the $v_{s1}(t)$ phasor:

\[
v_{s1}(t) = \frac{4v_g(t)}{\pi} \sin \left( \frac{\pi d(t)}{2} \right)
\]

then $V_{s1} = v_{s1}(t) \bigg|_{v_g = V_g, d = D} = \frac{4V_g}{\pi} \sin \left( \frac{\pi D}{2} \right)$

\[
A = \frac{\partial (v_{s1})}{\partial V_g} \bigg|_{v_g = V_g, d = D} = \frac{4}{\pi} \sin \left( \frac{\pi D}{2} \right)
\]

\[
B = \frac{\partial (v_{s1})}{\partial d} \bigg|_{v_g = V_g, d = D} = 2V_g \cos \left( \frac{\pi D}{2} \right)
\]
\[ V_{s1} = \frac{4V_g}{\pi} \sin\left(\frac{\pi D}{2}\right) + \frac{4}{\pi} \sin\left(\frac{\pi D}{2}\right) \cdot V_g + 2V_g \cos\left(\frac{\pi D}{2}\right) \cdot \hat{d} \]

Note that the phasor \( V_{s1} \) is purely real. This is because we define phase with respect to the switch voltage, so by definition \( V_{s1} \) has zero phase, and \( \text{Im}(V_{s1}) = 0 \).

Large-signal plus small-signal phasor-transformed model for the output of the switch network:

(b) Simulation model

We can follow the approach described in the last several pages of the packet 2 notes. It is desired to obtain both steady-state and small-signal ac models - the solution of the steady-state model is needed for evaluation of the coefficients of the small-signal model. For convenience, let's derive a single model that simultaneously works for both steady-state and small-signals. (It isn't required that this be done - two separate models are also OK)
Start with Eq. (12) of the packet 2 notes:
\[ v(t) = L \frac{d}{dt} \left( \frac{i(t)}{L} \right) + j \omega_s(t) L \frac{i(t)}{L} \]

Let \[ v(t) = V + \hat{V}(t), \quad i(t) = I + \hat{i}(t), \quad \omega_s(t) = \omega_s + \hat{\omega}_s(t) \]
with \[ ||V|| \gg ||\hat{V}(t)||, \quad ||I|| \gg ||\hat{i}(t)||, \quad \omega_s \gg |\hat{\omega}_s(t)| \].

Then
\[ V + \hat{V}(t) = L \frac{d}{dt} \left( I + \hat{i}(t) \right) + j \left( \omega_s + \hat{\omega}_s(t) \right) L \left( I + \hat{i}(t) \right) \]
\[ = L \frac{d}{dt} \left( I + \hat{i}(t) \right) + j \omega_s L (I + \hat{i}(t)) + j L \hat{I}_s(t) \hat{\omega}_s(t) \]

Equivalent circuit:

This is nearly the same as Fig. 4(b) of packet 2, except steady-state terms are also included.
Now separate into real and imaginary parts for SPICE simulation. Let
\[
\hat{V} = \hat{v} + j\hat{v}_2, \quad V = V_1 + jV_2, \quad \dot{I} = \dot{i}_1 + j\dot{i}_2, \quad I = I_1 + jI_2
\]
we get
\[
(V_1 + \hat{v}_1) + j(V_2 + \hat{v}_2) = \frac{d}{dt} \left[ (I_1 + \hat{i}_1) + j(I_2 + \hat{i}_2) \right] + j \omega_s L \left[ (I_1 + \hat{i}_1) + j(I_2 + \hat{i}_2) \right] + jL(I_1 + jI_2) \omega_s
\]
The real part is
\[
(V_1 + \hat{v}_1) = L \frac{d}{dt} (I_1 + \hat{i}_1) - \omega_s L (I_2 + \hat{i}_2) - L I_2 \omega_s
\]
The imaginary part is
\[
(V_2 + \hat{v}_2) = L \frac{d}{dt} (I_2 + \hat{i}_2) + \omega_s L (I_1 + \hat{i}_1) + L I_1 \omega_s
\]
These lead to the following equivalent circuits:

[Diagram of equivalent circuits]
These models are similar to Fig. 22 of packet 2, except that both steady-state and small-signal terms are included. The circuit reduces to the steady-state model by letting $L \rightarrow$ short circuit and perturbations $\rightarrow 0$. The circuit reduces to the small-signal model by letting steady-state components of the waveforms go to zero (but don't set the coefficients/gains of the sources, such as the $L I_2 \hat{w}_3$ source, to zero!). So the same model works for both steady-state and small signals.

The complete SPICE model for this inverter becomes:

\[
\begin{align*}
\text{real part} & \\
2V_0 \cos\left(\frac{\pi D}{2}\right) & = + V_{1-1} + V_{1-1} \\
\frac{4V_0}{\pi} \sin\left(\frac{\pi D}{2}\right) & = L_2 I_{2-2} \hat{w}_3 \\
\frac{4V_0}{\pi} \sin\left(\frac{\pi D}{2}\right) & = L_1 I_{1-2} \hat{w}_5
\end{align*}
\]
To find the small-signal variation in the output voltage envelope $\hat{V}_{2-env}$, we must evaluate the following block diagram.

To employ these models, one must first run SPICE to find the steady-state quantities. These values are then plugged into the generator coefficients of the small-signal model, and SPICE is run again to evaluate the small-signal transfer functions.