Resonant Switch Converters

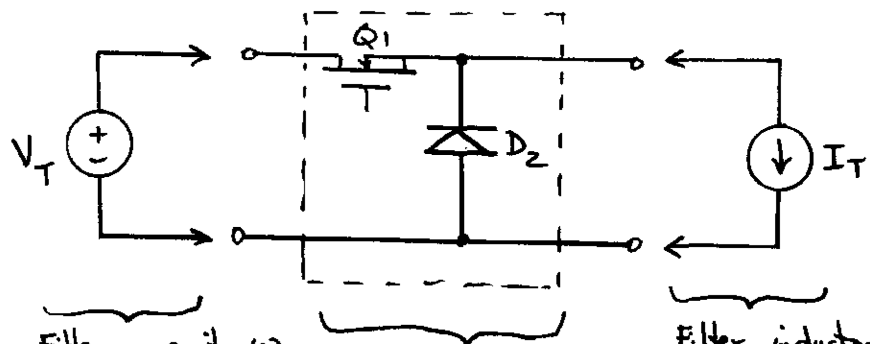
A class of converters in which a resonant tank is added to the switching elements in a conventional switched-mode converter, such that one or more of the switching loss mechanisms is eliminated. The result is a hybrid resonant / switched mode converter.

Such converters exhibit greatly reduced switching loss, at the expense of increased conduction loss. The resonant elements are operated in a manner similar to a series or parallel resonant converter discontinuous conduction mode. As in other resonant schemes, the objectives of designing such a converter are: (1) to obtain smaller transformer and low-pass filter elements via increased switching frequency, and/or (2) to reduce switching loss from component nonidealities such as transformer leakage inductance or winding capacitance and semiconductor device capacitances.

The resonant switch idea is quite general, and can be applied to a wide variety of topologies and applications.

The Zero Current Switch – "type a" (ZCS)

Standard PWM switch

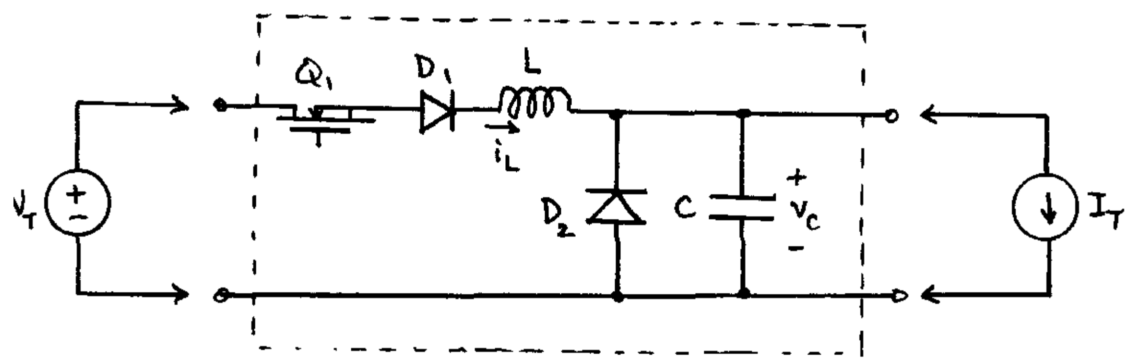


Filter capacitor(s) and/or V_g effectively apply voltage V_T to switch input

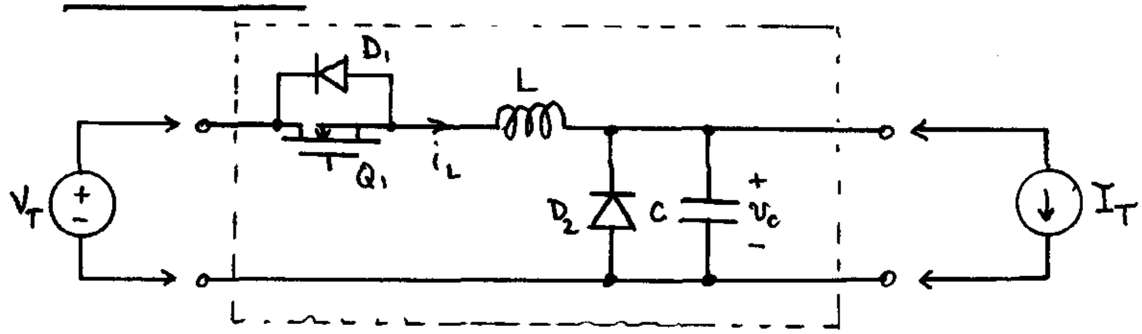
basic PWM switch cell

Filter inductor(s) of PWM converter effectively apply current I_T to switch output.

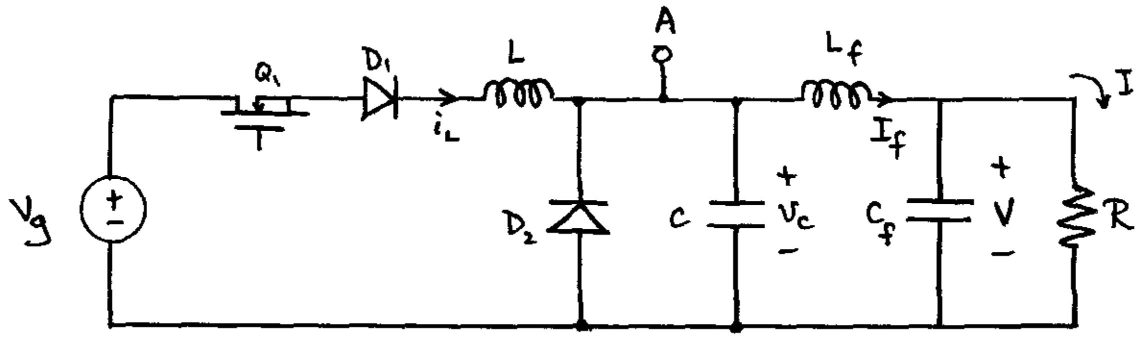
Half-wave ZCS



Full-wave ZCS



Buck converter example – half wave zero current switch



For this example, $V_T = V_g$ and $I_f = I_T$.
L_f and C_f are the (large) low-pass filter elements of the conventional buck converter. They are designed such that their ripple components are small. L, C, and D_1 are the additional resonant switch elements. L and C ring at a frequency comparable to or greater than the switching frequency, with piecewise sinusoidal and/or linear waveforms. Tank capacitor C may be connected from point A to any point at ac ground. The transistor is controlled such that it switches at zero current. Other versions, such as the "type b" switch, allow switching at zero voltage.

The zero current switch can be inserted into many other PWM topologies, such as boost, buck-boost, forward, flyback, Ćuk, etc.

Basic steady-state relations on L_f and C_f:

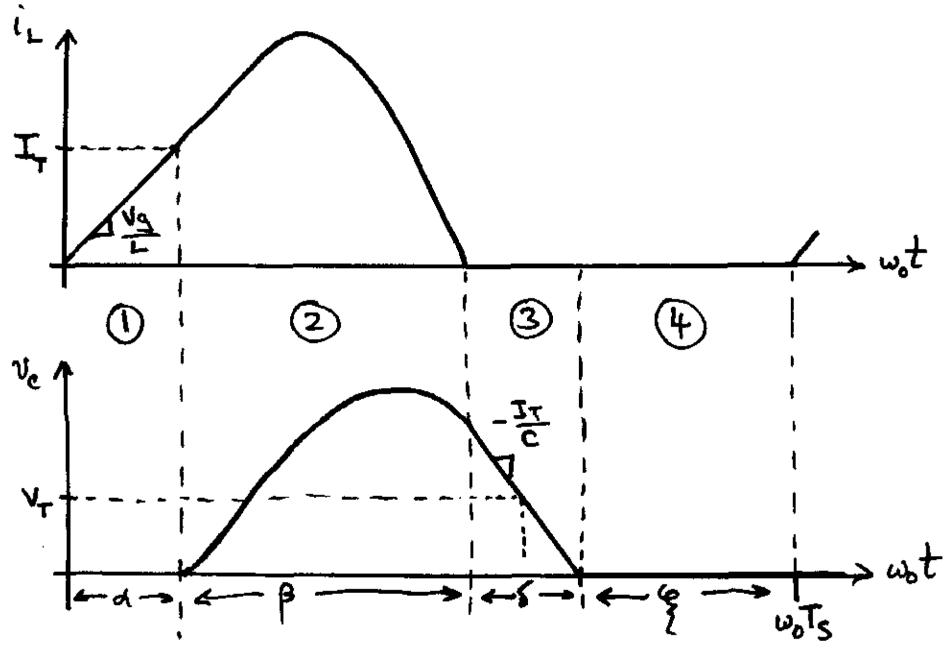
volt-second balance on L_f:

$$V = \langle v_c \rangle$$

charge balance on C_f:

$$I = I_f$$

<u>Tank waveforms</u> - half wave



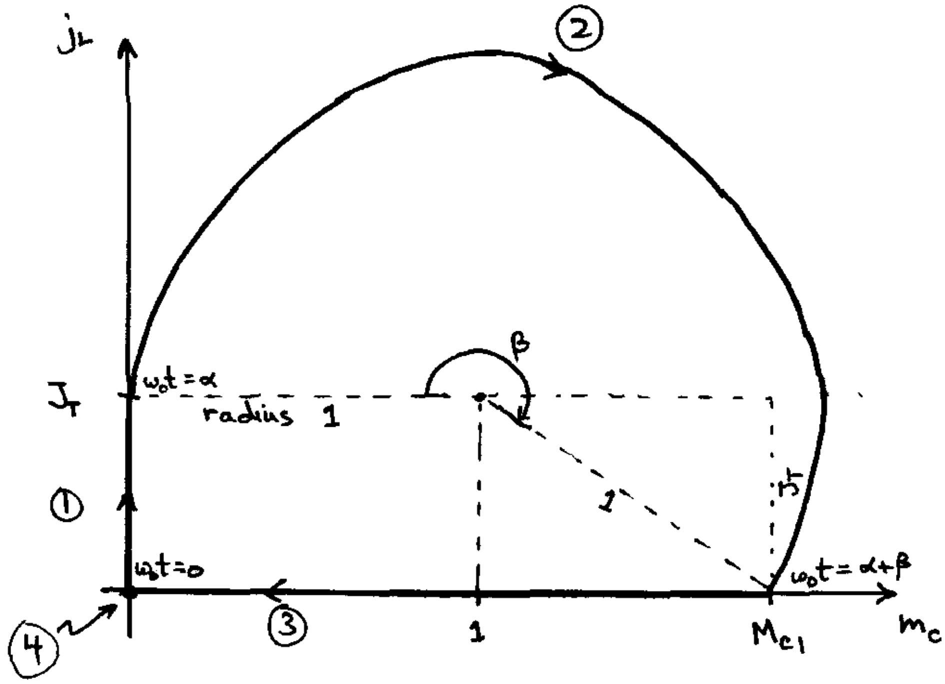
conducting devices: Q_1, D_1, D_2 | Q_1, D_1 | X | D_2

turn Q_1 on here

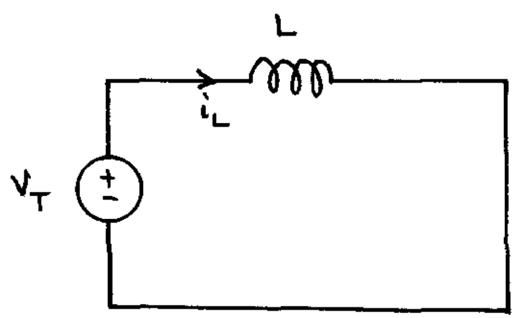
turn Q_1 off during this interval

control output by variation of dead time ξ, or equivalent variation of T_s or f_s.

normalized phase plane



interval 1 $\quad 0 \le \omega_0 t \le \alpha \quad$ Q_1, D_1, D_2 conduct



$$L \frac{di_L}{dt} = V_T \ , \quad i_L(0) = 0$$

$$v_c = 0$$

normalize using

$$V_{base} = V_T$$

$$R_{base} = R_o = \sqrt{\frac{L}{C}}$$

$$I_{base} = V_T / R_o$$

$$f_{base} = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

hence,

$$\frac{1}{\omega_o}\frac{dj_L}{dt} = 1 \quad , \quad j_L(0) = 0$$

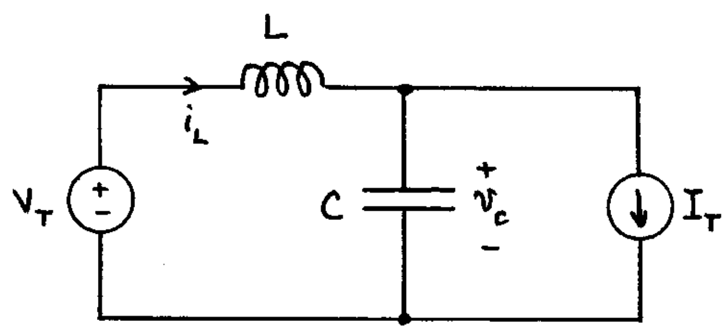
$$\Rightarrow \quad j_L(\omega_o t) = \omega_o t \quad , \quad m_c(\omega_o t) = 0$$

Diode D_2 cannot turn off until the resonant inductor current i_L equals I_T. Hence, the interval ends when

$$i_L = I_T \quad \text{at} \quad \omega_o t = \alpha$$

$$\Rightarrow \quad j_L(\alpha) = J_T = \alpha \qquad (\text{soln for } \alpha)$$

<u>interval 2</u> $\quad \alpha \le \omega_o t \le \alpha+\beta \quad$ Q_1, D_1 conduct



During the second interval, D_2 does not conduct. The L-C tank rings, and is excited by voltage V_T and current I_T.

We solved this circuit previously for the parallel resonant converter. The state equations are;

$$L \frac{di_L}{dt} = V_T - v_c$$

$$C \frac{dv_c}{dt} = i_L - I_T$$

normalize:

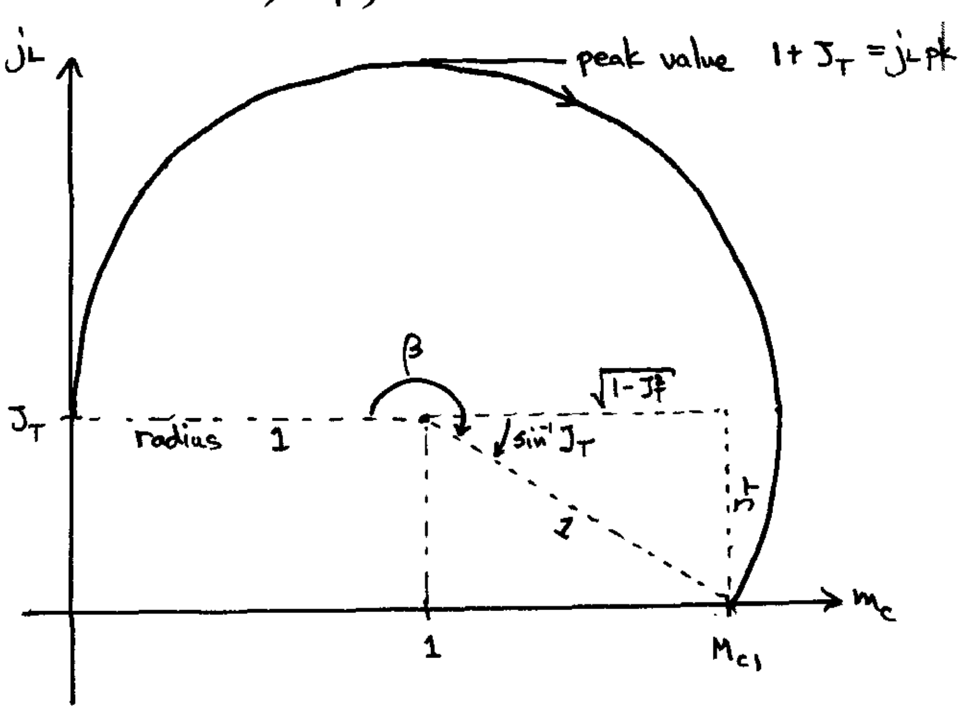
$$\frac{1}{\omega_0} \frac{dj_L}{dt} = 1 - m_c$$

$$\frac{1}{\omega_0} \frac{dm_c}{dt} = j_L - J_T$$

The state plane trajectory is a circular arc centered at

$$(m_c, j_L) = (1, J_T)$$

with initial value $(0, J_T)$



Solution of geometry:

length of ringing interval $\beta = \pi + \sin^{-1} J_T$

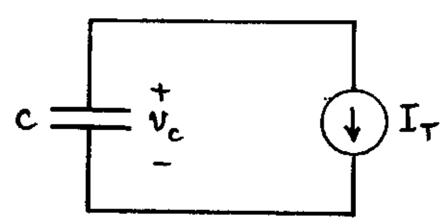
final values; $j_L(\alpha+\beta) = 0$

$m_c(\alpha+\beta) = M_{c1} = 1 + \sqrt{1 - J_T^2}$

This interval ends when $i_L(\alpha+\beta) = 0$. Diode D_1 then becomes reverse-biased.

Note that we require $J_T \le 1$. Otherwise, the ringing waveform never reaches the $j_L = 0$ axis, and the transistor does not switch off at zero current.

<u>interval 3</u> $\alpha+\beta \le \omega_0 t \le \alpha+\beta+\delta$ all semiconductors are off



The tank capacitor is discharged by I_T, so v_c decreases linearly to zero.

$$C\frac{dv_c}{dt} = -I_T, \quad v_c(\alpha+\beta) = V_{c1}$$

$i_L = 0$
$j_L = 0$

$$\Rightarrow \frac{1}{\omega_0}\frac{dm_c}{dt} = -J_T, \quad m_c(\alpha+\beta) = M_{c1}$$

<u>soln</u>: $m_c(\omega_0 t) = M_{c1} - J_T(\omega_0 t - \alpha - \beta)$

Interval 3 ends when v_c reaches zero; diode D_2 then becomes forward biased. Hence,

$$m_c(\alpha+\beta+\delta) = 0 = M_{c1} - J_T\delta$$

$$\Rightarrow \quad \delta = \frac{M_{c1}}{J_T} = \frac{1}{J_T}\left(1+\sqrt{1-J_T^2}\right)$$

<u>interval 4</u> $\quad \alpha+\beta+\delta \le \omega_0 t \le \alpha+\beta+\delta+\xi \quad$ diode D_2 conducts

The fourth interval, of length ξ, is identical to the $D'T_s$ interval in the conventional PWM converter. Diode D_2 conducts the filter inductor current I_T, and $v_c = 0$. Q_1 is off, so $i_L = 0$. In the phase plane, the trajectory remains at the origin.

The normalized switching period is

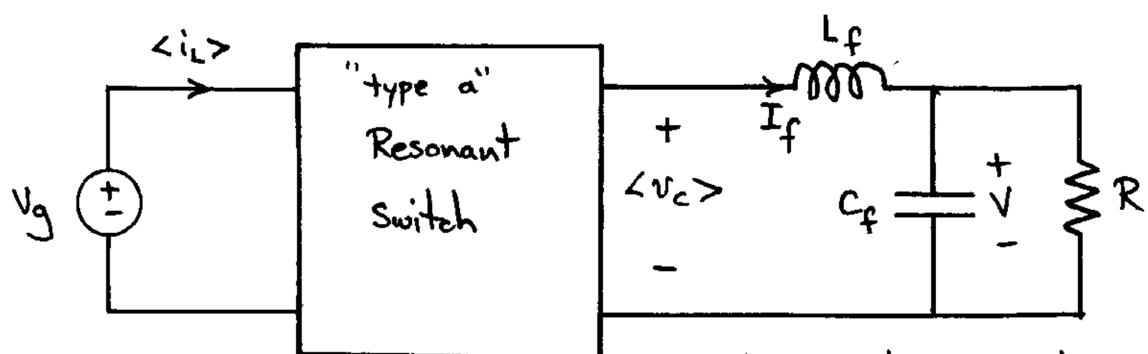
$$\omega_0 T_s = \alpha+\beta+\delta+\xi$$

Control of the switching frequency is equivalent to control of the length of the fourth interval ξ. ξ must be positive, and hence the minimum switching period is limited:

$$\omega_0 T_s \ge \alpha+\beta+\delta$$

The circuit averaging step

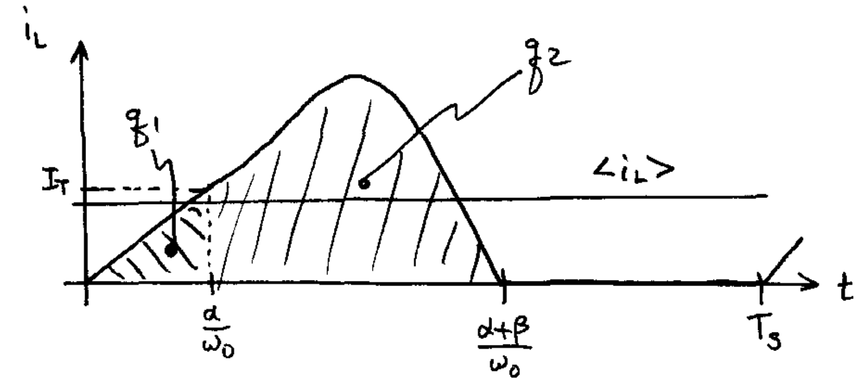
Assume that the natural time constants of the converter filter elements (L_f and C_f) are long compared to the switching period T_s, so that they respond only to the low frequency (averaged) components of the tank waveforms. Hence, it is valid to average the terminal waveforms of the resonant switch over one switching period since this does not significantly change the converter low frequency behavior.



(buck converter example)

Hence, we need to compute $\langle i_L \rangle$ and $\langle v_c \rangle$.

Average i_L can be computed using capacitor charge arguments.

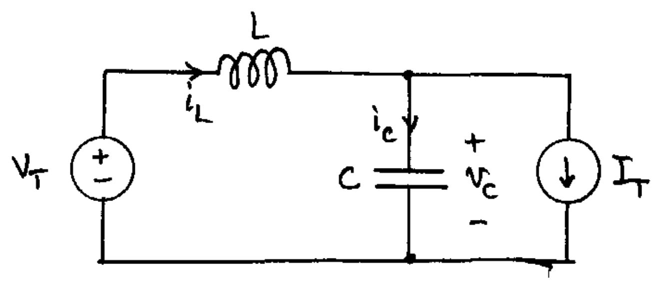


$$\langle i_L \rangle = \frac{1}{T_s}\int_0^{T_s} i_L \, dt = \frac{q_1 + q_2}{T_s}$$

with $$q_1 = \int_0^{\alpha/\omega_0} i_L \, dt = \frac{1}{2}\underbrace{\left(\frac{\alpha}{\omega_0}\right)}_{\text{base}}\underbrace{\left(I_T\right)}_{\text{height}}$$

$$q_2 = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_L \, dt$$

note during interval 2, $\quad i_L = i_c + I_T$



so $$q_2 = \underbrace{\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_c \, dt}_{\substack{= q_c = C\,\Delta V_c \\ = C(V_{c1} - 0)}} + \underbrace{\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} I_T \, dt}_{= I_T \frac{\beta}{\omega_0}} = C V_{c1} + I_T \frac{\beta}{\omega_0}$$

Total is

$$\langle i_L \rangle = \frac{1}{T_s}\left(\frac{1}{2}\frac{\alpha}{\omega_0} I_T + C V_{c1} + I_T \frac{\beta}{\omega_0} \right)$$

$$= \frac{1}{\omega_0 T_s}\left(\frac{1}{2}\alpha I_T + \omega_0 C V_{c1} + \beta I_T \right)$$

normalize:

$$\langle j_L \rangle = \frac{F}{2\pi} \cdot J_T \cdot \left(\frac{1}{2}\alpha + \beta + \frac{M_{c1}}{J_T} \right)$$

$$= J_T \cdot F \cdot \frac{1}{2\pi}\left[\frac{1}{2} J_T + \pi + \sin^{-1} J_T + \frac{1}{J_T}\left(1 + \sqrt{1 - J_T^2}\right) \right]$$

Average output voltage can be computed using

$P_{OUT} = P_{in}$

$I_T \langle v_c \rangle = V_T \langle i_L \rangle$

$\Rightarrow J_T \langle m_c \rangle = 1 \cdot \langle j_L \rangle$

so $\langle m_c \rangle = \frac{1}{J_T} \langle j_L \rangle$

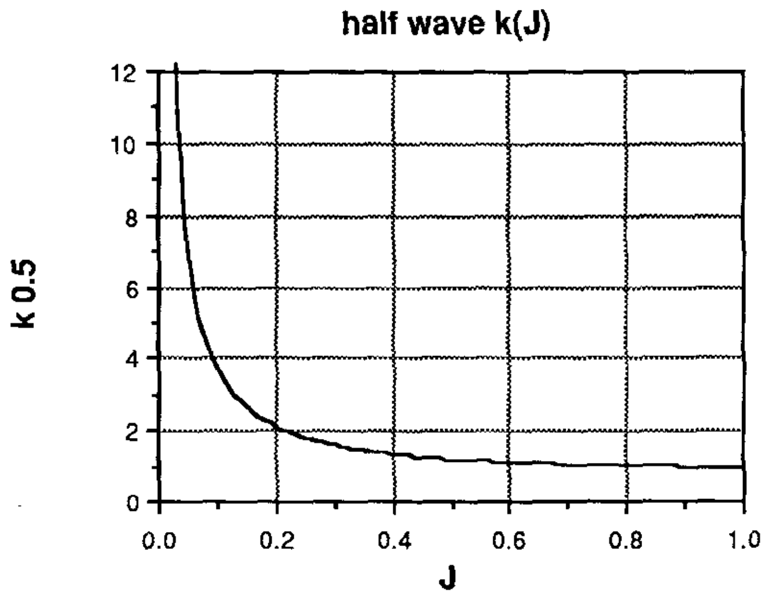
$$\langle m_c \rangle = F \cdot \underbrace{\frac{1}{2\pi}\left[\frac{1}{2} J_T + \pi + \sin^{-1} J_T + \frac{1}{J_T}\left(1 + \sqrt{1 - J_T^2}\right) \right]}_{P(J_T)}$$

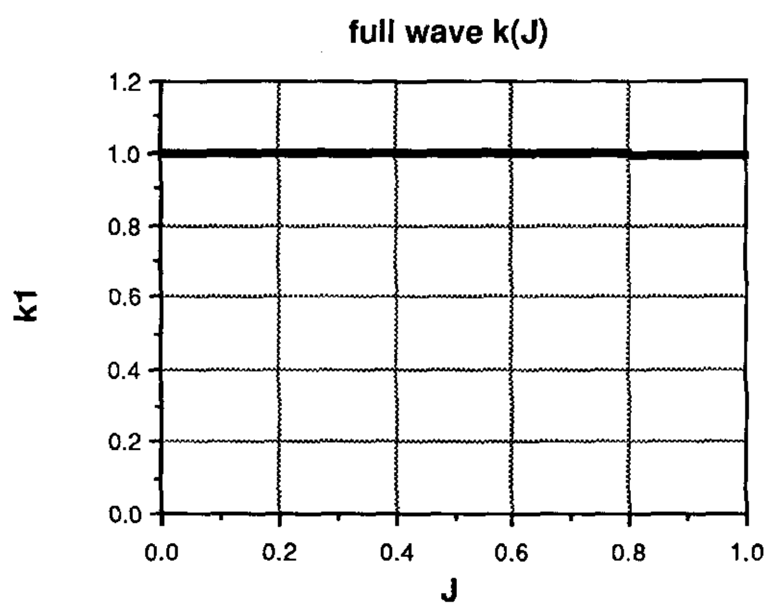
$$\langle m_c \rangle = F \cdot P(J_T)$$

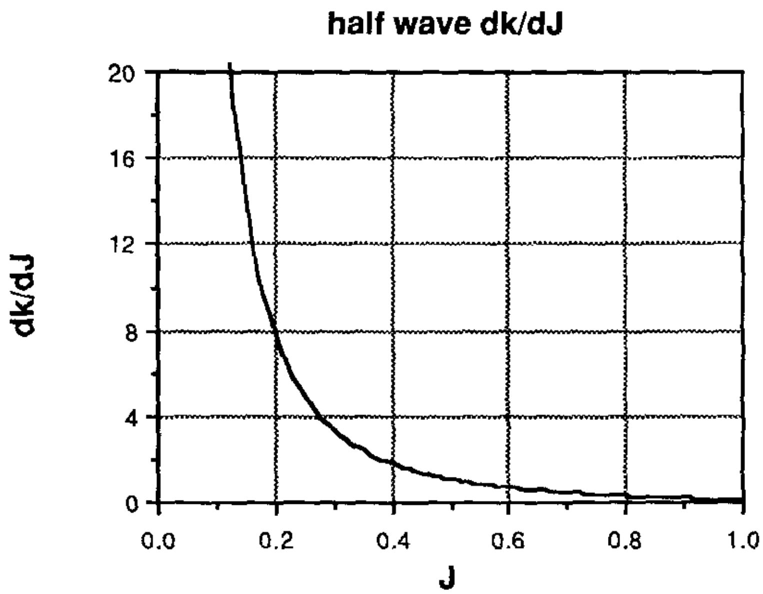
<u>Switch conversion ratio</u>

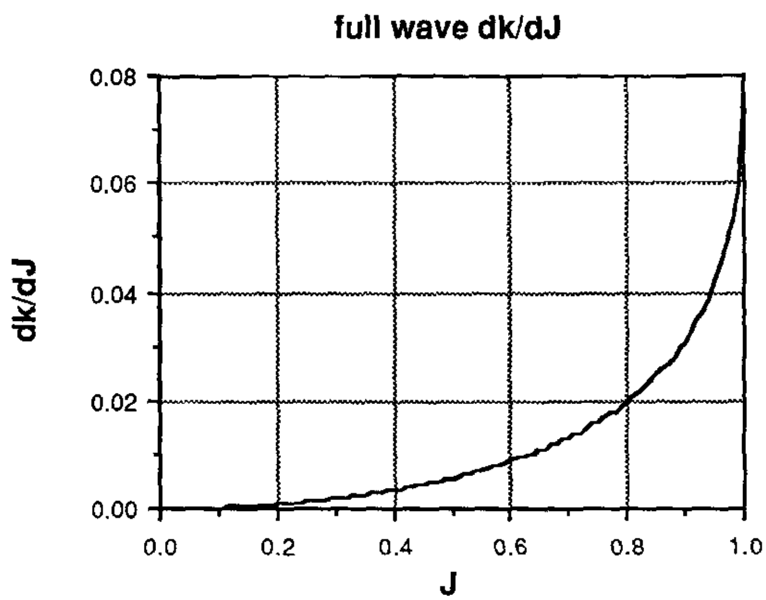
$$\mu = \frac{\langle v_c \rangle}{V_T} = \frac{\langle j_L \rangle}{J_T} = F\, P(J_T)$$

- controllable by variation of $F = \frac{f_s}{f_0}$, and also depends on $J_T = \frac{I_T R_0}{V_T}$.


half wave k(J)
12
10
8
6
4
2
0
k 0.5
0.0
0.2
0.4
0.6
0.8
1.0
J


full wave k(J)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
k1
0.0
0.2
0.4
0.6
0.8
1.0
J


half wave dk/dJ
20
16
12
8
4
0
dk/dJ
0.0
0.2
0.4
0.6
0.8
1.0
J


full wave dk/dJ
0.08
0.06
0.04
0.02
0.00
dk/dJ
0.0
0.2
0.4
0.6
0.8
1.0
J

Output plane

Mode boundaries :

1. $J_T \leq 1$ — otherwise no zero current switching

2. $\xi \geq 0$, $\Rightarrow \frac{2\pi}{F} \geq \alpha + \beta + \delta$ — minimum switching period

$$\Rightarrow \frac{2\pi}{F} \geq J_T + \pi + \sin^{-1} J_T + \frac{1}{J_T}\left(1+\sqrt{1-J_T^2}\right) = \frac{2\pi}{F}\langle M_c\rangle + \frac{1}{2} J_T$$

$$\Rightarrow \langle m_c \rangle \leq 1 - \frac{J_T F}{4\pi}$$

This is a value slightly less than one.

Hence, the allowed operating region is defined by

$$0 \leq \langle m_c \rangle \leq 1 - \frac{J_T F}{4\pi}$$

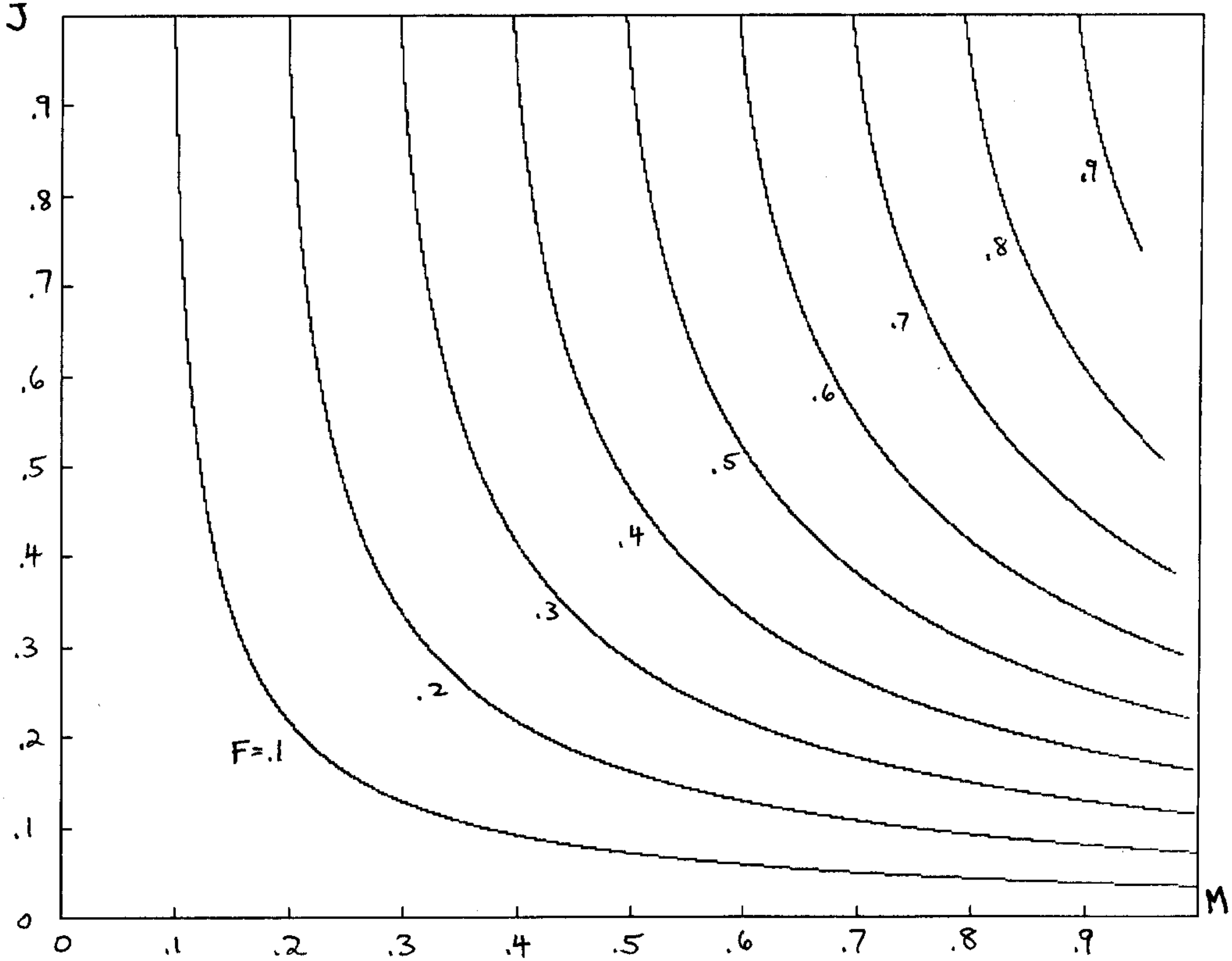
$$0 \leq J_T \leq 1$$

For the buck converter, $M = \frac{V}{V_g} = \langle m_c \rangle$

$$J = \frac{I R_o}{V_g} = J_T$$

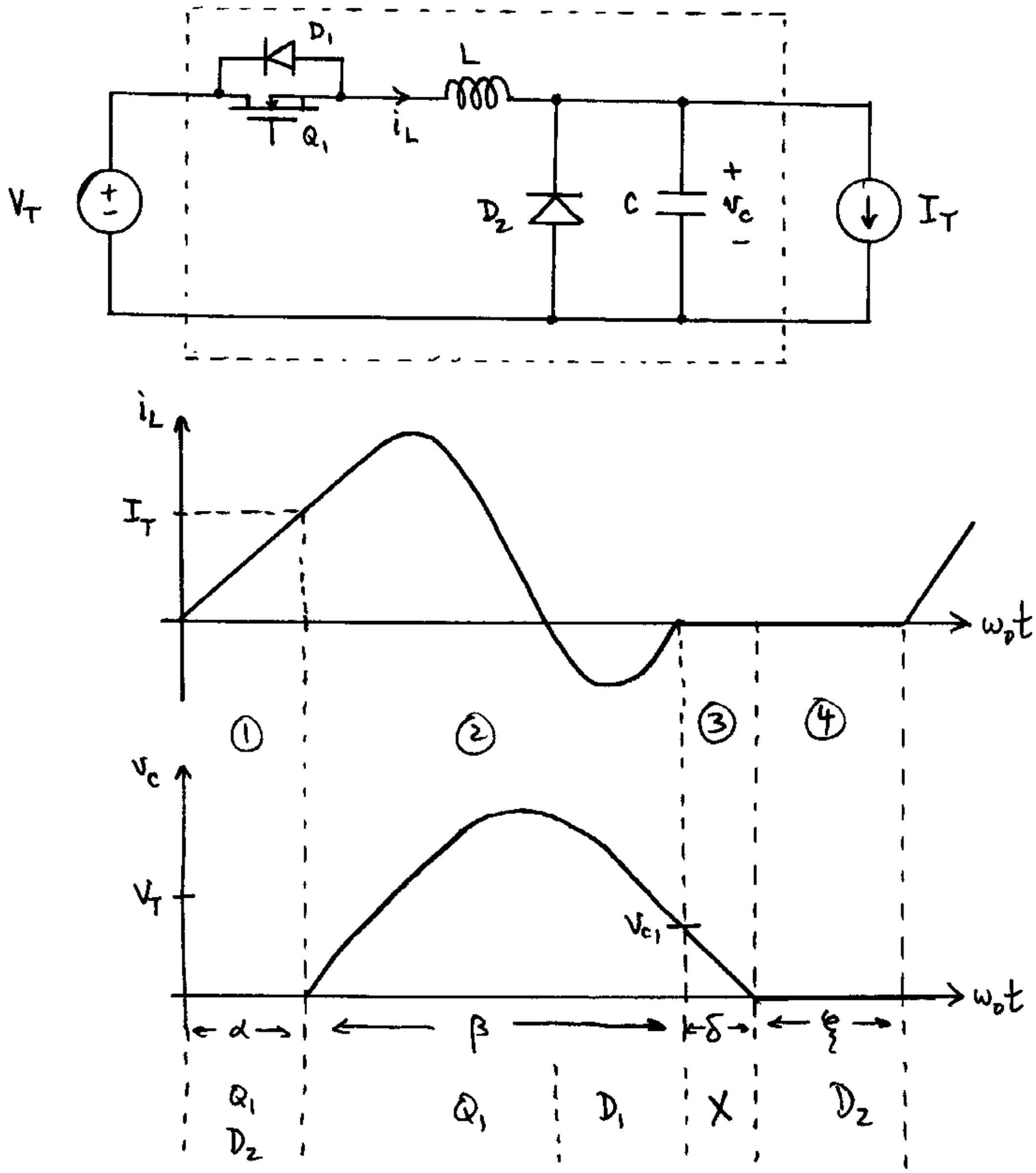
The complete output plane characteristics for the half-wave ZCS buck converter are given on the next page.

Output Plane of Buck Converter with Half-Wave Resonant Switch. Curves end near M=1 at mode boundary.

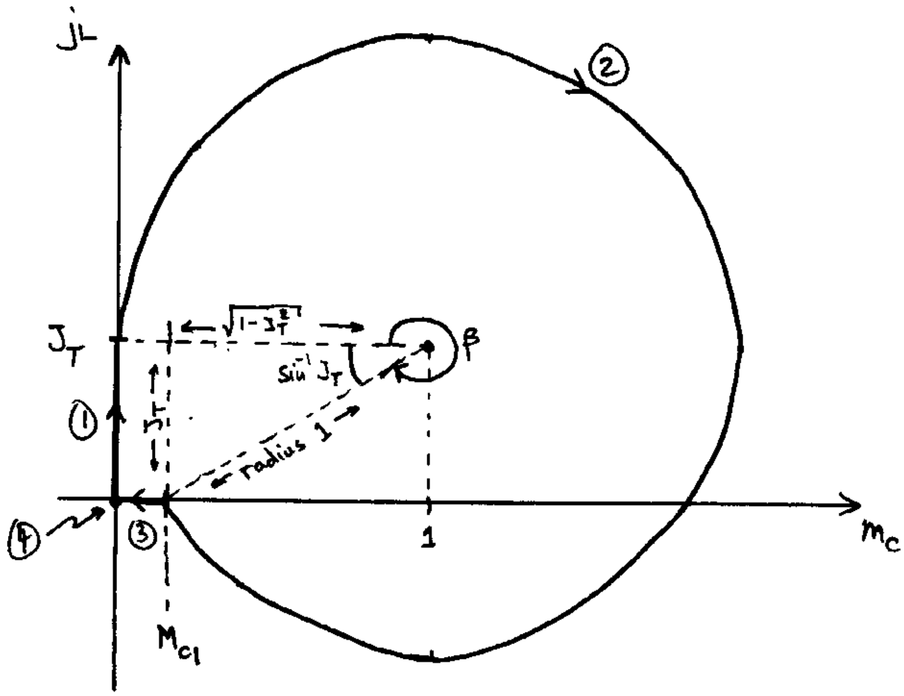


Full Wave "type a" zero current switch

Place D_1 antiparallel to Q_1, and turn off Q_1 at the second zero crossing of $i_L(t)$:



normalized state plane:



Analysis is nearly same as half-wave case, except for geometry during interval 2:

$$\beta = \begin{cases} \pi + \sin^{-1} J_T & \text{- half wave} \\ 2\pi - \sin^{-1} J_T & \text{- full wave} \end{cases}$$

$$M_{c1} = \begin{cases} 1 + \sqrt{1 - J_T^2} & \text{- half wave} \\ 1 - \sqrt{1 - J_T^2} & \text{- full wave} \end{cases}$$

In either case,

$$\langle m_c \rangle = F \cdot \frac{1}{2\pi} \left[\frac{1}{2}\alpha + \beta + \frac{M_{c1}}{J_T} \right]$$

Plug in full wave values;

$$\langle m_c \rangle = F \cdot \frac{1}{2\pi}\left[\frac{1}{2}J_T + 2\pi - \sin^{-1}J_T + \frac{1}{J_T}\left(1-\sqrt{1-J_T^2}\right)\right]$$

In general, the switch conversion ratio can be written in the form

$$\mu = \frac{\langle v_c \rangle}{V_T} = \langle m_c \rangle = F\, P(J_T)$$

However, the function $P(J_T)$ changes, depending on whether the full-wave, half-wave, or other mode is used.

<u>Summary: Switch Conversion Ratio results</u>

$$\mu = \frac{\langle v_c \rangle}{V_T} = \langle m_c \rangle = F \cdot P(J_T)$$

<u>half-wave</u>: $P(J_T) = k_{\frac{1}{2}}(J_T) \triangleq \frac{1}{2\pi}\left[\frac{1}{2}J_T + \pi + \sin^{-1}J_T + \frac{1}{J_T}\left(1+\sqrt{1-J_T^2}\right)\right]$

<u>full-wave</u>: $P(J_T) = k_1(J_T) \triangleq \frac{1}{2\pi}\left[\frac{1}{2}J_T + 2\pi - \sin^{-1}J_T + \frac{1}{J_T}\left(1-\sqrt{1-J_T^2}\right)\right]$

In the half-wave case, $P(J_T)$ is heavily dependent on J_T: $P \to \infty$ as $J_T \to 0$, and $P \to 1$ as $J_T \to 1$.

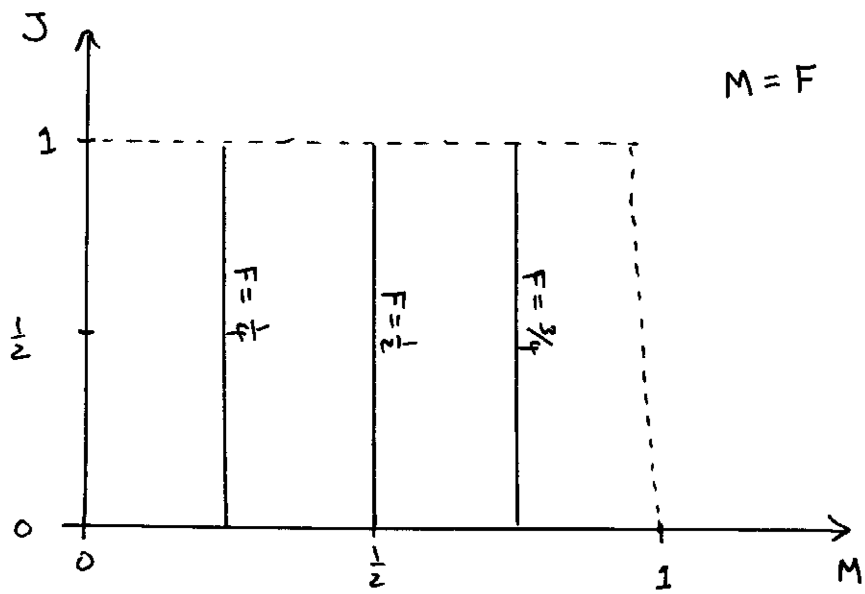
In the full-wave case, $P(J_T)$ is essentially independent of J_T: $P \approx 1$. The worst-case deviation occurs at $J_T \to 1$, where $P \to 0.96$. So P is within 4% of unity.

Hence, for the full wave case,

$$\mu \approx F = \frac{f_s}{f_0}$$

The switch then behaves as a voltage source, controllable by F. This is similar to the standard PWM case, with D replaced by F.

<u>Full-wave ZCS buck converter output characteristics:</u>



So the full-wave switch is well-behaved, and is relatively easy to control. For a voltage regulator application, the switching frequency typically varies over a much smaller range in full-wave mode than in half-wave mode.