Homework #4 [100 pts]

Due: Thursday, November 5, 2015

1. **Null polarimetry [20 pts]**
   Elliptically polarized light of unknown orientation and ellipticity is passed through a quarter wave plate and a linear polarizer, and a transmission null is searched for by arbitrarily rotating both the \( \lambda/4 \) plate and polarizer. The null is found when the \( \lambda/4 \) plate is orientated 30\(^\circ\) from \( x \) and when the linear analyzer is oriented at 60\(^\circ\) from \( x \). What is the orientation of the ellipse and the ellipticity of the incident light? Take the fast axis of the \( \lambda/4 \) plate to be along the \( x \) direction. The pass axis of the polarizer is at 60 degrees relative to \( x \).

2. **Opotisolator [20 pts]**
   Consider a system consisting of a perfect horizontal transmission polarizer, followed by a nominal quarter wave plate with its fast axis oriented at +45 degrees, followed by a retroreflecting mirror.
   (a) Assuming that the waveplate is exactly quarter wave at the operating wavelength, what is the reflection backwards through the polarizer? Why?
   (b) If the waveplate has a retardance that deviates away from \( \delta = 90 \) degrees by a small deviation \( \Delta \), eg \( \delta = 90 + \Delta \), solve approximately for the reflected power as a function of \( \Delta \) for small \( \Delta \).

3. **Jones to Orientation [20 pts]**
   Relate the ratio of the \( E_x \) and \( E_y \) components of a wave and their relative phase difference \( \delta \) to the orientation of the major axis of the polarization ellipse \( \Psi \) and the ellipticity \( \chi \) given by the arctan of the ratio of the major to the minor axis.

4. **Circular basis polarization decomposition [20 pts]**
   Show how to produce an arbitrary state of elliptical polarization from the superposition of weighted and phase delayed orthogonal circular polarizations. Illustrate graphically for an ellipse with an ellipticity angle \( \chi = 30^\circ \) and orientation of 30\(^\circ\) from \( x \) by drawing the ellipse and circular decomposition. Also show the construction on the Poincare sphere.

5. **Polychromatic Mueller calculus**
   For a polychromatic wave where each component has the same Stokes vector, the intensity linearity of the Stokes vectors indicates that the Mueller matrix should be represented by an integration of source spectrum times the spectrally dependent Mueller matrices.
   \[
   M = \int I(\nu)M(\nu)d\nu
   \]
   The Mueller matrix for a retarder is often wavelength dependent in proportion to the deviation from the design wavelength \( \lambda_o \) by
\[ \phi = \frac{\lambda_o}{\lambda} \phi_o = \frac{v_o}{v} \phi_o. \] For 45 degree polarized light passing through a quarter wave retarder at \( v_o \), find the output polarization and degree of polarization for the following spectral distributions:

(a) \( I(v) = \delta(v - v_o) \)

(b) \( I(v) = \frac{1}{v_o} \text{rect} \left( \frac{v - v_o}{v_o} \right) \) where \( \text{rect}(x) = 1 \) for \(-0.5 < x < 0.5\) and 0 otherwise.