Analysis of ZVS-QSW switch cell
(1-transistor case: current-bidirectional transistor + diode)
Single-transistor ZVS-QSW waveforms and state-plane analysis

\[ V_{b0x} = V_b = 0 \]
\[ I_{b0x} = V_b R_0 \]

\[ L_0 = \sqrt{\frac{L}{C}} \]

\[ \omega_o T_s \]

\[ j_1 \]

\[ m_c \]

\[ \omega_o t \]

Devices conducting
Interval 1: $Q_1$ conduction

\[ V = <v_c> = \mu \cdot V_1 \]
\[ \text{by def.} \]

\[ \begin{align*}
L \frac{di_L}{dt} &= V_1 - V > 0 \implies i_L \text{ ramps up as a lin. of time.} \\
\text{at the end of the interval } \Delta \\
i_L(t_\Delta) &= I_{L_1} = \frac{V_1 - V}{L} \cdot t_\Delta, \quad i_L(0) = 0
\end{align*} \]

\[ J_{L_1} = (1 - \mu) \Delta \]
Interval 2: transition

\[ r_1^2 = J_{l_1}^2 + (1-\mu)^2 \]
\[ r_1^2 = J_{l_2}^2 + \mu^2 \]

\[ J_{l_1}^2 + (1-\mu)^2 = J_{l_2}^2 + \mu^2 \]
\[ J_{l_1}^2 + 1 - 2\mu = J_{l_2}^2 \]

\[ J_{l_2} = \sqrt{J_{l_1}^2 + (1-2\mu)} > 0 \]

\[ \beta = \beta_1 + \beta_2 = \tan^{-1} \frac{1-\mu}{J_{l_1}} + \tan^{-1} \frac{\mu}{J_{l_2}} \]
Interval 3: $D_2$ conduction

\[ L \frac{di_L}{dt} = -V \]

$I_{L2}$ initial value

\[ J_{L2} = \mu \cdot \delta \]
Interval 4: transition

\[ V = \mu V_1 \]

ZVS condition: \( \mu > 0.5 \)

\[ J_{L3}^2 + (1-\mu)^2 = \mu^2 \]

\[ J_{L3}^2 + 1 - 2\mu = 0 \]

\[ J_{L3} = 2\mu - 1 \]

\[ J_{L3} = \sqrt{2\mu - 1}, \quad \mu > 0.5 \]

\[ \xi = \pi - \tan^{-1} \frac{3J_{L3}}{1-\mu} \]

resonant transition
Interval 5: $D_1$ conduction

\[ i_L + \quad \mu \quad i_L - \quad V_1 \quad \text{initially} < 0 \quad V = \mu V_1 \quad J_{L3} \to 0 \]

\[ J_{L3} = (1-\mu)\beta \]
Average output current

Approaches to solving for $\mu$

1. $V = \mu V_A = \langle V_c \rangle$
   \[ \mu = \frac{\langle V_c \rangle}{V_A} = \langle m_c \rangle \]

2. $I = \langle i_L \rangle$
   $J = \langle \dot{V}_L \rangle$
   $\langle i_L \rangle = \mu \cdot J$

\[ I = \frac{1}{T_s} \int_0^{T_s} i_L(t) \, dt = \frac{1}{T_s} \int_2^4 i_L(t) \, dt + \frac{1}{T_s} \int_4^6 i_L(t) \, dt \]

\[ J = \frac{F}{2\pi} \left[ (L_1 - L_3)(\alpha + \xi) + J_{L_2} \xi \right] \]

\[ I = \frac{I_{L_1} - I_{L_3}}{2} \cdot \frac{\alpha + \xi}{\omega_0 T_s} + \frac{I_{L_2}}{2} \cdot \frac{\xi}{\omega_0 T_s} \]

$D_1$ and/or $Q_1$ are on. \[ \alpha + \xi = \Theta \]
$\Theta$ or $F$ are control variables.
Control input: transistor/diode conduction angle $\theta$

Transistor/diode $Q_i/D_i$ conduction angle

Define $\theta = \alpha + \psi$

The transistor/diode duty cycle can be defined as

$$d = \frac{\theta}{\omega_0 T_s} = \frac{\theta F}{2\pi}$$

$\theta$ could be viewed as a control input.
A way to solve and plot the characteristics

Given \( \mu \) and \( \Theta \), we can find \( J_2 \) without iteration (and also \( F \)), and then plot the output characteristics of the switch. The relevant equations are:

\[
(1) \quad J_{L3} = \sqrt{2\mu - 1}
\]

\[
(2) \quad J_{L1} = -J_{L3} + \Theta (1-\mu)
\]

which follows from the slope of the current during the transistor conduction interval:

\[
\text{So} \quad I_{L1} = -I_{L3} + \frac{V_d - V}{L} \frac{1}{\omega_c}
\]

normalize to get (2)
\[ J_{L_2} = \sqrt{1 - 2\mu + J_{L_1}^2} \]  

(4) \[ \beta = \tan^{-1}\left(\frac{1 - \mu}{J_{L_1}}\right) + \tan^{-1}\left(\frac{\mu}{J_{L_2}}\right) \]  

(5) \[ \delta = \frac{J_{L_2}}{\mu} \]  

(6) \[ \xi = \pi - \tan^{-1}\left(\frac{J_{L_3}}{1 - \mu}\right) \]  

(7) \[ F = \frac{2\pi}{\theta + \beta + \delta + \xi} \]  

(8) \[ J = \frac{F}{4\pi} \left[(J_{L_1} - J_{L_3})\theta + J_{L_2}\delta\right] \]  

So given \( \mu \) and \( \theta \), we can evaluate the above equations in order, to find the normalized dc inductor current without iteration.
Results: output-plane characteristic of the switch conversion ratio \( \mu \) with \( F \) as the parameter

*Basic single-transistor resonant switch*
Output-plane characteristic of the switch conversion ratio $\mu$ with $\theta$ as the parameter

Course website contains Excel spreadsheet (with function macros) that evaluates the above equations and can plot the above characteristic.

Basic single-transistor switch

Transistor $(Q_1/D_1)$ conduction angle $\theta$
Summary of 1-transistor ZVS-QSW characteristics

Conversion ratio $\mu$ vs. $J$

$\mu$ vs. $F$

$F \to 0$

CCM.
Boundaries of ZVS operation

- $J = 0$: zero DC inductor current.

Diagram:
- $F < 1$
- $\delta = 0$: $D_2$ does not conduct at all.
- $\Theta = 0$: $Q_1$ never conducts.
- $\delta = 0$: $D_2$ never conducts.

- $J = 0$: at the end of the power switch.

- $F = 1$: