20.1.1 Waveforms of the half-wave ZCS quasi-resonant switch cell

The half-wave ZCS quasi-resonant switch cell, driven by the terminal quantities \( \langle v_1(t) \rangle_{TS} \) and \( \langle i_2(t) \rangle_{TS} \). Waveforms:

\[ V_1 \quad i_1(t) \quad i_2(t) \quad v_2(t) \]

Switch network

Subinterval:

Conducting devices:

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Lecture 32
State-plane diagram: ZCS half-wave

Half-wave ZCS quasi-resonant switch cell

Switch network

Conducting devices:

ECEN 5817
1. \( \frac{V_1}{L} \cdot t_1 = \frac{I_z}{L} \)

2. \( \frac{V_1}{R_0} \cdot \omega_0 t_1 = \frac{I_z}{L} \)

3. \( \frac{I_z}{V_1 R_0} = J_s \)

4. \( d = J_s \)

5. \( M_{c1} \geq 1 \)

6. \( (M_{c1} - 1)^2 + J_s^2 = 1 \)

7. \( M_{c1} = 1 + \sqrt{1 - J_s^2} \)

8. \( \beta = \pi + \sin^{-1} J_s \)

9. \( J_s \cdot \delta = M_{c1} \)

10. \( \delta = \frac{M_{c1}}{J_s} = \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right) \)

remaining part of the arc period.
Boundary of zero current switching

If the requirement

\[ I_2 < \frac{V_1}{R_0} \]

is violated, then the inductor current never reaches zero. In consequence, the transistor cannot switch off at zero current.

The resonant switch operates with zero current switching only for load currents less than the above value. The characteristic impedance must be sufficiently small, so that the ringing component of the current is greater than the dc load current.

Capacitor voltage at the end of subinterval 2 is

\[ v_2(\alpha + \beta) = V_{c1} = V_1 \left( 1 + \sqrt{1 - \left(\frac{I_2 R_0}{V_1}\right)^2} \right) \]
Maximum switching frequency: \( \omega_0 \cdot T_s = \frac{2\pi f_0}{F_s} = \frac{2\pi}{F} \)

\[
\alpha + \beta + \delta + \frac{1}{2} \geq \frac{2\pi}{F} \quad \text{normalized period.}
\]

\[
\frac{2\pi}{F} \geq \alpha + \beta + \delta
\]

\[
\frac{2\pi}{F} \geq J_s + \pi + \sin^{-1} J_s + \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right)
\]

\( F_{\text{max}} \)
Maximum switching frequency

The length of the fourth subinterval cannot be negative, and the switching period must be at least long enough for the tank current and voltage to return to zero by the end of the switching period.

The angular length of the switching period is

$$\omega_0 T_s = \alpha + \beta + \delta + \xi = \frac{2\pi f_0}{f_s} = \frac{2\pi}{F}$$

where the normalized switching frequency $F$ is defined as

$$F = \frac{f_s}{f_0}$$

So the minimum switching period is

$$\omega_0 T_s \geq \alpha + \beta + \delta$$

Substitute previous solutions for subinterval lengths:

$$\frac{2\pi}{F} \geq \frac{I_2 R_0}{V_1} + \pi + \sin^{-1} \left( \frac{I_2 R_0}{V_1} \right) + \frac{V_1}{I_2 R_0} \left( 1 \pm \sqrt{1 - \left( \frac{I_2 R_0}{V_1} \right)^2} \right)$$
Averaged switch modeling: we need to determine the average values of $i_1(t)$ and $v_2(t)$. The average switch input current is given by

$$\langle i_1(t) \rangle_{T_s} = \frac{1}{T_s} \int_{t}^{t+T_s} i_1(t) \, dt = \frac{q_1 + q_2}{T_s}$$

$q_1$ and $q_2$ are the areas under the current waveform during subintervals 1 and 2. $q_1$ is given by the triangle area formula:

$$q_1 = \int_{0}^{\frac{\alpha}{\omega_0}} i_1(t) \, dt = \frac{1}{2} \left( \frac{\alpha}{\omega_0} \right) (I_2)$$
\[ \mu = \frac{1}{T_s} \int_0^{T_s} (g_1 + g_2) dt = \frac{1}{T_s I_2} \left( g_1 + g_2 \right) \]

\[ g_1 = \frac{I_2 \cdot t_1}{2} \]

\[ \frac{g_1}{T_s I_2} = \frac{X_2 \cdot t_1}{2 T_s \cdot I_2} = \frac{\omega_0 t_1}{2 \omega_0 T_s} = \frac{\alpha}{4\pi} \cdot F \]

\[ g_2 = \int_{t_1}^{t_1 + t_2} i_1(t) dt = \int_{t_1}^{t_1 + t_2} (V_{c1} + I_2) dt \]

\[ g_2 = C_r V_{c1} + I_2 t_2 \]

\[ \frac{g_2}{I_2 T_s} = \frac{C_r V_{c1}}{I_2 T_s} + \frac{I_2 t_2}{I_2 T_s} = \]

\[ = \frac{\omega_0 C_r V_{c1}}{\omega_0 T_s \cdot I_2} + \frac{\beta}{2\pi F} = \]

\[ = \rho \frac{C_{c1}}{\nu_0} \cdot \frac{1}{2\pi F} + \rho \frac{\beta}{2\pi F} \]
\[ M = \frac{F}{2\pi} \left[ \frac{d}{2} + \frac{M_{cl}}{J_S} + \beta \right] \]

\[ M_{cl} = 1 + \sqrt{1 - J_S^2} \]

\[ d = J_S \]

\[ \beta = \pi + \sin^{-1} J_S \]

*Ex: Duty cycle of the TCS-QR switch.

*Back example:

\[ M = \frac{V}{V_g} = M, \quad J_S = \frac{I}{V_g/R_o} \]

*Normalized load current.
Charge arguments: computation of $q_2$

$$q_2 = \int_{\alpha / \omega_0}^{\alpha + \beta / \omega_0} i_1(t) \, dt$$

Node equation for subinterval 2:

$$i_1(t) = i_C(t) + I_2$$

Substitute:

$$q_2 = \int_{\alpha / \omega_0}^{\alpha + \beta / \omega_0} i_C(t) \, dt + \int_{\alpha / \omega_0}^{\alpha + \beta / \omega_0} I_2 \, dt$$

Second term is integral of constant $I_2$

$$\int_{\alpha / \omega_0}^{\alpha + \beta / \omega_0} I_2 \, dt = I_2 \frac{\beta}{\omega_0}$$

Circuit during subinterval 2
Charge arguments continued

\[ q_2 = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha + \beta}{\omega_0}} i_c(t) \, dt + \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha + \beta}{\omega_0}} I_2 \, dt \]

First term: integral of the capacitor current over subinterval 2. This can be related to the change in capacitor voltage:

\[ \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha + \beta}{\omega_0}} i_c(t) \, dt = C \left( v_2\left(\frac{\alpha + \beta}{\omega_0}\right) - v_2\left(\frac{\alpha}{\omega_0}\right) \right) \]

\[ \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha + \beta}{\omega_0}} i_c(t) \, dt = C \left( V_{c1} - 0 \right) = CV_{c1} \]

Substitute results for the two integrals:

\[ q_2 = CV_{c1} + I_2 \frac{\beta}{\omega_0} \]

Substitute into expression for average switch input current:

\[ \langle i_1(t) \rangle_{T_s} = \frac{\alpha I_2}{2\omega_0 T_s} + \frac{CV_{c1}}{T_s} + \frac{\beta I_2}{\omega_0 T_s} \]
Switch conversion ratio $\mu$

$$\mu = \frac{\langle i_1(t) \rangle_{T_s}}{I_2} = \frac{\alpha}{2\omega_0 T_s} + \frac{CV_{c1}}{I_2 T_s} + \frac{\beta}{\omega_0 T_s}$$

Eliminate $\alpha$, $\beta$, $V_{c1}$ using previous results:

$$\mu = F \frac{1}{2\pi} \left[ \frac{1}{2} J_s + \pi + \sin^{-1}(J_s) + \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right) \right]$$

where

$$J_s = \frac{I_2 R_0}{V_1}$$
Analysis result: switch conversion ratio $\mu$

Switch conversion ratio:

$$\mu = F \frac{1}{2\pi} \left[ \frac{1}{2} J_s + \pi + \sin^{-1}(J_s) + \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right) \right]$$

with $J_s = \frac{I_2 R_0}{V_1}$

This is of the form

$$\mu = FP_{\frac{1}{2}}(J_s)$$

$$P_{\frac{1}{2}}(J_s) = \frac{1}{2\pi} \left[ \frac{1}{2} J_s + \pi + \sin^{-1}(J_s) + \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right) \right]$$

Graph showing $P_{\frac{1}{2}}(J_s)$ against $J_s$.
Characteristics of the half-wave ZCS resonant switch

Switch characteristics:

\[ \mu = F P_{1/2}(J_s) \]

Mode boundary:

\[ J_s \leq 1 \]

\[ \mu \leq 1 - \frac{J_s F}{4\pi} \]
Buck converter containing half-wave ZCS quasi-resonant switch

Conversion ratio of the buck converter is (from inductor volt-second balance):

\[ M = \frac{V}{V_g} - \mu \]

For the buck converter,

\[ J_s = \frac{IR_0}{V_g} \]

ZCS occurs when

\[ I \leq \frac{V_g}{R_0} \]

Output voltage varies over the range

\[ 0 \leq V \leq V_g - \frac{FIR_0}{4\pi} \]
ZCS half-wave quasi-resonant boost converter example

For the boost converter,

\[ M = \frac{V}{V_g} = \frac{1}{1 - \mu} \]

\[ J_s = \frac{I_2 R_0}{V_1} = \frac{I_g R_0}{V} \]

\[ I_g = \frac{I}{1 - \mu} \]

Hal-wave ZCS equations:

\[ \mu = FP_\frac{1}{2} (J_s) \]

\[ P_{\frac{1}{2}} (J_s) = \frac{1}{2\pi} \left[ \frac{1}{2} J_s + \pi + \sin^{-1}(J_s) + \frac{1}{J_s} \left( 1 + \sqrt{1 - J_s^2} \right) \right] \]
20.1.3 The full-wave ZCS quasi-resonant switch cell

Half wave

Switch network

Full wave

Switch network

ECEN 5817
Full-wave ZCS state-plane diagram
Analysis in the full-wave case is nearly the same as in the half-wave case. The second subinterval ends at the second zero crossing of the tank inductor current waveform. The following quantities differ:

\[
\beta = \begin{cases} 
\pi + \sin^{-1}(J_s) & \text{(half wave)} \\
2\pi - \sin^{-1}(J_s) & \text{(full wave)}
\end{cases}
\]

\[
V_{c1} = \begin{cases} 
V_1 \left(1 + \sqrt{1 - J_s^2}\right) & \text{(half wave)} \\
V_1 \left(1 - \sqrt{1 - J_s^2}\right) & \text{(full wave)}
\end{cases}
\]

In either case, \( \mu \) is given by

\[
\mu = \frac{\langle i_1(t) \rangle_{T_s}}{I_2} = \frac{\alpha}{2\omega_0 T_s} + \frac{CV_{c1}}{I_2 T_s} + \frac{\beta}{\omega_0 T_s}
\]
Full-wave cell: switch conversion ratio $\mu$

$$P_1(J_s) = \frac{1}{2\pi} \left[ \frac{1}{2} J_s + 2\pi - \sin^{-1}(J_s) + \frac{1}{J_s} \left( 1 - \sqrt{1 - J_s^2} \right) \right]$$

$$\mu = FP_1(J_s)$$

Full-wave case: $P_1$ can be approximated as

$$P_1(J_s) \approx 1$$

so

$$\mu \approx F = \frac{f_s}{f_0}$$