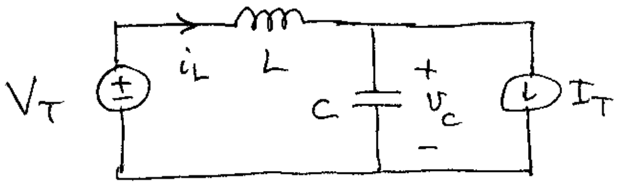
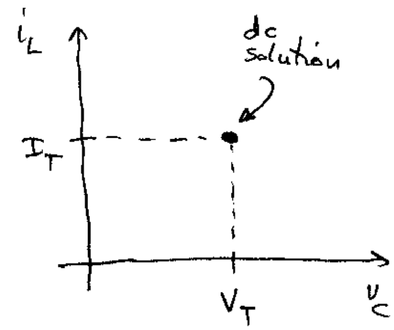
- State plane trajectories of the basic L-C resonant circuit
- Flyback converter example

The circuit studied so far:



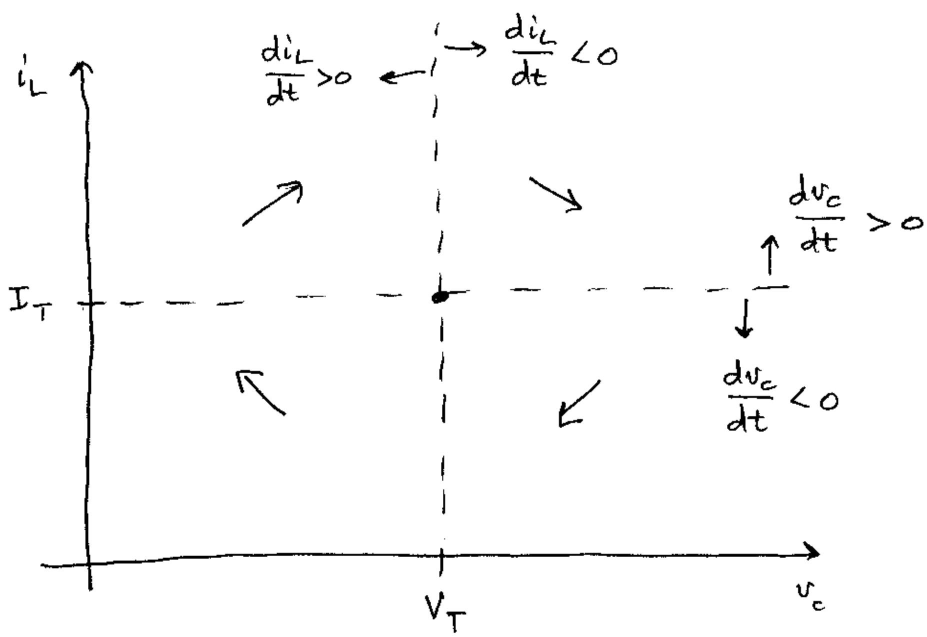
has dc solution $i_L = I_T$, $v_c = V_T$
(also called a center, or an equilibrium point)



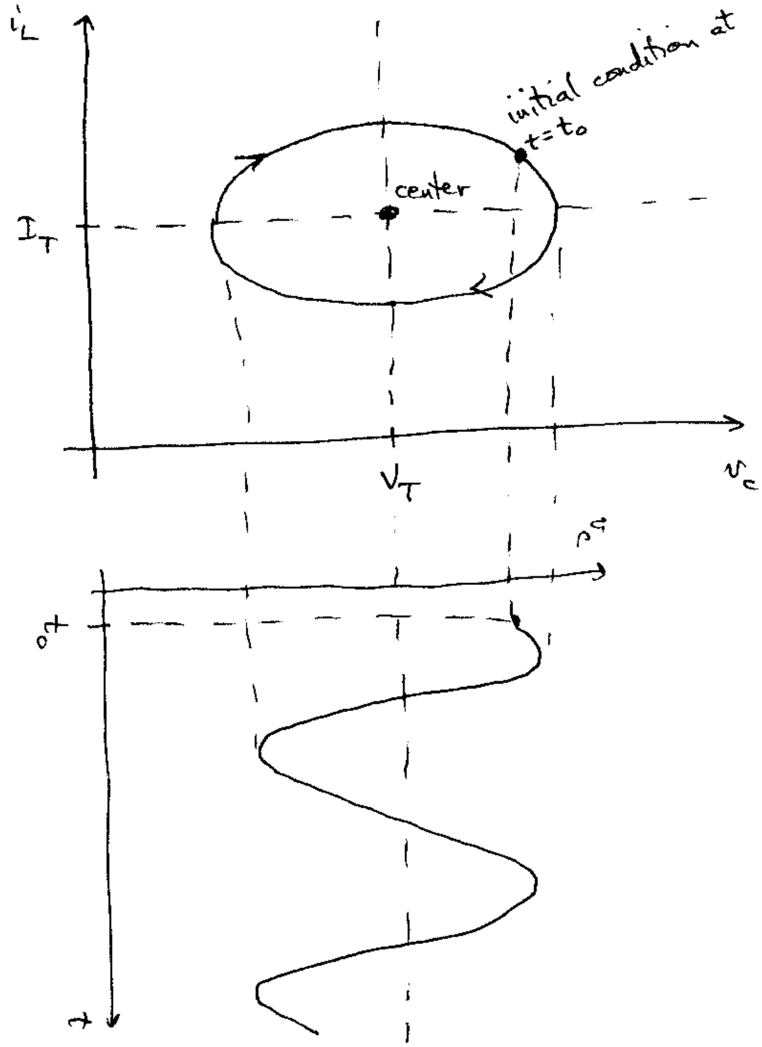
Circuit differential equations are

$$L \frac{di_L}{dt} = V_T - v_c$$

$$C \frac{dv_c}{dt} = i_L - I_T$$



The solutions follow ellipses that are centered on the dc solution. The size of the ellipse depends on the initial conditions



The eccentricity of the ellipse (ie, the relative sizes of the major and minor axes) depends on the scales chosen for the i_L and v_c axes and also on the characteristic impedance $R_0 = \sqrt{L/C}$.

The ellipse becomes a circle (that is easier to analyze) when we normalize the waveforms according to the next page.

Normalization of waveforms, and notation

Voltages: divide by $V_{base} = V_T$ (a somewhat arbitrary choice)

replace "v" by "m"

so $$m_c(t) = \frac{v_c(t)}{V_{base}} = \frac{v_c(t)}{V_T}$$

$$M_T = \frac{V_T}{V_{base}} = 1$$

Currents: divide by $I_{base} = \frac{V_{base}}{R_o} = \frac{V_T}{R_o}$

(use of R_o causes the state plane trajectories to become circular)

replace "i" by "j"

so $$j_L(t) = \frac{i_L(t)}{I_{base}} = \frac{i_L(t)\,R_o}{V_T}$$

$$J_T = \frac{I_T}{I_{base}} = \frac{I_T\,R_o}{V_T}$$

with $R_o = \sqrt{\frac{L}{C}} = \omega_o L = \frac{1}{\omega_o C}$, $\omega_o = \frac{1}{\sqrt{LC}}$

Note: in a transformer-isolated converter, it may be helpful to choose different base quantities on the two sides of the transformer, to account for how voltages and currents are altered by the turns ratio.

Time/angle correspondence: let $\theta = \omega_o t$, $t = \theta/\omega_o$

so $d\theta = \omega_o\,dt$

Angle around the circle is directly related to time

Normalization of the differential equations:

$$L \frac{di_L}{dt} = V_T - v_c \qquad \text{becomes}$$

$$L \frac{d}{dt}\left(j_L I_{base}\right) = V_{base}\left(1 - m_c\right)$$

$$\Rightarrow \quad \frac{L\, I_{base}}{V_{base}} \frac{dj_L}{dt} = 1 - m_c$$

note $$\frac{L\, I_{base}}{V_{base}} = L\left(\frac{V_T}{R_0}\right)\frac{1}{V_T} = \frac{L}{R_0} = \frac{L}{\omega_0 L} = \frac{1}{\omega_0}$$

so we get

$$\frac{1}{\omega_0}\frac{dj_L}{dt} = 1 - m_c \quad . \qquad \text{But } \omega_0 dt = d\theta, \text{ so}$$

$$\boxed{\frac{dj_L}{d\theta} = 1 - m_c}$$

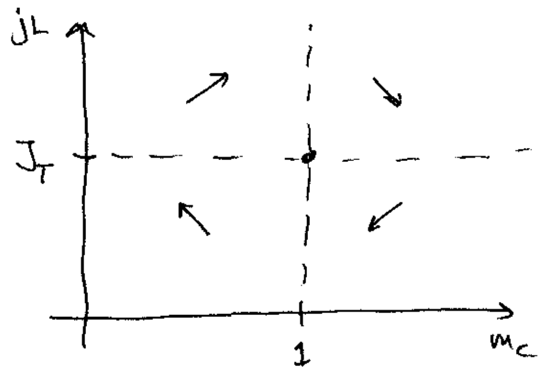
In a similar manner,

$$C \frac{dv_c}{dt} = i_L - I_T \qquad \text{becomes}$$

$$\boxed{\frac{dm_c}{d\theta} = j_L - J_T}$$

Exercise for the student: derive this one

Normalized state plane:



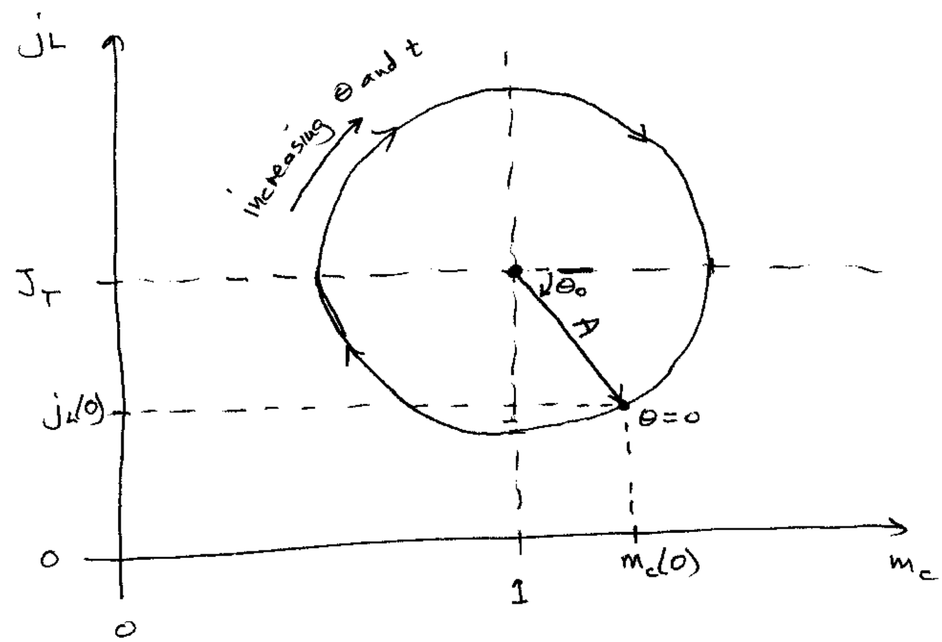
Solution is of the form

$$m_c(\theta) = 1 + A\cos(\theta + \theta_0)$$
$$j_L(\theta) = J_T - A\sin(\theta + \theta_0)$$

where A and θ_0 depend on the initial conditions.

Exercise for the student: plug these solutions into the differential equations of the previous page, to prove that the solution is valid.

The solution above describes a circle of radius A, centered at $(m_c, j_L) = (1, J_T)$:

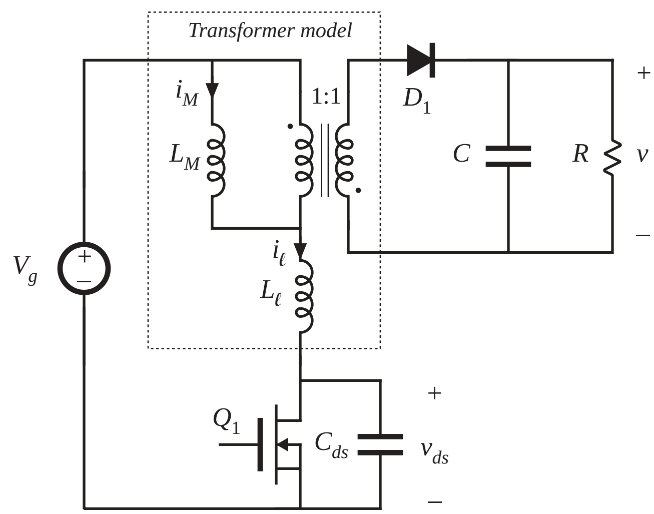


- As time increases, the solution progresses around the circle in the clockwise direction. The circle is called the "trajectory" of the solution
- Initial conditions determine the radius A
- Angle $\theta = \omega_0 t$
- Peak m_c is $1 + A$, and occurs at $\theta = 2\pi - \theta_0$
  $\Rightarrow$ peak $v_c = m_c V_{base} = (1+A)V_T$, occurs at $t = \frac{2\pi - \theta_0}{\omega_0}$
- Peak $j_L = J_T + A \Rightarrow$ peak $i_L = I_T + A\frac{V_T}{R_0}$

1. Transistor voltage spike in the flyback converter (30 points)

The flyback converter illustrated below is a conventional hard-switched PWM converter. The transformer is modeled as illustrated: the model contains an ideal 1:1 transformer, a magnetizing inductance L_M, and a leakage inductance L_ℓ. The MOSFET contains an output capacitance C_{ds} that can be modeled as a conventional linear capacitance having constant value C_{ds}. The converter operates in the continuous conduction mode with small ripples in the magnetizing current and output voltage. You may also assume that V and I_M are related to V_g, R, and D by the usual ideal CCM PWM converter equations, which neglect losses, ringing, and other nonidealities.

When the transistor turns off, the leakage inductance L_ℓ causes a voltage spike to be observed across Q_1. The voltage $v_{ds}(t)$ then exhibits overshoot and ringing.



(a) Sketch the waveforms of $v_{ds}(t)$ and $i_\ell(t)$, including the ringing that occurs when the transistor turns off. Label salient features.

(b) Sketch the state plane trajectory for the ringing interval.

(c) Derive an expression for the peak MOSFET voltage $v_{ds,pk}$. Express $v_{ds,pk}$ as a function of V_g, R, R_0, and D, where R_0 is the characteristic impedance of the tank circuit formed by L_ℓ and C_{ds}.

(continued on next page)

This converter is constructed with the following element values and specifications:

$150\text{ V} \le V_g \le 200\text{ V}$

$L_\ell = 10\ \mu\text{H}$

$C_{ds} = 500\text{ pF}$

Load power: $10\text{ W} \le P \le 100\text{ W}$

The controller adjusts D to regulate the output voltage to $V = 120$ V.

The converter always operates in the continuous conduction mode with small ripple in v and i_M.

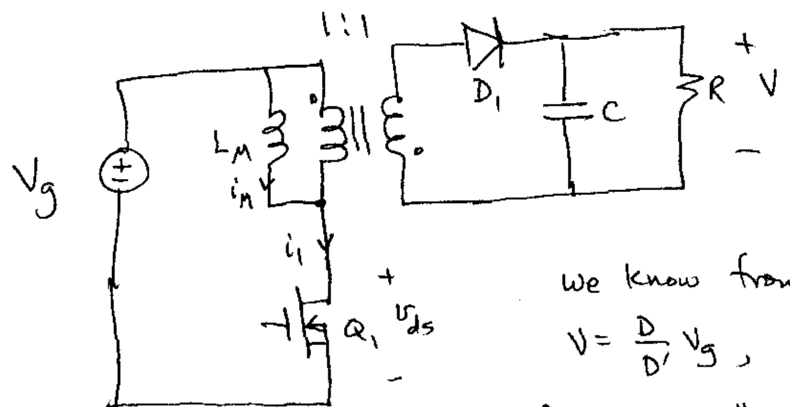
(d) Over what range with the numerical value of $v_{ds,pk}$ vary?

Flyback converter example

Given as midterm exam problem, Spring semester 2000

- Understanding what really happens during the MOSFET turn-off transition in the flyback converter
- How the leakage inductance induces switching loss
- Ringing and peak voltage stress
- Dependence on converter operating point
- Attempts to mitigate switching loss are usually unsuccessful unless they are based on detailed knowledge of the switching loss mechanisms

Ideal dc-dc flyback converter:



continuous conduction mode

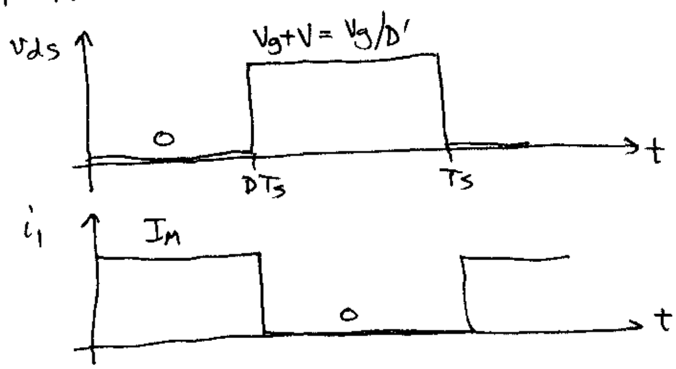
We know from Power Electronics I that

$$V = \frac{D}{D'} V_g, \quad I_M = \frac{V}{D'R} = \frac{D V_g}{D'^2 R}$$

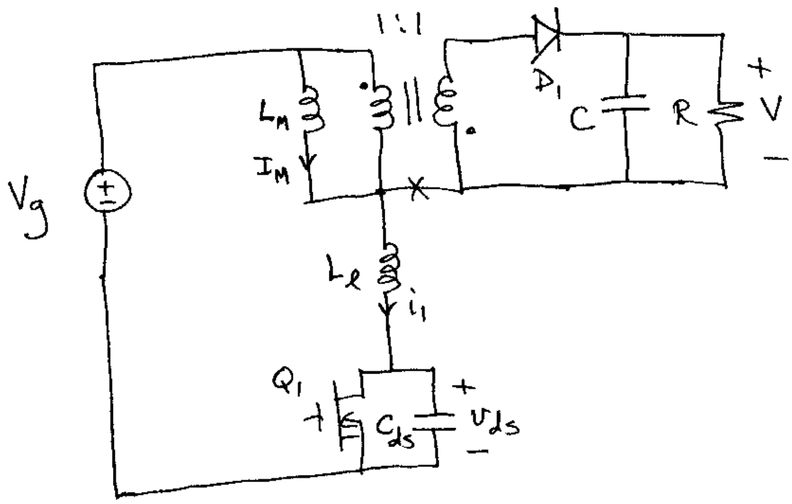
Assume small ripples in $i_m(t)$ and $v(t)$:

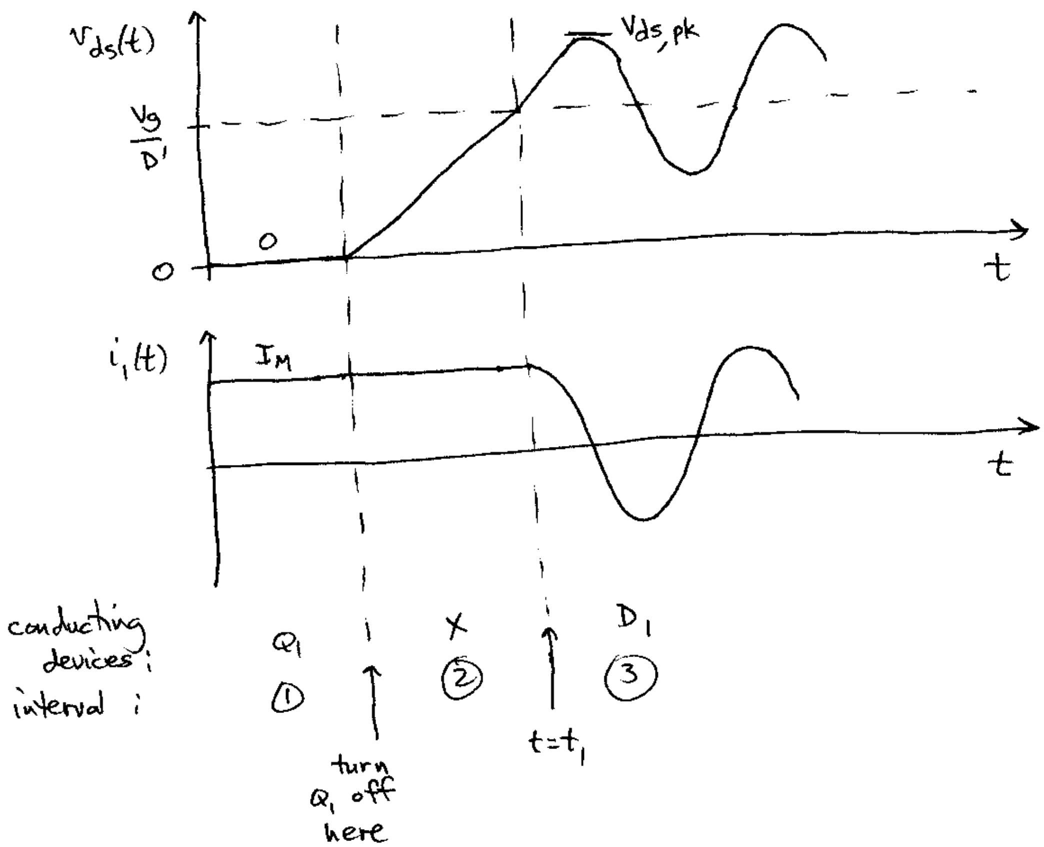
$i_m(t) \approx I_M, \; v(t) \approx V$

ideal MOSFET waveforms:

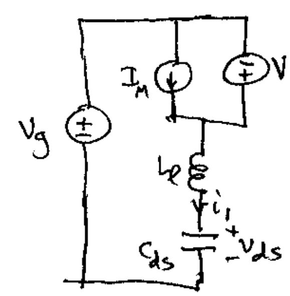


Nonideal flyback – transformer leakage inductance L_ℓ and MOSFET output capacitance C_{ds} cause ringing and switching loss at MOSFET turn-off transition





(magnified time scale, to show details of MOSFET turn-off transition)



Ringing interval is interval 3, where the circuit is

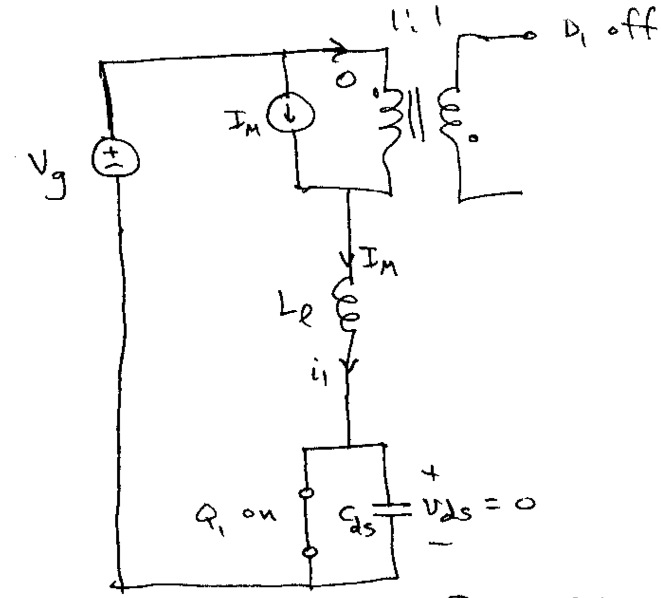
$\Rightarrow V_T = V_g + V, \quad I_T = 0.$ So define $V_{base} = V_g + V$

$$I_{base} = V_{base}/R_0$$

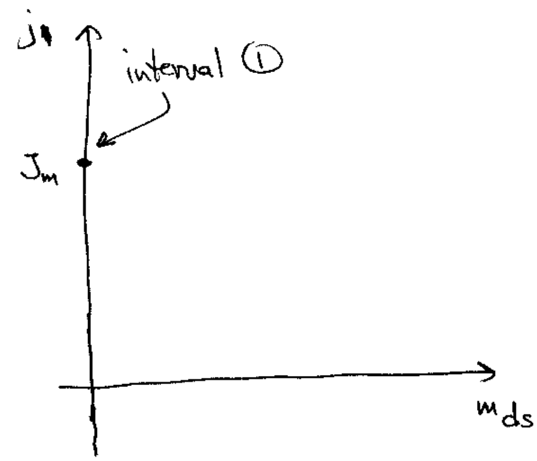
$$R_0 = \sqrt{L_\ell / C_{ds}}$$

Interval ①: Q_1 on, D_1 off

$i_M \cong I_M$, $v \cong V$



normalized state plane: $m_{ds} = \dfrac{v_{ds}}{V_{base}}$, $j_1 = i_1 / I_{base}$



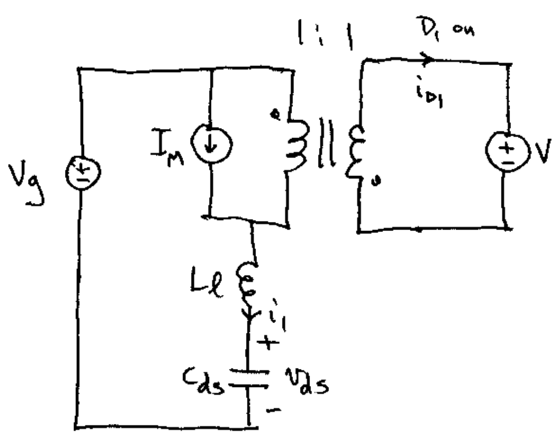
$$m_{ds} = 0,\quad j_1 = J_M = \frac{I_M}{I_{base}} = \frac{DV_g}{D'^2 R\,(V+V_g)}\,R_0$$

Interval ②: Q_1 is off. Does D_1 turn on yet?

At the beginning of interval ②, $v_{ds} = 0$ and $i_1 = I_M$.

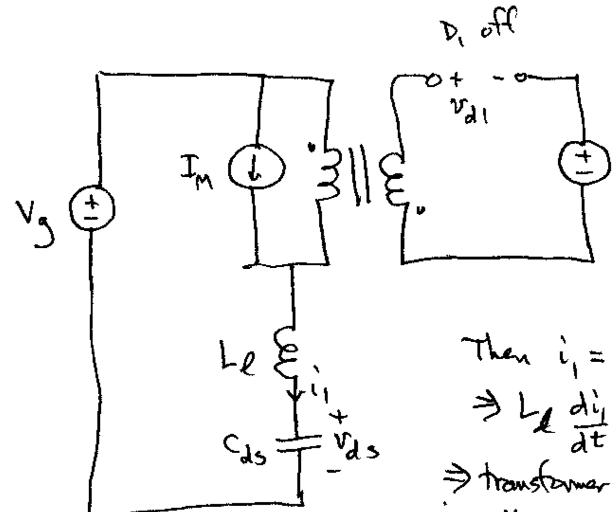
There are two possibilities:

D_1 turns on



Then $i_{D1} = I_M - i_1$, which must be positive if the diode is forward-biased. And a constant voltage of $V_T = V_g + V$ is applied across the tank.

D_1 doesn't turn on



Then $i_1 = I_M$

$\Rightarrow L_\ell \dfrac{di_1}{dt} = 0$

$\Rightarrow$ transformer voltage is $V_g - v_{ds}$

$\Rightarrow v_{d1} = -V - V_g + v_{ds}$

Diode remains reverse-biased as long as $v_{ds} < V + V_g$

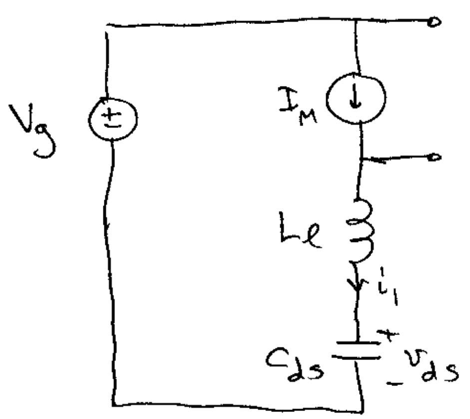
Correct answer is — diode is reverse-biased, and remains off as long as $v_{ds} < V + V_g = \frac{V_g}{D'}$.

The left-hand circuit, for the hypothetical case when the diode turns on, doesn't work because it predicts negative current through the diode:

$i_{d1} = I_M - i_1$ with i_1 initially equal to I_M, so i_{d1} is initially zero. The circuit predicts that i_1 increases, since $L_\ell \frac{di_1}{dt} = V_g + V - v_{ds}$ in this hypothetical circuit, with v_{ds} initially equal to zero.

If i_1 increases, then $i_{d1} = I_M - i_1$ decreases and becomes negative. But negative i_{d1} would reverse-bias the diode. So D_1 can't turn on yet.

So for interval 2, we have

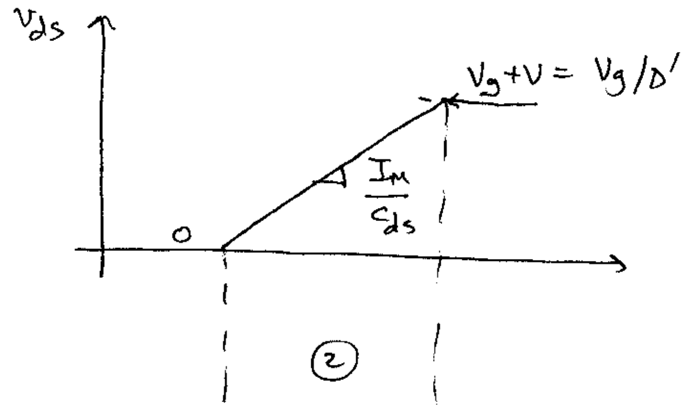


D_1 off
$\Rightarrow$ transformer is open-circuited

$$i_1 = I_M = \text{constant}$$

$$C_{ds} \frac{dv_{ds}}{dt} = i_1 = I_M$$

$\Rightarrow v_{ds}$ increases with slope $\frac{I_M}{C_{ds}}$

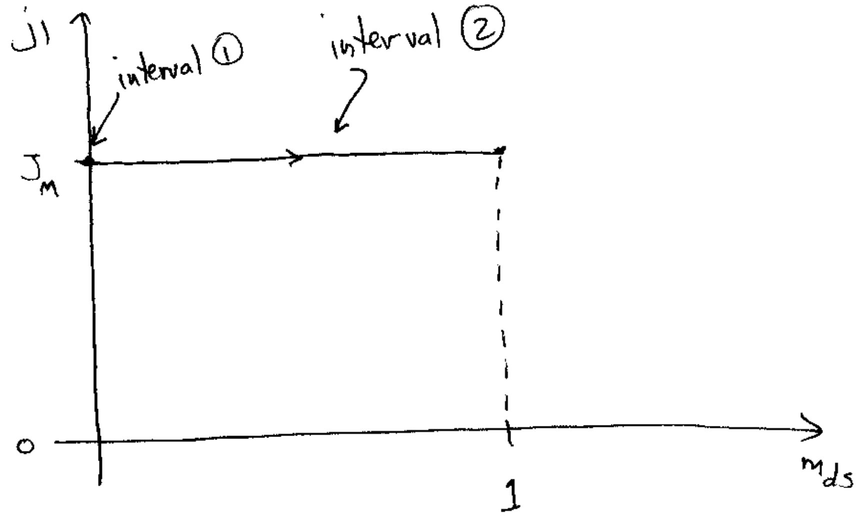


at end of interval 2:

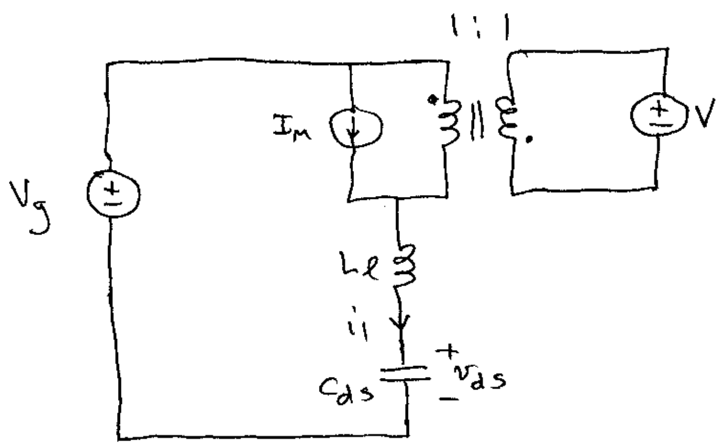
$v_{ds} = V_g + V$

$m_{ds} = \frac{V_g + V}{V_{base}} = 1$

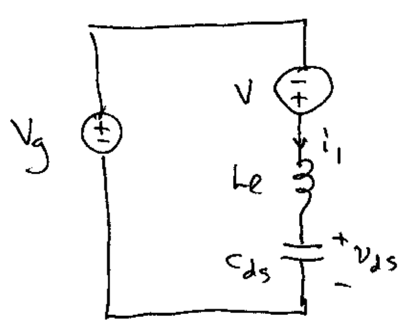
State plane



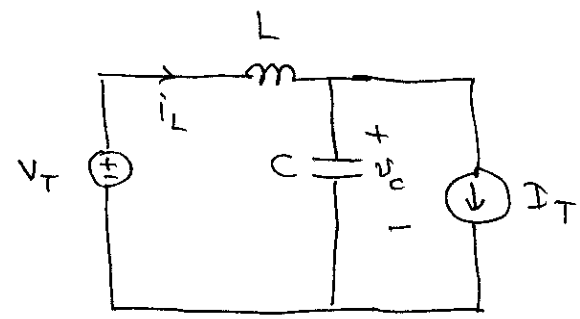
<u>Interval ③</u> D_1 becomes forward-biased, and circuit becomes



Note that the I_M and V sources are effectively in parallel, and the I_M source has no effect.



which is of the form →



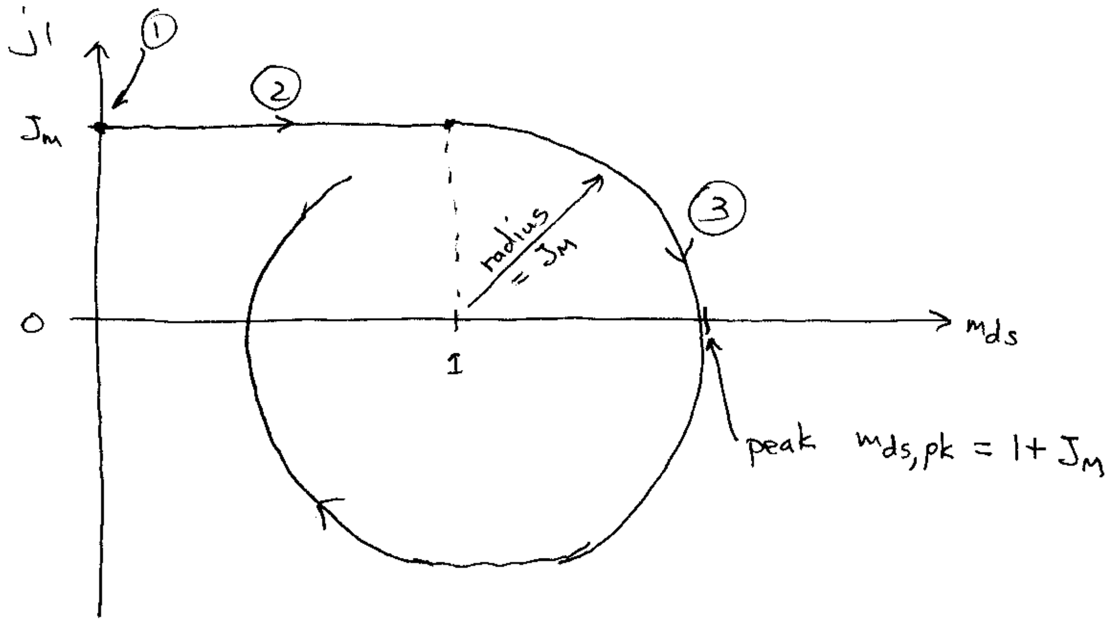
with $V_T = V_g + V$, $I_T = 0$

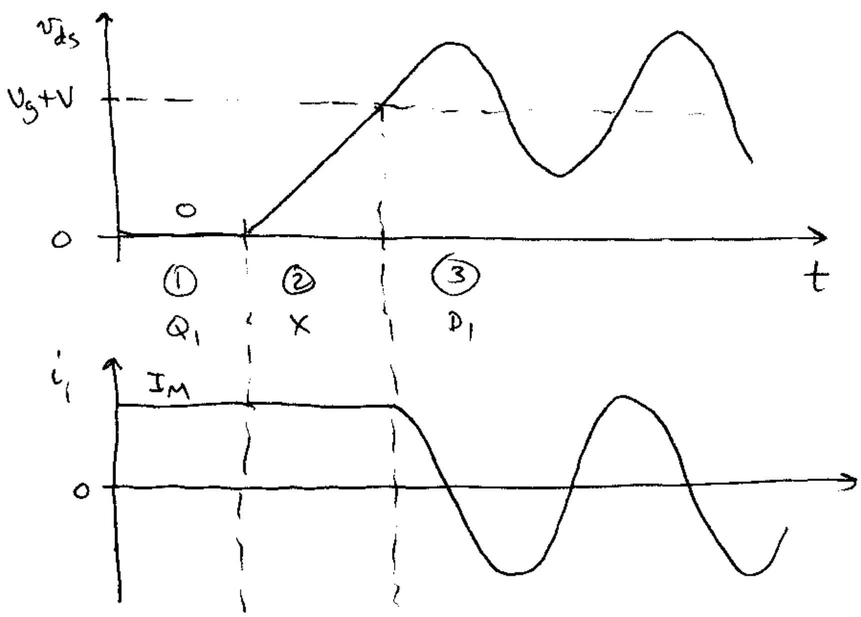
$i_L = i_1$, $v_c = v_{ds}$

Use previous result:

State plane trajectory is circle centered at

$$m_{ds} = 1, \quad j_1 = 0$$





Peak transistor voltage $v_{ds,pk} = V_{base}(1 + J_M) = (V + V_g)\left(1 + \frac{DV_g}{D'^2 R}\frac{R_0}{V + V_g}\right)$

$$= V + V_g + \frac{D}{D'^2}\frac{R_0}{R}V_g$$

The above result predicts that the parasitic ringing will last forever. The reason for this is that no losses have been modeled, and so the circuit is undamped.

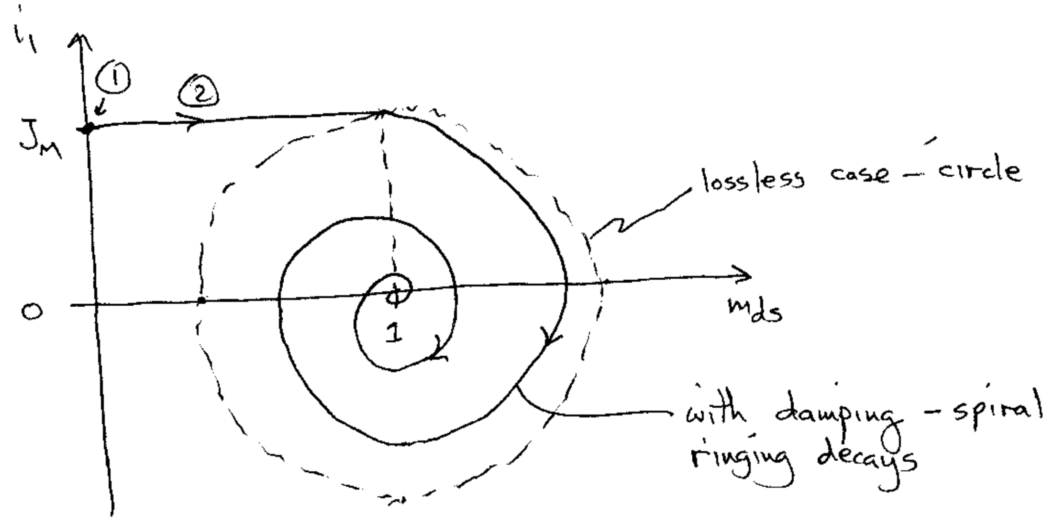
The above result also shows that energy is trapped in the tank elements, which rings "forever" and is not transferred to the load during the diode conduction interval 3.

The leakage inductance L_ℓ initially stores energy

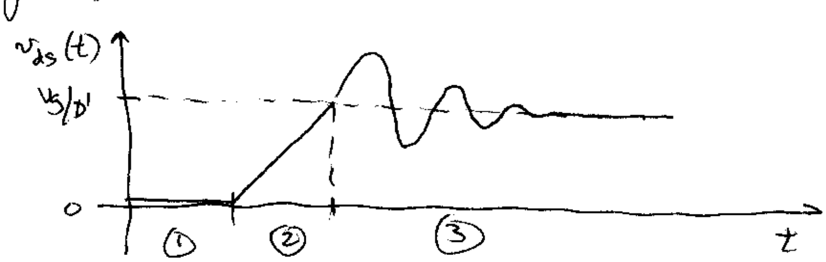
$$\frac{1}{2} L_\ell I_M^2$$

During interval 3, this energy circulates between the tank elements L_ℓ and C_{ds}, causing the ringing.

In practice, the losses are not zero, and there is at least a little bit of damping. This damping causes the radius of the state plane trajectory to decrease over time, so that it spirals in towards the center:



corresponding v_{ds} waveform:

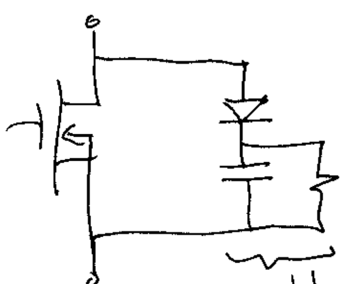


whatever parasitic loss elements damp the ringing will dissipate the energy, resulting in an average power loss of

$$\underbrace{\left(\tfrac{1}{2} L_\ell I_M^2\right)}_{\text{energy lost per switching period}} \underbrace{(f_s)}_{\text{switching frequency}}$$

Sometimes, people attempt to recover and recycle this energy, through a "soft switching" or "lossless snubber" circuit. Such a circuit must capture the energy of the switching before it has been substantially damped, and then transfer it to the source or load. Of course, these circuits must operate with an efficiency less than one.

A conventional dissipative snubber such as



will capture the energy and dissipate it in the resistor. Additional energy must usually also be dissipated, so that the resistor power loss is greater than or equal to the average power loss above.