Conoscopic interferometer: optically anisotropic specimen placed between two crossed polarizers

- Conoscopic interference patterns used to study crystals, identify minerals, and explore structure of biological specimens.

- Focusing light beam probes angular variation of crystal.
- Input light decomposed into eigenmodes (depends on direction of incident light), which gain a phase shift of $\delta$ in crystal.
- Eigenmodes depend on incident angle and recombine at output polarizer, producing interference pattern.
- Eigenmodes separated by angle $\Psi$.

\[ \Psi \]
To solve

Project input polarization into eigenmodes of propagation which vary with propagation angle (uniaxial crystal)

\[ \vec{E}_{in} = [\vec{y} \cdot \hat{\sigma}(\theta, \phi)] \hat{\sigma}(\theta, \phi) + \\
[\vec{y} \cdot \tilde{\sigma}(\theta, \phi)] \tilde{\sigma}(\theta, \phi) \]

= extraordinary component delayed by \( S \)

\[ \vec{E}_{out} = [\vec{y} \cdot \hat{\sigma}(\theta, \phi)] \hat{\sigma}(\theta, \phi) e^{i \phi(\theta, \phi)} \\
+ [\vec{y} \cdot \tilde{\sigma}(\theta, \phi)] \tilde{\sigma}(\theta, \phi) \]

> Output passed through horizontal polarizer (input is vertically polarized)

\[ \vec{E}_{final} = [\vec{y} \cdot \hat{\sigma}(\theta, \phi)] [\hat{\sigma}(\theta, \phi) \cdot \hat{x}] e^{i \phi(\theta, \phi)} \\
+ [\vec{y} \cdot \tilde{\sigma}(\theta, \phi)] [\tilde{\sigma}(\theta, \phi) \cdot \hat{x}] \]

Solution:

\[ I = |E_0 \cos \Psi \sin \Psi - E_0 \sin \Psi \cos \Psi e^{i \phi}|^2 \]

\[ I = I_0 \sin^2 2\Psi \sin^2 \Psi \frac{1}{2} \]

Isogyres dark fringes where 2 eigenmodes coincide with transmission axes of polarizers. Isonochromatics bright fringes where the optical path length matches the wavelength.

Conoscopic setup is sensitive to phase difference as a function of angle. See 2 slips (cross pattern modulated by Fresnel zone plate fringes).

\[ \sin \theta = n_0 \sin \theta_0 \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \left[ d(\theta_0) n_0(\theta_0) \right. \]

\[ - d(\theta_0) n_0(\theta_0) \]

\[ \Delta \phi = (R_0 - R_e) \cdot \hat{n} \cdot L \]

For small \( \theta \)

\[ \Delta \phi \propto \theta^2 \rightarrow \text{Fresnel zone plate rings} \]

Explanations:

\[ d_0 = L/\cos \theta_0 \]

\[ d_e = L/\cos \theta_e \]

\[ t = x \sin \theta = L (\tan \theta_e - \tan \theta_0) \sin \theta \]

\[ \Delta \phi = \frac{2\pi}{\lambda} \left[ t \cdot l - n_0 d_0 - n_e(\theta) d_e \right] = \frac{2\pi}{\lambda} L \left( \frac{n_0 \cos \theta_0 - n_e(\theta) \cos \theta_e}{n_e(\theta)} \right) \]
\[ \Delta \phi = \frac{2\pi}{\lambda} \left[ n_0 \cos \theta_0 - n_e \cos \theta \right] \]

\[ \phi = \sin^{-1} \left( \frac{\sin \theta}{n_e} \right) \quad \theta_0 = \sin^{-1} \left( \frac{\sin \theta_0}{n_e} \right) \]

Expand for small \( \theta \):

\[ \Delta \phi \approx b \theta^2 = \frac{2\pi L}{\lambda} \frac{(n_0 - n_e)(n_0 + n_e)}{2n_0 n_e^2} \theta^2 \]

Calculate:

\[ n_0 = 1.658 \]
\[ n_e = 1.486 \]
\[ \lambda = 589 \text{ nm} \]
\[ L = 1 \text{ cm} \]

\[ \Delta \phi \approx 2.4 \theta^2 \text{ deg} \]

**Dark rings**

\[ \frac{\Delta \phi}{2\pi} = m \quad \Theta_{\text{deg}} = \sqrt{\frac{2\pi m}{2\pi}} = 0, 1.62, 2.28, 2.8, \ldots \]

**Bright rings**

\[ \Delta \phi = (2m+1) \frac{\pi}{2} \]

\[ \Theta_{\text{deg}}^{\text{bright}} = \sqrt{\frac{2\pi (m+\frac{1}{2})}{2\pi}} = 1.44, 1.98, 2.55, 3.02, \ldots \]

**LiNbO\(_3\)**

\[ \lambda = 632.8 \text{ nm} \]
\[ n_0 = 2.288 \]
\[ n_e = 2.2 \]
\[ L = 1.2 \text{ cm} \]

\[ b = 0.635 \Theta_{\text{deg}}^{\text{dark}} = 0, 3.14, 4.45, \ldots \]
Polarizing elements using anisotropic media

Ref: Longhurst, Propagation of light in crystals

Nicol prism produce single plane polarized beam of light

exploit double refraction in calcite \( (n_o = 1.656) \) \( (n_e = 1.49) \)

Principal section

Calcite crystal is cut on diagonal
- two halves cemented together with Canada balsam (intermediate in between 0 & E waves of calcite) \( \Rightarrow \) Ne of Canada balsam = 1.55
- E wave transmitted, O wave is totally internally reflected & absorbed by black coating on edge of prism
- O & E wave only separated for small range of entrance angles

- can use 2 in series
  1st - polarize light
  2nd - analyzer