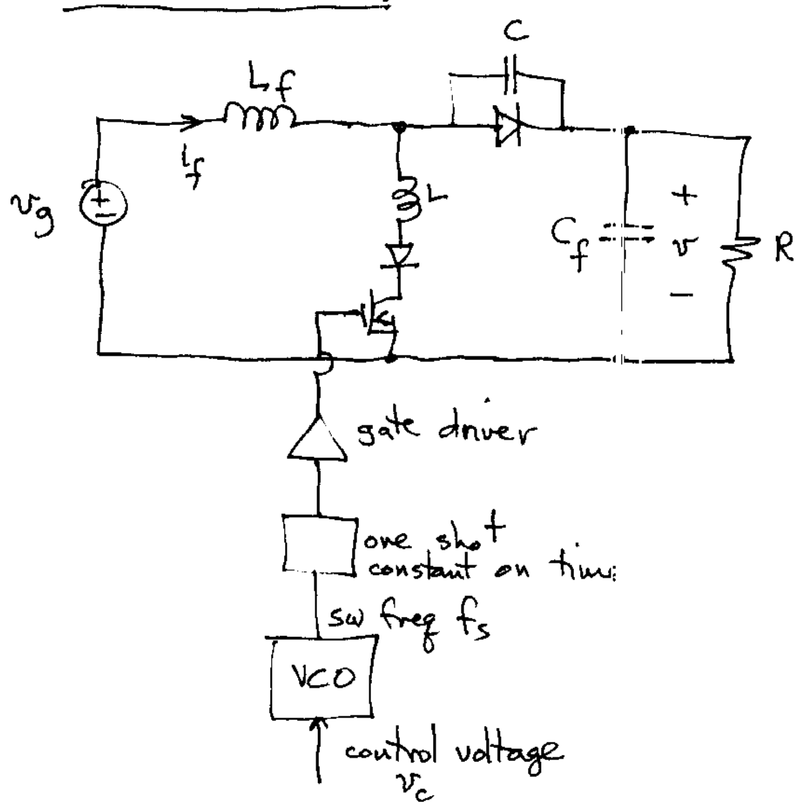
Example: Derivation of the small-signal control-to-output transfer function of the boost converter with half-wave ZCS resonant switch
(see results given in supplementary notes, "Extension of State Space Averaging to Model Some Other Types of Switches" pp. 12 ff)

Converter circuit

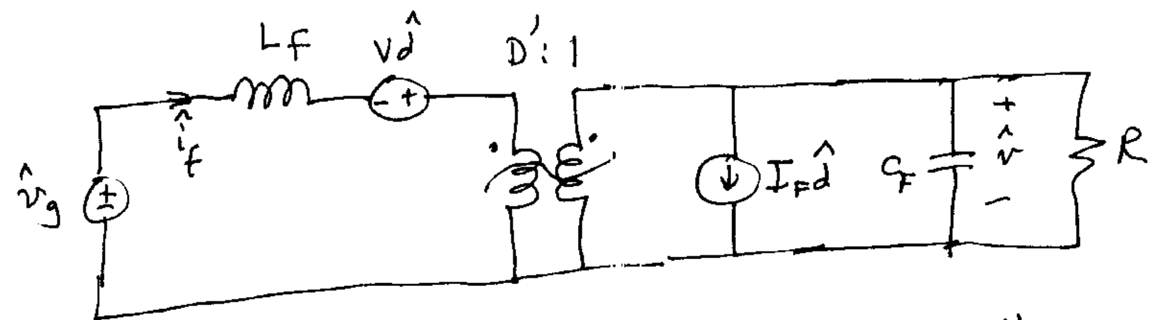


We know that this resonant switch can be modeled using the conversion ratio $\mu = F\,P_{\frac{1}{2}}(J)$ derived in class and in the notes, where

$$F = \frac{f_s}{f_o} \qquad f_o = \frac{1}{2\pi\sqrt{LC}}, \qquad J = \frac{i_f R_o}{v}$$

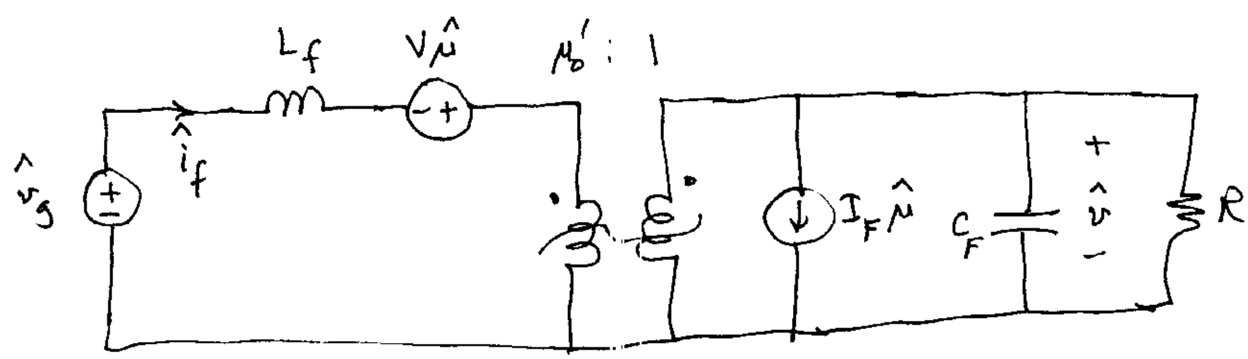
$$R_o = \sqrt{\frac{L}{C}}$$

This expression can be substituted for d in previous analyses of the PWM boost converter. For example, a small-signal model of the PWM boost converter is given on p. 210 of the textbook – Fig. 7.17 (b):



with the dc quantities defined by $V = \frac{V_g}{D'}$, $I_F = \frac{V}{D'R}$

So a low-frequency model of our quasi-resonant boost converter is



with $V = \frac{V_g}{\mu_0'}$, $I_F = \frac{V}{\mu_0' R}$

$$\mu(t) = \mu_0 + \hat{\mu}(t)$$
$$\mu_0' = 1 - \mu_0$$

(i.e., replace d with μ)

The expression for μ is perturbed and linearized as on pp. 13–14 of the supplementary notes:

$$\mu = F P_{\frac{1}{2}}(J) = \frac{f_s}{f_o} P_{\frac{1}{2}}\left(\frac{i_f R_o}{v}\right)$$

$$\mu = \mu_o + \hat{\mu}$$
$$f_s = F_{so} + \hat{f}_s$$
$$i_f = I_f + \hat{i}_f$$
$$v = V + \hat{v}$$

$$\hat{\mu} = K_c \hat{f}_s + K_i \hat{i}_f + K_v \hat{v}$$

with

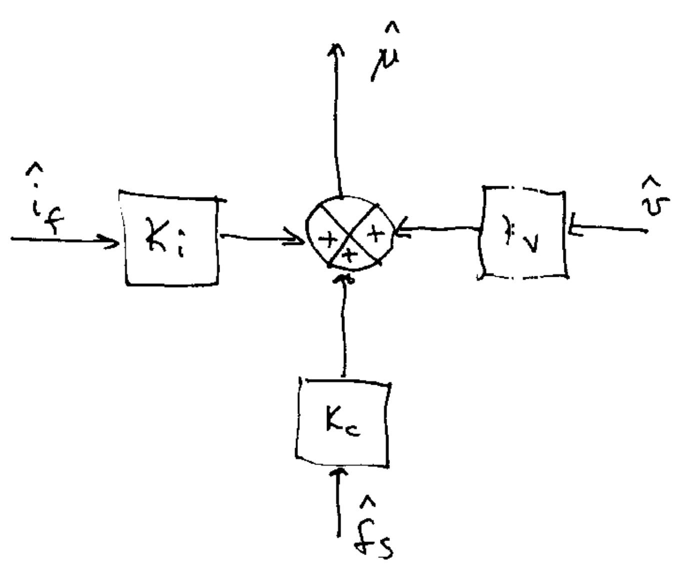
$$K_c = \frac{\mu_o}{F_{so}}$$

$$K_i = -\frac{\partial \mu}{\partial J} \frac{R_o}{V}$$

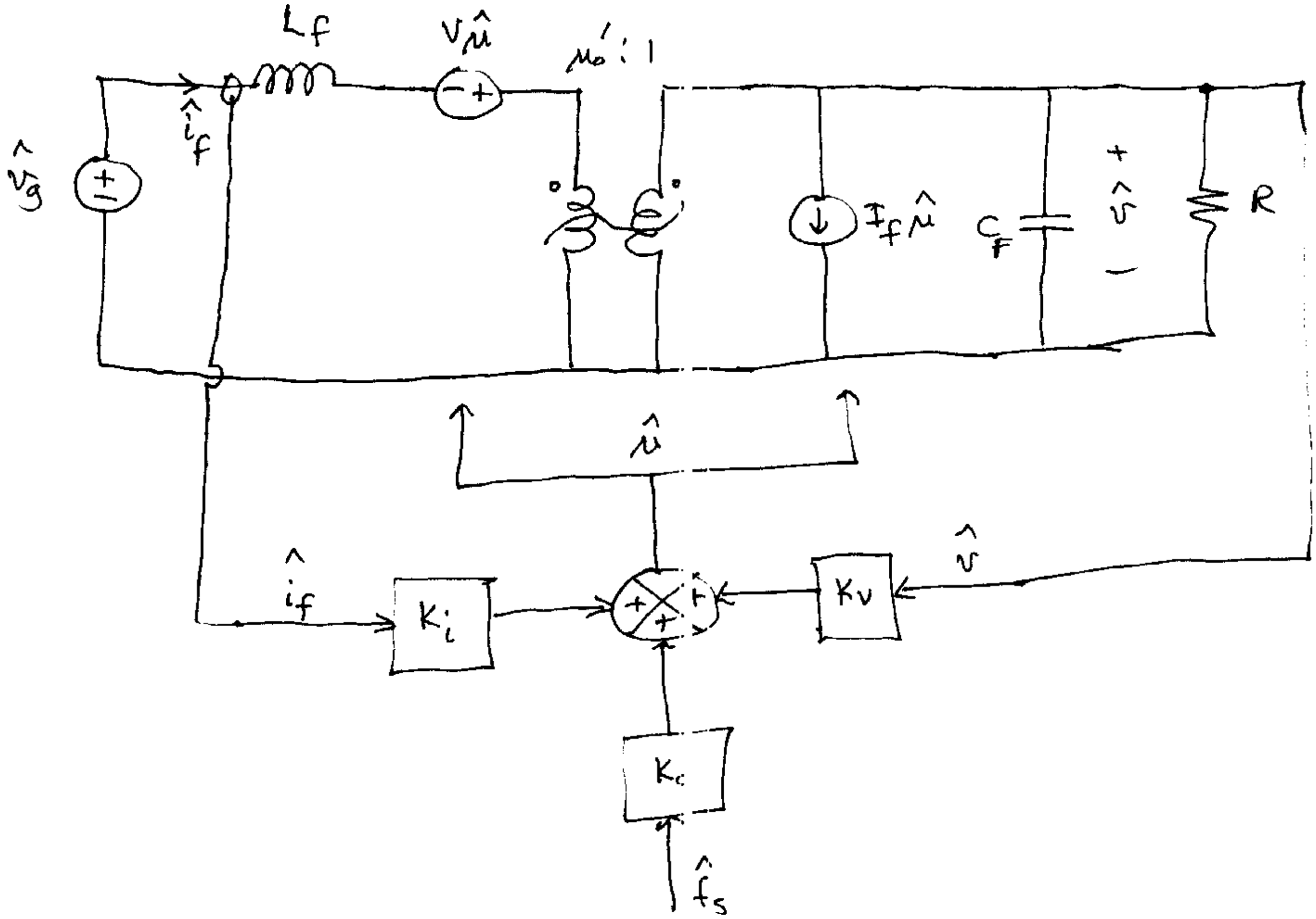
$$K_v = -\frac{\partial \mu}{\partial J} \frac{R_o I_f}{V^2}$$

$$\frac{\partial \mu}{\partial J} = \frac{F_{so}}{f_o} \frac{1}{2\pi} \left[\frac{1}{2} - \frac{1 - \sqrt{1 - J^2}}{J^2} \right], \text{ evaluated with } J = \frac{I_f R_o}{V}$$

Block diagram:



Connect block diagram to circuit model:



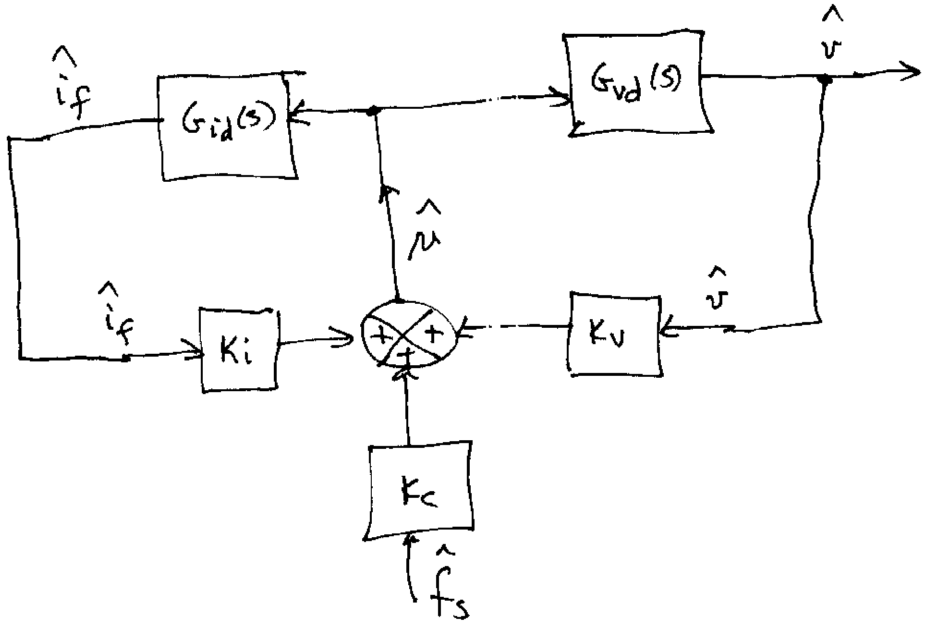
To find $G_{vc}(s) = \dfrac{\hat{v}}{\hat{f}_s}$, set the independent input $\hat{v}_g$ to zero, then solve for $\hat{v}$.

The transfer function from $\hat{\mu}$ to $\hat{v}$ is the same as the transfer function $G_{vd}(s) = \dfrac{\hat{v}}{\hat{d}}$ of the original PWM hard-switched boost converter:

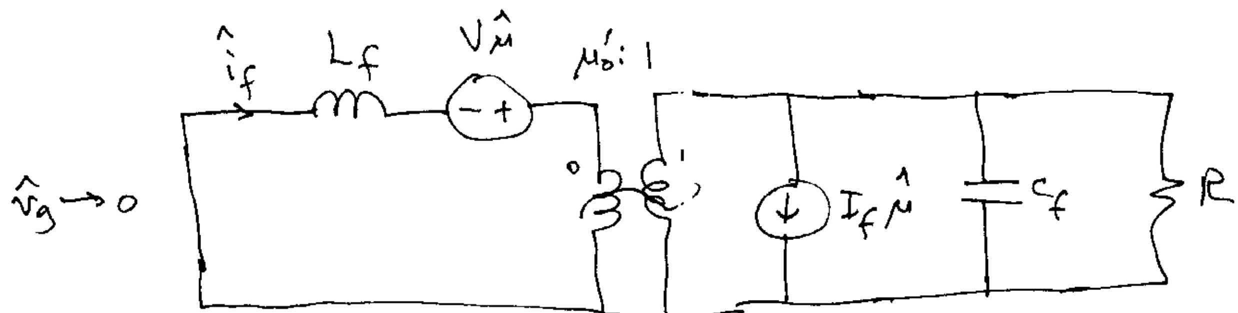
$$G_{vd}(s) = \frac{\hat{v}}{\hat{\mu}} = \frac{V}{\mu_0'} \frac{1 - \dfrac{sL_f}{\mu_0'^2 R}}{1 + \dfrac{sL_f}{\mu_0'^2 R} + \dfrac{s^2 L_f C_f}{\mu_0'^2}}$$

(see pp. 292-293 of text)

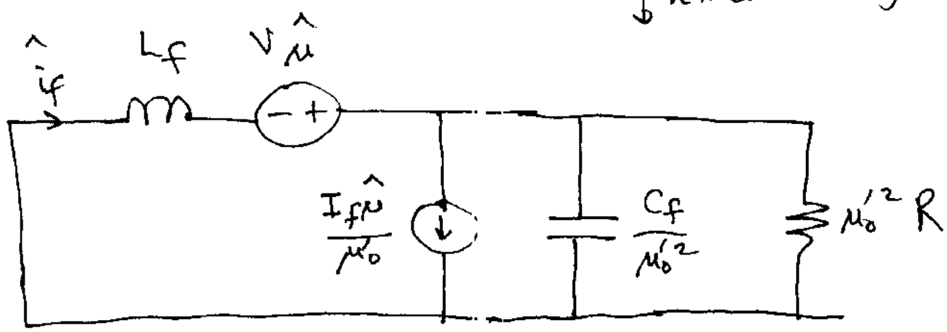
A block diagram of the system is therefore



We need to also work out $G_{id}(s)$, which is the transfer function from $\hat{\mu}$ to $\hat{i}_f$ in the original PWM converter model:



↓ reflect through transformer



Superposition

$$\hat{i}_f = V\hat{\mu} \frac{1}{sL_f + \mu_0'^2 R \| \frac{\mu_0'^2}{sC_f}} + \frac{I_f \hat{\mu}}{\mu_0'} \cdot \frac{\mu_0'^2 R \| \frac{\mu_0'^2}{sC_f}}{sL_f + \mu_0'^2 R \| \frac{\mu_0'^2}{sC_f}}$$

simplify:

$$G_{id}(s) = \frac{\hat{i}_f}{\hat{\mu}} = \frac{2V}{\mu_0'^2 R} \frac{\left(1 + s\frac{RC_f}{2}\right)}{1 + \frac{sL_f}{\mu_0'^2 R} + \frac{s^2 L_f C_f}{\mu_0'^2}}$$

Now solve block diagram:

$$\hat{\mu} = K_i \hat{i}_f + K_v \hat{v} + K_c \hat{f}_s$$

with $\hat{i}_f = G_{id} \hat{\mu}$ and $\hat{v} = G_{vd} \hat{\mu}$

eliminate $\hat{i}_f$ and $\hat{\mu}$: $\quad \hat{i}_f = \frac{G_{id}}{G_{vd}} \hat{v}$

$$\hat{v} = G_{vd}\hat{\mu} = G_{vd}\left(K_i \frac{G_{id}}{G_{vd}} \hat{v} + K_v \hat{v} + K_c \hat{f}_s\right)$$

Solve for $\hat{v}$:

$$\hat{v}\left(1 - K_i G_{id} - K_v G_{vd}\right) = G_{vd} K_c \hat{f}_s$$

$$G_{vc}(s) = \frac{\hat{v}}{\hat{f}_s} = \frac{K_c G_{vd}}{1 - K_i G_{id} - K_v G_{vd}}$$

Now plug in expressions for G_{vd} and G_{id}, and simplify.

Result:

$$G_{vc}(s) = G_{oc} \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_p} + \left(\frac{s}{\omega_p}\right)^2}$$

with $$G_{oc} = \frac{\mu_0' K_c V}{\mu_0'^2 - V K_{v_i} \mu_0' + \frac{2 V K_i}{R}}$$

$$\omega_z = \frac{\mu_0'^2 R}{L_f}$$

$$\omega_p = \frac{1}{\sqrt{L_f C_f}} \sqrt{\mu_0'^2 - V K_v \mu_0' + 2 V K_i / R}$$

$$Q = \frac{\sqrt{\mu_0'^2 - V K_v \mu_0' + 2 V K_i / R}}{\frac{1}{R}\sqrt{\frac{L_F}{C_F}}\left(1 - \frac{V K_v}{\mu_0'}\right) + V K_i \sqrt{\frac{C_F}{L_F}}}$$