\[
\cos \varphi = \cos \frac{\gamma}{2} + j \sin \frac{\gamma}{2}
\]

\[
J_{L1} = -\frac{\sin \varphi}{\cos \frac{\gamma}{2}}
\]

\[
J_{L0} = -(J^2 - 1) \tan \frac{\gamma}{2}
\]

\[
M_{C0} = -\frac{J \sin \varphi}{\cos \frac{\gamma}{2}}
\]

\[
M = \left(\frac{2}{\gamma}\right) \left(\varphi - \frac{\sin \varphi}{\cos \frac{\gamma}{2}}\right)
\]
State plane trajectory: DCVM

As predicted by CCM analysis

DCVM trajectory
disc. capacitor voltage mode

DCVM occurs when
\[ J_{L1} < J \]
Discontinuous conduction mode (DCVM)

- Occurs at heavy load and low output voltage
- During the “discontinuous” interval, all four output diodes are forward-biased
- The capacitor voltage waveform exhibits an additional subinterval with $v_c = 0$
CCM/DCVM boundary

CCM solution:

\[
\cos \frac{\theta}{2} + J \sin \frac{\theta}{2} = \cos \gamma
\]

\[
J_{L1} = -\frac{\sin \varphi}{\cos \frac{\gamma}{2}}
\]

at the boundary:

\[
J_{L1} = J
\]

\[
J \cos \frac{\theta}{2} = -\sin \gamma
\]

\[
J^2 \cos^2 \frac{\theta}{2} + (\cos \frac{\theta}{2} + J \sin \frac{\theta}{2})^2 = \sin^2 \gamma + \cos^2 \gamma = 1
\]

\[
J^2 \cos^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} + 2J \sin \frac{\theta}{2} \cos \frac{\theta}{2} + J^2 \sin^2 \frac{\theta}{2} = 1
\]

\[
J^2 + J \sin \gamma + \sin^2 \frac{\theta}{2} = 0
\]

\[
J_{\text{crit}}(\gamma) = -\frac{1}{2} \sin (\gamma) + \sqrt{\sin^2 \left(\frac{\gamma}{2}\right) + \frac{1}{4} \sin^2(\gamma)}
\]

Solution is \( J = J_{\text{crit}} \) at the CCM/DCVM boundary

\[
J > 0
\]

\[
\theta = \frac{\pi}{4}, \gamma = \frac{\theta}{2}
\]
CCM/DCVM Boundary in the Output Plane

Fig. 5.16. Output characteristics of the parallel resonant converter in the continuous conduction mode. The solid portions of the lines are valid CCM characteristics; shaded portions are invalid.
DCVM solution

Mode boundary

\[ J > J_{\text{crit}}(\gamma) \quad \text{for DCM} \]
\[ J < J_{\text{crit}}(\gamma) \quad \text{for CCM} \]

\[ J_{\text{crit}}(\gamma) = -\frac{1}{2} \sin(\gamma) + \sqrt{\sin^2\left(\frac{\gamma}{2}\right) + \frac{1}{4} \sin^2(\gamma)} \]

Steady-state solution (see notes)

\[ M_{C0} = 1 - \cos(\beta) \]
\[ J_{L0} = J + \sin(\beta) \]
\[ \cos(\alpha + \beta) - 2 \cos(\alpha) = -1 \]
\[ - \sin(\alpha + \beta) + 2 \sin(\alpha) + (\delta - \alpha) = 2J \]
\[ \beta + \delta = \gamma \]
\[ M = 1 + \left(\frac{2}{\gamma}\right)(J - \delta) \]

Output characteristics
Solid lines: CCM
Dashed lines: DCVM

[Graph showing mode boundary and steady-state solution with marked points and lines indicating different states.]
Using output-plane characteristic to design a parallel resonant converter

Design:

Given $V_g$, $V$, $P$ or $I$

- $L$, $C$, $n$, $f_s$

- Low-pass filter

Example 1: $n = 1$, $\Rightarrow L$, $C$, $f_s$