Reflected & Transmitted amplitudes

S polarization (TE)

- If \( u_1 = u_2 \)
  \[ B^t \cos \Theta_0 = B^i \cos \Theta_i - B^r \cos \Theta_r \]

- Use Snell's Law \( \Rightarrow \Theta_i = \Theta_r \)

\[ B^i = E^i \frac{n_1}{c} \quad B^r = E^r \frac{n_1}{c} \quad B^t = E^t \frac{n_2}{c} \]

\[ \cos \Theta_i \frac{n_1}{c} (E^i + E^r) = \frac{n_2}{c} E^t \cos \Theta_t \]

amplitude of incident field + amplitude of reflected field + amplitude of transmitted field

If medium is lossless

TE: \( E^i + E^r = E^t \) (from Boundary conditions)

Solving for ratios of reflected & transmitted fields yields:

\[ r_s = r_s = r_i = \frac{E^r}{E^i} = \frac{n_1 \cos \Theta_i - n_2 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_t} \]

\[ t_s = t_s = t_t = \frac{E^t}{E^i} = \frac{2n_1 \cos \Theta_i}{n_1 \cos \Theta_i + n_2 \cos \Theta_t} \]

OR alternate form

\[ r_s = -\frac{\sin (\Theta_i - \Theta_t)}{\sin (\Theta_i + \Theta_t)} \quad t_s = +\frac{2 \sin \Theta_t \cos \Theta_i}{\sin (\Theta_i + \Theta_t)} \]

\[ BC: E^i + E^r = E^t \quad H^i \cos \Theta_i - H^r \cos \Theta_r = H^t \cos \Theta_t \]
Alternate expressions (in terms of impedance)

From Electromagnetic Waves
Staelin/Morgenthaler/Kong

**TE waves**

\[ \Gamma_{TE} = \frac{Z_{n,TE} - 1}{Z_{n,TE} + 1} \]

\[ T_{TE} = \frac{2 Z_{n,TE}}{Z_{n} + Z_{n,TE}} \]

\[ Z_{n,TE} = \text{normalized wave impedance} = \frac{U_t / k_t z}{u_i / k_i z} \]

\[ k_{i z} = w \sqrt{u_i e_i} \cos \Theta_i \]

\[ k_{t z} = \sqrt{k_t^2 - k_x^2} = \sqrt{w^2 u_t e_t - k_i^2 \sin^2 \Theta_i} \]

**TM waves**

\[ \Gamma_{TM} = -\left( \frac{Y_{n,TM} - 1}{Y_{n,TM} + 1} \right) = \frac{E_{t,TM}}{E_{i,TM}} \]

\[ T_{TM} = \frac{2}{Y_{n,TM} + 1} = \frac{E_{t,TM}}{E_{i,TM}} \]

\[ Y_{n} = \frac{E_{t} / k_{t z}}{E_{i} / k_{i z}} \]
Deriving $\mathbf{B}$ from $\mathbf{E}$

$\nabla \times \mathbf{E} = -j \omega \mathbf{B}$

For plane wave, this simplifies to

$\mathbf{k} \times \mathbf{E} = -\omega \mathbf{B}$

\begin{align*}
\begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
-k_z & k_y & k_z \\
0 & E_y & 0 \\
\end{vmatrix}
\end{align*}

$\mathbf{k} \times \mathbf{E} = \hat{x} (-k_z E_y) + \hat{y} (0 + \hat{z} k_x E_y) = -E_y (\hat{x} k_z + \hat{z} k_x)$

$k_x = k \sin \Theta$  \quad $k = \omega \sqrt{\mu} = \frac{\omega \eta}{c}$

$k_z = k \cos \Theta$

$\mathbf{k} \times \mathbf{E} = -E_y \frac{\omega \eta}{c} (\hat{x} \sin \Theta + \hat{z} \cos \Theta)$

$\mathbf{B} = E_y \frac{n}{c} (\hat{x} \sin \Theta + \hat{z} \cos \Theta)$

$|\mathbf{B}| = E_y \frac{n}{c} \sqrt{\sin^2 \Theta + \cos^2 \Theta} = E_y \frac{n}{c}$
Reflected & Transmitted Waves

(P polarization) (nonmagnetic \( n_1 = n_2 \))

\[
\begin{align*}
\Gamma_p &= \pm \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i} \\
\Gamma_p &= \frac{R_p}{\tan (\theta_i - \theta_t)} \\
T_p &= \frac{R_p^2}{\tan (\theta_i + \theta_t)} \\
T_p &= \frac{t_{II}}{E_t} = (1 - R_p) \frac{\cos \theta_i}{\cos \theta_t}
\end{align*}
\]

Boundary conditions

\[\begin{align*}
&\text{tangential} \\
E_t &\cos \theta_t = E_r \cos \theta_r = E_t \cos \theta_t \\
&\text{continuity} \\
&H_t + H_r = H_t
\end{align*}\]

Conservation of Power

\[1 + \Gamma_p = \frac{t_{II}}{E_t} + \frac{n_2}{n_1} \frac{t_{II}}{E_t} = 1 + \Gamma_p \frac{n_2}{n_1} \]

Intensity \[I = \frac{n \varepsilon_0 c}{2} |E|^2, \text{ energy flux across boundary scaled by beam sectional area} \]

reflectance \[R = \frac{I_r}{I_i} = \frac{n_1 \varepsilon_0 c}{2} |E_r|^2 = \left| R \right|^2 \frac{\cos \theta_t}{\cos \theta_i} \]

transmittance \[T = \frac{I_t \cos \theta_t}{I_i \cos \theta_i} = \frac{n_2 \varepsilon_0 c}{2} |E_t|^2 \]

\[T = \frac{n_2 \cos \theta_t}{n_1 \cos \theta_i} \left| \frac{1}{E_t} \right|^2 \]

\[R + T = 1\]
Energy incident, reflected & transmitted per unit area given by component of Poynting vector normal to surface:

\[ Q_i = S_i \cos \theta_i = \frac{n_1}{2n_0} |A_i|^2 \cos \theta_i \]

\[ Q_r = S_r \cos \theta_r = \frac{n_1}{2n_0} |A_r|^2 \cos \theta_r \]

\[ Q_t = S_t \cos \theta_t = \frac{n_2}{2n_0} |A_t|^2 \cos \theta_t \]

**Brewster's Angle**

Look for zero reflection cases, w/ \( \theta_r = \theta_i = \theta_t \)

TE: \( k_1 \cos \theta_i = k_2 \cos \theta_t \)

w/ Snell's Law

\( \tan \theta_i = \tan \theta_t \)

\( \Theta_i = \Theta_t \Rightarrow E_t = E_i \) (not interesting)
Tm waves (p polarization)

\[ k_i \cos \Theta_i = k_t \cos \Theta_t \]

Thus, Snell's Law yields

\[ \Theta_i + \Theta_t = \pi/2 \]

\[ \tan \Theta_i = \tan \Theta_B = \sqrt{\varepsilon_t / \varepsilon_i} \]

\[ \Theta_B = \text{Brewster's angle} = \tan^{-1} \sqrt{\varepsilon_t / \varepsilon_i} \]

when \( \omega_t = \omega_i \)

See slide on Brewster's angle

(lp. 155 of EM waves Staelin/Morgenthaler/Kung)

Geometry, when transmuted dipoles w/ polarization \( \vec{E}_t \) are parallel to reflected \( \vec{K}_t \)

(Dipoles do not radiate on axis)

---

Total Internal Reflection & Critical Angle

\( n_1 < n_2 < n_3 \)

At critical angle of incidence, transmitted wave is maximally bent away from normal & propagates \( \perp \) to boundary surface

\( \Theta_t = 90^\circ \)

\[ \frac{\sin \Theta_i}{\sin \Theta_t} = \frac{n_i}{n_t} \quad (\text{Snell's Law}) \]

If \( \Theta_t = 90^\circ \)

\[ \Theta_c = \sin^{-1} \left( \frac{n_t}{n_i} \right) \]

\( \Theta_i > \Theta_c, \quad k_{tx} > |k_t| \Rightarrow \text{not possible for real values of } k_t \)
Dispersion relation for transmitting medium:

\[ k_t^2 = \omega^2 u_t \varepsilon_t = k_t x^2 + k_t z^2 \]

At \( \Theta_c \),

\[ \Theta_t = 90^\circ, \ k_t z = 0, \ k_x = k_t x = \omega \sqrt{u_t \varepsilon_t} \]

For \( \Theta_i > \Theta_c \)

\[ k_t z = k_t^2 - k_x^2 < 0 \]

\[ k_t z = \pm j \sqrt{k_x^2 - k_t^2} = \pm j \alpha_z \]

where \( \alpha_z \) is a positive real quantity;

\( k_x = k_x i; \ k_t = \omega \sqrt{u_t \varepsilon_t} \)

\[ -j k_t x - \alpha_z z \]

\[ E_t = \hat{y} T E_i e \]

- Wave exponentially decays in 2nd medium \( \Rightarrow \) no average power transmitted in 2nd medium

\( \Rightarrow \) all incident light is reflected \( \Rightarrow \) total internal reflection (\( \Gamma_{TE} \) or \( \Gamma_{TM} = 1 \))

**See slide on TIR**

- Characteristic decay distance into 2nd medium \( < x_o/2 \)

\( n_1 > n_2 \)

**S polarization** (\( u_1 = u_2 \))

\[ \cos \Theta_t = \frac{\sin \Theta_i}{n_2/n_1} \]

\[ \frac{\cos \Theta_i - i \sqrt{\sin^2 \Theta_i - (n_2/n_1)^2}}{\cos \Theta_i + i \sqrt{\sin^2 \Theta_i - (n_2/n_1)^2}} = \frac{+ j \sqrt{\sin^2 \Theta_i - 1}}{\sin \Theta_i} \]
Goos-Hänchen Phase Shift

(Ret & Hau's Waves & Fields in Optoelectronics, Kang EM wave theory)

- Incident field reflects off TIR boundary with phase shift
- Incident & reflected light

\[ E_1(x, z) = \begin{cases} E_0 \exp \left( -j(k_x x + k_z z) \right) \\ + E_0 \exp \left( -j(k_x x - k_z z - \delta) \right) \exp(-jwt) \end{cases} + \text{c.c.} \]

\[ E_1(x, z) = 2 E_0 \cos(k_z z + \delta/2) \exp(-j(k_x x - wt - \delta/2)) \]

+ c.c.

- Transmitted light (continuity at boundary requires)

\[ E_2(x, z) = \begin{cases} 2 E_1 \exp \left( -j \delta/2 \right) \cos(\delta/2) \\ \exp(-j\delta z) \exp(j(k_x x - wt)) \end{cases} \]

- Bigger phase shift for p than s since \( \delta \) is bigger

- Phase of Fresnel reflection coefficient at total internal reflection is

\[ \Gamma = 1 - \exp(j2\delta) \]

\[ \phi = \text{Goos-Hänchen shift} = \frac{1}{2} \arg(\Gamma^2) \]

(See slide on Goos-Hänchen shift)
\[ |\rho_0|^2 = 1 \]
\[ \rho_s = \frac{a - ib}{a + ib} \text{ (form)} = \frac{|1| e^{i\alpha}}{|1| e^{-i\alpha}} = e^{i2\alpha} \]

Phase: \[ \tan^{-1} \left( \frac{b}{a} \right) = \frac{\delta_s}{2} \angle \text{angle} \]

\[ \tan \left( \frac{\delta_s}{2} \right) = \frac{\text{Im} (a - ib)}{\text{Re} (a - ib)} = -\frac{\sqrt{\sin^2 \theta_i - (n_2/n_1)^2}}{\cos \theta_i} \]

\[ P \text{ polarization} \]

\[ \rho_p = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \]

\[ \rho_p = \frac{(n_2/n_1)^2 \cos \theta_i - \sqrt{\sin^2 \theta_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cos \theta_i + \sqrt{\sin^2 \theta_i - (n_2/n_1)^2}} \]

Similar to \( s \) polarization:

\[ \tan \left( \frac{\delta_p}{2} \right) = \frac{\text{Imaginary (Numerator)}}{\text{Real (Numerator)}} \]

\[ = \frac{\sqrt{\sin^2 \theta_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cos \theta_i} \]

See slide on TIR

Relative Phase:

Difference between \( s \) & \( p \):

\[ \delta = \delta_s - \delta_p \]

\[ \tan \frac{\delta}{2} = \tan \frac{\delta_s}{2} - \tan \frac{\delta_p}{2} \]

\[ 1 + \tan \frac{\delta_s}{2} \tan \frac{\delta_p}{2} \]
\[ \tan \frac{\delta}{2} = - \frac{\cos \theta i \sqrt{\sin^2 \theta i - \left(\frac{n_2}{n_1}\right)^2}}{\sin^2 \theta i} \]

For \( \theta > \theta_c \)

\[ k_{te} = \pm j \sqrt{k_x^2 - k_t^2} = \pm j \, \alpha z \]
\[ k_x = k_{x1} = k_0 n_1 \sin \theta i \]
\[ k_t = k_0 n_2 \]
\[ \pm j \, \alpha z = \pm j \, k_0 \sqrt{n_1^2 \sin^2 \theta i - n_2^2} \]
\[ \pm j \, \alpha z = \pm j \, k_0 n_1 \sqrt{\sin^2 \theta i - \left(\frac{n_2}{n_1}\right)^2} \]

Compare w/ phase

\[ \tan \frac{\delta_s}{2} = - \frac{\sqrt{\sin^2 \theta i - \left(\frac{n_2}{n_1}\right)^2}}{\cos \theta i} \]

\[ = - \frac{\alpha z}{k_0 n_1 \cos \theta i} = - \frac{\alpha z}{k_i z} \]

\[ \tan \frac{\delta_p}{2} = \frac{\sqrt{\sin^2 \theta i - \left(\frac{n_2}{n_1}\right)^2}}{\left(\frac{n_2}{n_1}\right)^2 \cos \theta i} \]

\[ = \frac{\alpha z}{k_0 n_1 \left(\frac{n_2}{n_1}\right)^2 \cos \theta i} = \frac{\alpha z}{k_i z \left(\frac{n_2}{n_1}\right)^2} \]
Waves in Lossy Media

References: Griffiths, Intro to Electrodynamics; Kono, EM wave theory

\[ \vec{E} = \sigma \vec{E}_0, \text{ no tensors (isotropic media)} \]

Rederive wave equation w/ loss

1) Curl of Faraday's Law

\[ \nabla \times (\nabla \times \vec{E}) = -\mu \frac{d}{dt} (\nabla \times \vec{H}) \]

2) Substitute Ampère's Law

\[ \nabla \times \vec{H} = \vec{G} + \frac{d\vec{D}}{dt} \]

\[ \nabla \times (\nabla \times \vec{E}) = -\mu \frac{d}{dt} \left[ \vec{G} + \frac{d\vec{D}}{dt} \right] \]

\[ = -\mu \frac{d}{dt} \left[ \sigma \vec{E} + \varepsilon \frac{d\vec{E}}{dt} \right] \]

3) Identity

\[ \nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \]

no enclosed charge

\[ \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu \sigma \frac{d\vec{E}}{dt} - \mu \varepsilon \frac{d^2\vec{E}}{dt^2} \]

\[ \nabla^2 \vec{E} - \frac{\mu \varepsilon}{\varepsilon \sigma} \frac{d^2\vec{E}}{dt^2} - \mu \sigma \frac{d\vec{E}}{dt} = 0 \]

Wave equation with loss

Solution to wave equation with loss:

\[ \vec{E}(z, t) = \hat{x} \vec{E}_0 \exp(-j(kz - \omega t)) + \text{CC} \]

\[ = \hat{x} \vec{E}_0 \exp(-kz) \cos(k \omega t) \]

\[ k^2 = \mu \varepsilon \omega^2 + j \sigma \omega \]

\[ (0) \]
\[ k = k_R + jk_I \]
\[ k^2 = k_R^2 - k_I^2 + 2jk_I k_R \]

\[ \Rightarrow \]
\[ k_R^2 - k_I^2 = \omega^2 u \epsilon \] \hspace{1cm} (1)
\[ 2k_I k_R = \omega u \sigma \] \hspace{1cm} (2)

Square (1) and (2) and then adding yields:

\[ (k_I^2 + k_R^2)^2 = (\omega^2 u \epsilon)^2 + (\omega u \sigma)^2 \]

\[ k_I^2 + k_R^2 = \sqrt{(\omega^2 u \epsilon)^2 + (\omega u \sigma)^2} \]

Taking square root of (1) yields:

\[ k_R = \omega \sqrt{u \epsilon} \left( \frac{1}{2} \left( \sqrt{1 + \frac{\sigma^2}{\epsilon^2 u \omega^2}} + 1 \right) \right)^{1/2} \]

\[ k_I = \omega \sqrt{u \epsilon} \left( \frac{1}{2} \left( \sqrt{1 + \frac{\sigma^2}{\epsilon^2 u \omega^2}} - 1 \right) \right)^{1/2} \]

Using electric field from previous page, magnetic field is:

\[ \overline{H}(z,t) = \frac{\hat{y}}{u \mu} \frac{E_0}{\epsilon} \exp(-k_I z) \{ k_R \cos(k_R z - \omega t) \}
\]

Or

\[ \overline{H}(z,t) = \frac{\hat{y}}{u \mu} \frac{k E_0}{\epsilon} \exp(-k_I z) \exp(-j(k_R z - \omega t)) \] + CC

Poynting vector power density is:

\[ \overline{S}(r,t) = \frac{\hat{z}}{u \mu} \frac{E_0^2}{\epsilon} \exp(-2k_I z) \{ k_R \cos^2(k_R z - \omega t) - k_I \sin(k_R z - \omega t) \cos(k_R z - \omega t) \} \]
\[
\langle S(r, t) \rangle = \frac{1}{2\pi} \int_0^{2\pi} d(\omega t) \overline{E} \times \overline{H}
\]

**Power density**

\[
\langle S(r, t) \rangle = \frac{k_R}{\omega_0} E_0^2 \exp(-2k_Iz)
\]

Attenuating & propagating in \( z \) direction

Skin depth (distance it takes to reduce amplitude by factor (1/e)) \( \sim (1/\lambda) \)

\[
d = \frac{1}{k_I}
\]

- Measure of depth to which electromagnetic wave penetrates a good conductor
- Real part of propagation constant determines propagation speed & index of refraction

\[
\lambda = \frac{2\pi}{k_R}, \quad \nu = \frac{\omega}{k_R}, \quad \eta = \frac{c k_R}{\omega}
\]

- Consider 2 limiting cases

  **Poor conductor**

  \( \sigma \ll \omega \varepsilon \)

  \[
k_R \propto \omega \sqrt{\sigma} \quad k_I = \frac{\sigma}{2\sqrt{\varepsilon}}
\]
Good conductor \( \sigma > \omega \sigma \)

\[ k_R \approx k_I = \sqrt{\frac{\omega \sigma}{\mu}} \]

Skin depth decreases with increasing frequency

\[ d = \frac{\lambda}{2\pi} \] (skin depth)

Monochromatic plane waves in conducting media

\[ k = k_R + jk_I = |k| e^{j\phi} \]

\[ \phi = \tan^{-1} \left( \frac{k_I}{k_R} \right) \]

Complex amplitudes of \( E \) & \( H \) reveal that fields are no longer in phase and differ by \( \phi \):

\[ E = \frac{\hat{E}_0}{|k|} e^{j\phi} \exp(j\omega t) \exp(jk_R z) \]

\[ H = \frac{j\hat{E}_0}{\omega}\frac{1}{|k|} \exp(j\phi) \exp(jk_R z) \]

\[ x \quad \hat{E} \quad \hat{H} \quad z \]

\[ y \]
Reflection & Transmission at a Conducting Surface

Boundary conditions w/ surface charge & surface current

(i) \( e_1 E_{in} - e_2 E_{zn} = 0 \) surface charge

(ii) \( B_{in} = B_{2n} \)

(iii) \( E_{1t} = E_{2t} \)

(iv) \( \frac{B_{1t}}{u_1} - \frac{B_{2t}}{u_2} = H_{1t} - H_{2t} = \bar{q}_s \times \hat{n} \) surface current

Example (optional if lack of time)

\[
\begin{align*}
\overline{E}_t(z,t) &= \hat{y} \left\{ E_0 \exp \left( j \left( k_1 z - wt \right) \right) + cc \right\} \\
\overline{H}_t(z,t) &= \left( \hat{k} \times \hat{y} \right) \left\{ \frac{E_0 \exp \left( -j \left( k_1 z - wt \right) \right) + cc}{n_1} \right\} \\
\overline{E}_R(z,t) &= \hat{y} \left\{ E_{0R} \exp \left( -j \left( k_1 z - wt \right) \right) + cc \right\} \\
\overline{B}_R(z,t) &= - \left( \hat{k} \times \hat{y} \right) \left\{ \frac{E_{0R} \exp \left( -j \left( k_1 z - wt \right) \right) + cc}{n_1} \right\}
\end{align*}
\]
\[
\text{Transmitted}
\begin{align*}
E_T(z,t) &= \gamma \mathcal{E}_0 T \left\{ \exp \left( j \left( k z - \omega t \right) \right) + c_c \right\} \\
B_T(z,t) &= (\mathbf{k} \times \mathbf{E}) T \left\{ \frac{\mathcal{E}_0 T}{n_2^2} \exp \left( -j \left( k z - \omega t \right) \right) + c_c \right\}
\end{align*}
\]

Boundary conditions:

\[
\mathbf{E}_0 + \mathbf{E}_0 R = \mathbf{E}_0 T \quad \text{tangential } \mathbf{E} \text{ continues}
\]

\[
\frac{\mathbf{E}_0}{n_1} - \frac{\mathbf{E}_0 R}{n_1} - \frac{\mathbf{E}_0 T}{n_2} = 0 \quad \text{tangential } \mathbf{H} \text{ continues}
\]

\[
\mathbf{B} = \frac{\mathbf{u}_1 c \mathbf{k}_2}{n_1 u_2 w}, \quad k_2 \text{ is complex}
\]

\[
\Rightarrow \quad \Pi = \frac{\mathbf{E}_0 R}{\mathbf{E}_0} = \left( \frac{1 - B}{1 + B} \right) \quad \text{or use impedance definition of } \Pi
\]

\[
T = \frac{\mathbf{E}_0 T}{\mathbf{E}_0} = \frac{2}{1 + B}
\]

**Perfect conductor**

\[
\sigma = \infty \Rightarrow B = \infty
\]

\[
\mathbf{E}_0 R = -\mathbf{E}_0 \quad \mathbf{E}_0 T = 0
\]

Wave is totally reflected w/ \( \pi \) phase shift

**Good conductor** \( B \) so large

\[
\left( \frac{1 - B}{1 + B} \right) = \left( \frac{1 - \frac{1}{\mathcal{B}}}{1 + \frac{1}{\mathcal{B}}} \right) \approx \left( 1 - \frac{1}{\mathcal{B}} \right)^2 \approx \frac{2}{\mathcal{B}} - 1
\]

\[
\mathcal{R} \approx 1 - \frac{2}{1 + \mathcal{B}^2}
\]
Plasma assembly of positive and negative charged particles where time-averaged charge density is zero.

Example: Neutralized gas; free electrons and positive ions.

- Ions are much heavier than electrons.
- Consider only interaction between EM wave and electrons.

\[ \overline{\mathbf{J}} = N_q \overline{\mathbf{v}} \] electron velocity

\[ \nabla \times \overline{\mathbf{B}} = \frac{\varepsilon_0}{\mu_0} \frac{\partial \overline{\mathbf{E}}}{\partial t} + \overline{\mathbf{J}} \]

Derive wave equation:

\[ (\nabla^2 - \mu_0 \varepsilon_0 \frac{1}{\mu_0} \frac{\partial^2}{\partial t^2} - \mu_0 \frac{N_q e^2}{m}) \overline{\mathbf{E}} = 0 \]

Substitute:

\[ \overline{\mathbf{E}}(z, t) = \mathbf{E}_0 \exp(-ik_I z) \cos(k_R^2 - \omega t) \]

Dispersion relation:

\[ k_R^2 - k_I^2 = \omega^2 \mu_0 \varepsilon_0 (1 - \frac{\omega_p^2}{\omega^2}) \]

\[ 2k_R k_I = 0 \]

\[ \omega_p = \sqrt{\frac{N_q e^2}{m \varepsilon_0}} \]

\[ \omega > \omega_p \]

\[ \begin{cases} k_R^2 = \omega^2 \mu_0 \varepsilon_0 (1 - \frac{\omega_p^2}{\omega^2}) \\ k_I = 0 \end{cases} \]
\[ w < w_p \]

\[
k_I^2 = w^2 w_0 \xi_0 \left( \frac{w_p^2}{w^2} - 1 \right)
\]

\[ k_R = 0 \]

Evanescent wave \( \Rightarrow \) no power flow!