Announcements

• On-campus students: turn in HW1

• Off-campus students turn in HW1 via D2L (link on the course website); grace period ends 5pm Friday, Feb.3

• HW2 (3 textbook problems from Chapter 19) is due Monday, Feb.6. The grace period for off-campus students ends 5pm Monday, Feb.13
Section 19.1: modeling based on sinusoidal approximation

A resonant dc-dc converter:

If tank responds primarily to fundamental component of switch network output voltage waveform, then harmonics can be neglected.
19.1.4 Solution of converter
voltage conversion ratio $M = \frac{V}{V_g}$

\[
M = \frac{V}{V_g} = \left( R \right) \left[ \frac{2}{\pi} \right] \left[ \frac{1}{R_e} \right] \left| H(s) \right|_{s=j\omega} \left[ \frac{4}{\pi} \right]
\]

\[
\left( \frac{V}{I} \right) \left( \frac{I}{I_{R1}} \right) \left( \frac{I_{R1}}{V_{R1}} \right) \left( \frac{V_{R1}}{V_{s1}} \right) \left( \frac{V_{s1}}{V_g} \right)
\]

Eliminate $R_e$.

\[
\frac{V}{V_g} = \left| H(s) \right|_{s=j\omega}
\]

Voltage-driven → voltage located tank.
19.2 Examples

19.2.1 Series resonant converter

\[ M = \|H(j\omega_s)\| = \sqrt{V_g} \]

\[ H(s) = \frac{R_e}{\frac{1}{sC_s} + sL + R_e} = \frac{R_e}{Z_i} \]
Model: series resonant converter

\[
\begin{align*}
2I_{s1} & \cos (\phi_s) \\
4V_g & \sin (\omega_s t)
\end{align*}
\]
Construction of $Z_i$ – resonant (high Q) case

$C = 0.1 \, \mu F$, $L = 1 \, mH$, $R_e = 10 \, \Omega$

\[
\frac{1}{\omega C}, \quad \omega L
\]

**Diagram:**
- Circuit diagram showing a series resonant circuit with $L = 1 \, mH$, $C = 0.1 \, \mu F$, and $R_e = 10 \, \Omega$.
- Resonance frequency $f_0 = \frac{1}{2\pi \sqrt{LC}} = 16 \, kHz$.
- Reactance $X_L = \omega L$ and $X_C = 1/\omega C$.
- Quality factor $Q = 10 \rightarrow 20 \, dB$.
- Impedance $Z_0 = \sqrt{\frac{L}{C}} = 100 \, \Omega$.
Construction of $H = V / V_g$ – resonant (high Q) case

$C = 0.1 \, \mu F, \, L = 1 \, \text{mH}, \, R_e = 10 \, \Omega$

\[ ||H|| = \frac{R_e}{||z_{ii}||} \leq 1 \]

Control variable: $= \text{sw. figury}$

\[ f_0 = \frac{1}{2\pi V_{LC}} \]

\[ ||H|| = M = \frac{V}{V_g} \]

Design decision: $= \text{impacts sign in feedback loop}$
Series resonant converter: DC conversion ratio

\[ H(s) = \frac{R_e}{Z_i(s)} = \frac{R_e}{R_e + sL + \frac{1}{sC}} = \frac{R_e}{1 + \left(\frac{s}{Q_e\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \]

\[ R_0 = \sqrt{\frac{L}{C}} \]

\[ Q_e = \frac{R_0}{R_e} \]

\[ M = \|H(j\omega_s)\| = \frac{1}{\sqrt{1 + Q_e^2\left(\frac{1}{F} - F\right)^2}} \]

\[ F = \frac{f_s}{f_0} \]

\[ F < 1 \text{ below res.} \]

\[ F > 1 \text{ above res.} \]
19.2.2 Subharmonic modes of the SRC

Example: excitation of tank by third harmonic of switching frequency

Can now approximate \( v_s(t) \) by its third harmonic:

\[
v_s(t) \approx v_{sn}(t) = \frac{4V_g}{n\pi} \sin (n\omega_s t)
\]

Result of analysis:

\[
M = \frac{V}{V_g} = \left| \frac{H(jn\omega_s)}{n} \right|
\]
SRC DC conversion ratio $M$

Subharmonic modes

Positive feedback

Desired control loop for operation below resonance

Limit VCO
19.2.3 Parallel resonant dc-dc converter

Differs from series resonant converter as follows:

Different tank network

Rectifier is driven by sinusoidal voltage, and is connected to inductive-input low-pass filter

Need a new model for rectifier and filter networks
Model of uncontrolled rectifier with inductive filter network – input port

Fundamental component of $i_R(t)$:

$$i_{R1}(t) = \frac{4I}{\pi} \sin (\omega_s t - \varphi_R)$$

$$i_{R1}(t) = \frac{v_{R1}(t)}{R_e}$$

$$R_e = \frac{\pi^2}{8} R$$

Fundamental component of $i_R(t)$:
Model of uncontrolled rectifier with inductive filter network – output port

Output inductor volt second balance: dc voltage is equal to average rectified tank output voltage

\[ \langle v_x \rangle = V \] by volt. sec. balance on \( L_F \)

\[ V = \frac{2}{\pi} V_{R1} \]

\[ I_{R1} = \frac{4 I}{\pi} \]

\[ R_e = \frac{V_{R1}}{I_{R1}} = \frac{\pi^2}{8} R \]
Effective resistance $R_e$

Again define

$$R_e = \frac{v_{R1}(t)}{i_{R1}(t)} = \frac{\pi V_{R1}}{4I}$$

In steady state, the dc output voltage $V$ is equal to the average value of $|v_R|$:

$$V = \frac{2}{T_s} \int_0^{T_s/2} V_{R1} \left| \sin (\omega_s t - \phi_R) \right| dt = \frac{2}{\pi} V_{R1}$$

For a resistive load, $V = IR$. The effective resistance $R_e$ can then be expressed

$$R_e = \frac{\pi^2}{8} R = 1.2337R$$
Equivalent circuit model of uncontrolled rectifier with inductive filter network

Output port modeled as a dependent voltage source based on rectified tank voltage, in contrast to SRC where output port is modeled as dependent current source based on rectified tank current.
Equivalent circuit model
Parallel resonant dc-dc converter

\[ \frac{2I_{s1}}{\pi} \cos (\varphi_s) \]

\[ \frac{4V^g}{\pi} \sin (\omega_s t) \]

\[ v_{s1}(t) = \frac{V}{V_g} = \frac{V}{V_{R1}} \cdot \frac{V_{R1}}{v_{s1}} \cdot \frac{v_{s1}}{V_g} = \]

\[ = \frac{2}{\pi} \| H(j\omega_s) \| \cdot \frac{4}{\pi} = \frac{8}{\pi^2} \| H(j\omega_s) \| \]

PRC \[ M \]