Additional Homework Problems for ECEN 6144

1-13 Certain resonant materials (so-called Lorentz media) exhibit a dielectric susceptibility expressible in the frequency domain as

\[ \hat{\chi}_e(\omega) = \frac{\chi_{e0}\omega_r^2}{\omega_r^2 - \omega^2 + 2j\omega\Gamma} \]

where \( \chi_{e0} \) is the susceptibility at zero frequency (DC), \( \omega_r \) is the resonant frequency and \( \Gamma \) is a constant related to the rate of absorption loss—all three of these constants assumed real and \( > 0 \). Obtain expressions for the time-domain susceptibility function \( \chi_e(t) \), for the case \( \omega_r > \Gamma \) and for the case \( \omega_r < \Gamma \). Comment on the meaning of your result in each case. [Hint: a result analogous to that of Example 13 of Appendix A may prove useful.]

1-14 A thin layer of material lies between \( z = -h/2 \) and \( z = +h/2 \), with permittivity given by some function \( \epsilon(z) \) and permeability given by another function \( \mu(z) \), both functions of \( z \) but not \( x \) or \( y \). Show that an approximate boundary condition for this thin layer is given by (1.138), but that now the surface susceptibilities are given by expressions containing certain integrals of \( \epsilon(z) \) and \( \mu(z) \), and give derivations for these expressions. Paul Drude investigated reflection and transmission from such a transition layer in the late 1800s (see his book *The Theory of Optics*, New York, Dover, 1959, pp. 287-295), but the formulation of equivalent boundary conditions for the layer seems never to have been published.

5-19 A uniform, time-harmonic line source of \( z \)-directed electric current \( I_0 \) is located at the coordinates \((\rho_0, \phi_0)\) in the presence of the PEC cylinder shown in Fig. 5.5 \((\rho_0 > a)\). Construct the solution for the scattered field \( E_s^z \) in a form analogous to that of eqn. (5.53). Obtain a far-field expression for the scattered field analogous to (5.54), and find the limiting form of the radiation pattern \( F(\phi) \) for the case \( ka \ll 1 \). Do not make any assumption about the size of \( k\rho_0 \).

5-20 A dielectric sphere of radius \( a \) and permittivity \( \epsilon \) is placed into a uniform, electrostatic incident electric field \( E^i = u_z E^i_z \) in free space. Use spherical harmonic expansions to obtain expressions for the scattered electrostatic potential \( \Phi^s \), and thereby the scattered electrostatic field \( E^s \), produced both inside and outside this object. From this result, find the induced electric dipole moment \( p = \int_{\text{sphere}} \mathcal{P} \, dV \) of the sphere.
6-7 Repeat problem 6-1, but replace the line current by a line charge $\rho_l$ C/m, and find the electrostatic field $E$ instead of $B$. 
7-13 Let the incident wave in problem 7-11 be a parallel-polarized plane wave incident from the upper half-space:

\[ \mathbf{E}^i = E_0 (u_x \cos \theta + u_z \sin \theta) e^{-jk(x \sin \theta - z \cos \theta)} \]

Determine the short-circuit current density \( J_{sc}^S \) at the location of the slot, and find an expression for the voltage \( V_{slot} = \int_{-a}^{a} E_x \, dx \) across the slot due to this incident wave.

A-12 Obtain an expression for \( \delta(\sin x) \) as a weighted sum of terms proportional to \( \delta(x - a_i) \), where the \( a_i \) are real constants.

A-13 Let

\[ g(x) = \frac{a}{\sqrt{\pi}} e^{-a^2 x^2} \]

where \( a \) is a real, positive constant.

(a) Obtain an expression for the \( n \)th order moments \( g_n \) of \( g(x) \) that appear in equation (A.19).

(b) Use the result of part (a) to show that

\[ \lim_{a \to \infty} g(x) \approx \delta(x) \]

in the sense of generalized functions.