4.3.7 Small signal model

For a small signal operation, diode biased to a point on the forward I-V characteristic. A small AC voltage is superimposed on DC quantities.

1. Determine DC operating points (using one of above models)
2. Model small signal behavior by slope (1/\(r_d\))

**Fig 4.13**

\[ V_D = \text{DC} \]

\[ V_d(t) = \text{Time varying signal} \]

In absence of signal \(V_d(t)\)

\[ I_D = I_S e^{V_d/V_T} \]

with \(V_d(t)\) superimposed

\[ V_D(t) = V_D + V_d(t) \]

\[ I_d(t) = I_S \exp \left( \frac{V_d(t)}{V_T} \right) \]

\[ I_D(t) = I_S \exp \left( \frac{V_d(t)}{V_T} \right) \exp \left( \frac{V_d(t)}{V_T} \right) \]

\[ I_d(t) = I_D \exp \left( \frac{V_d(t)}{V_T} \right) \]

Small signal means \(\frac{V_d}{V_T} \ll 1\)
\[ e^x \approx 1 + x \text{ when } x \ll 1 \]

\[ i_d(t) = I_D \left( 1 + \frac{V_d}{V_T} \right) \text{ small signal approximation} \]

valid for signal amplitudes \(< 5 \text{ mV} \]
and \( V_T = 25 \text{ mV} \) (room temperature)

\[ i_d(t) = \frac{I_D}{V_T} \cdot V_d(t) \]

\[ \Rightarrow (i_d(t) \propto V_d(t)) \]

\[ \frac{I_D}{V_T} = \text{ small signal conductance} \]

\[ R_d = \frac{V_T}{I_D} \text{ small signal resistance} \]

or incremental resistance

**Fig 4.13**

Small enough region of \( i \cdot v \) =

Graphs look linear
Can approximate by tangent = slope

\[ g_d = \left[ \frac{\delta i_d}{\delta v_d} \right]^{-1} \]
\[ i_d = I_d \]

- Quiescent points = DC bias points, \( V_d \) & \( I_d \)
- Small signal quantities \( v_d(t), i_d(t) \)
  \[ \Rightarrow \text{separated from DC parameters} \]

Steps to solve problems

1) DC analysis to determine quiescent points

2) Eliminate all DC sources, replace diodes with resistors
  - Current source is open
  - Voltage source is shorted

Ex 4.5 (modified)

\[ V^+ \uparrow \]
\[ \begin{array}{c}
  \text{+}\frac{1}{10k2} \\
  v_d \\
  \text{=}
\end{array} \]

\[ V^+ = 10V + 1V \sin(60Hz \cdot 2\pi t) \]

10V supply w/ 60Hz ripple

I) Calculating operating point (DC)

using following circuit (assume diode has 0.7V at 1mA of current)
1. \( I_D = \frac{10V - V_D}{R} \)

2. \( I_D = I_S \exp \left( \frac{V_D}{V_T} \right) \)
   \[ I_S = I_D \exp \left( -\frac{V_D}{V_T} \right) \]
   \[ = (1 \text{ mA}) \exp \left( -\frac{0.7V}{0.025V} \right) \]
   \[ = 0.91 \times 10^{-16} \text{ A} \]

\[ V_D = V_T \ln \left( \frac{I_D}{I_S} \right) = 2.3 V_T \log_{10} \left( \frac{I_D}{I_S} \right) \]

Start with \( V_D = 0.7V \) in ①

\[ I_D = \frac{(10 - 0.7V)}{10k} = 0.93 \text{ mA} \]

Substitute into ②

\[ V_D = 2.3 \times 0.025 \log_{10} \left( \frac{0.93 \text{ mA}}{0.91 \times 10^{-13} \text{ mA}} \right) \]

\[ V_D = 0.6974 \text{ V} \]

Substitute into ①

\[ I_D = \left( \frac{10 - 0.6974}{10k} \right) V = 0.9303 \text{ mA} \]

Substitute into ②

\[ V_D = 2.3 \times 0.025 \log_{10} \left( \frac{0.9303 \text{ mA}}{0.91 \times 10^{-13} \text{ mA}} \right) \]

\[ V_D = 0.6974 \text{ V} \Rightarrow \text{ converged!} \]
2) Small signal analysis

Calculate $R_d$

$$R_d = \frac{V_T}{I_0} = \frac{0.025V}{0.9303 \times 10^{-3} A} = 26.95\Omega$$

Construct small signal equivalent circuit replace diode with $R_d$

$$V_{d\text{ (peak)}} = V_s \left( \frac{R_d}{R_d + R} \right)$$

$$V_{d\text{ (peak)}} = 1V \left( \frac{0.0269 \Omega}{0.0269 \Omega + 10 \Omega} \right)$$

$$V_{d\text{ (peak)}} = 2.7 mV$$

Value of $V_d(t)$ is small enough to justify use of small signal model

$$V_0 = 0.7 \pm 0.0027V$$

Section 4.3.9 Voltage regulation

Voltage regulator $\Rightarrow$ provide constant DC voltage on output terminals in presence of:

1) Changes in load current drawn from regulator

2) Changes in DC power supply voltage
Diode forward voltage drop of 0.7V over large range of current
For larger output voltage, use several diodes in series

Ex. 4.6 Consider circuit below
String of 3 diodes provide constant voltage of 2.1V. Calculate change in regulated voltage caused by:

1) 10% change in power supply voltage
2) Connecting 1kΩ load resistance

\[ I = \frac{(10 - 2.1V)}{1k\Omega} = 7.9 \text{ mA} \]

Each diode will have:

\[ R_d = \frac{V_I}{I} = \frac{0.025V}{0.0079A} \]

\[ R_d = 3.15\Omega \]
3 diodes in series

\[ R_{\text{tot}} = 3R_d = 9.6 \Omega \]

\[ \text{Delta supply voltage} = \pm 1V \Rightarrow \Delta V_0 = \frac{2R_{\text{tot}}}{R_{\text{tot}} + 1k\Omega} \]

\[ \Delta V_0 = 2 \left( \frac{0.0096k\Omega}{1 + 0.0096k\Omega} \right) = 19 \text{mV peak-to-peak} \]

Change of \( \pm 1V \) in supply voltage causes \( \pm 9.5 \text{mV} \)

Change in \( V_0 \) (\( \pm 0.5 \% \)) \( \Rightarrow \) \( \pm 3.2 \text{mV/diode} \)

so small signal model is justified

Correct load resistance of 1k\( \Omega \)

- Draw current of \( \sim 2.1 \text{mA} \)
- Current through diodes decreases by \( 2.1 \text{mA} \), resulting in voltage decrease across the diodes of:

\[ \Delta V_0 = -2.1R_{\text{tot}} = (-2.1 \text{mA})(9.6 \Omega) \]

\[ = -20 \text{mV} \]

- Implies that voltage across each diode decreases by \( 6.7 \text{mV} \Rightarrow \)

small signal model is justified
- Detailed calculation using exponential model yields ~23mV, not too different from small signal model

Summary: 4 different models

1) Exponential
   \[ I = I_s \left[ \exp \left( \frac{V}{V_T} \right) - 1 \right] \]

2) Constant voltage drop
   \[ V \geq 0.7V \]
   \[ I = \infty \]

3) Ideal diode
   \[ V \geq 0 \]
   \[ I = \infty \]

4) Small signal
   \[ R_a = \frac{V_T}{I_D} \]

Section 4.4: Zener diodes

Steep i/v in breakdown region enables voltage regulation \( \Rightarrow \) zener diode

[Diagram of zener diode with breakdown and reverse bias regions]
\[ I_{zk} = \text{knee current} \]

For \( i > I_{zk} \), \( i/V \) is nearly straight line

Zener diode characteristic is specified by \( I_{zT} \) @ \( V_z \) (test current)

- As current changes, voltage drop changes but only slightly
- \( \Delta V = r_z \Delta I \) 
  \( r_z = \text{inverse slope} \) @ \( V_z \)
  Incremental dynamic resistance

- Typically, \( r_z \sim \) few or few 10's of ohms
  Smaller \( r_z \) is better \( \Rightarrow \) decreased sensitivity to current changes

\[
\begin{align*}
\text{Avoid} & \quad r_z \text{ increases dramatically near} \\
& \quad I_{zk} \\
V_z & \sim \text{range of few V to hundreds of volts depending on device}
\end{align*}
\]

Also specified: max power Zener can dissipate

\[
\begin{align*}
\text{Equivalent Circuit Model (Fig 4.18)}
\end{align*}
\]
Assume linear i-v

\[ V_Z = V_{Z0} + R_Z \cdot I_Z \quad \text{for} \quad I_Z > I_{ZK} \quad \text{and} \quad V_Z > V_{Z0} \]

**Example**

Given: 
- \( V_Z = 5.1 \text{V} \)
- \( R_Z = 0 \)
- \( V_{DC} = 11 \text{V} \)
- \( R_L = 1 \text{k}\Omega \)

Find \( R \) that minimum zener diode reverse current \( I_Z \geq I_{Z\text{min}} = 1 \text{mA} \)

Redraw with model of zener diode

\[ I_R = I_{Z\text{min}} + I_L \]
\[ I_L = \frac{5.1 \text{V}}{1 \text{k}\Omega} = 5.1 \text{mA} \]
\[ I_{R\text{-min}} = 1 \text{mA} + 5.1 \text{mA} = 6.1 \text{mA} \]
\[ R = \frac{11 - 5.1 \text{V}}{6.1 \text{mA}} = 9.67 \Omega \]
Example 4.7

\( V = 10 \pm 1 \text{V} \)

\( I \downarrow \uparrow 0.5 \text{k}\Omega \)

\( V_0 \)

\( R_L \)

6.8V zener

\( I_Z \)

6.8V zener diode

\( V_Z = 6.8 \text{V} \)

\( I_Z = 5 \text{mA} \)

\( R_L = 20 \Omega \)

\( I_{ZK} = 0.2 \text{mA} \)

a) Find \( V_0 \) with no load and \( V^+ \) at nominal value

**Step I** Find \( V_{Z0} \)

\[ V_Z = V_{Z0} + R_Z I_Z \]

\[ V_{Z0} = V_Z - R_Z I_Z \]

\[ V_{Z0} = (6.8 \text{V} - \frac{(20-5)(5 \text{mA})}{1000}) \]

\[ V_{Z0} = 6.7 \text{V} \]

**Step II** For no load, find current through zener

\[ I_Z = \frac{V^+ - V_{Z0}}{R + R_Z} = \frac{(10 - 6.7 \text{V})}{(0.5 + 0.02) \text{k}\Omega} \]

\[ I_Z = 6.35 \text{mA} \]
Step III

\[ V_0 = V_{20} + I_Z R_Z \]

\[ = 6.7 \text{V} + (0.35 \text{mA})(0.02 \text{k\Omega}) \]

\[ V_0 = 6.83 \text{V} \]

b) Find change in \( V_0 \) from \( +1 \text{V} \) swing on supply voltage.

\[ V_0 = V_{20} + (V^+ - V_{20}) \left( \frac{R_Z}{R + R_Z} \right) \]

\[ \Delta V_0 = (\Delta V^+) \frac{R_Z}{R_Z + R} = \pm 1 \text{V} \left( \frac{20 \Omega}{20 \Omega + 500 \Omega} \right) \]

\[ \Delta V_0 = \pm 1 - 38.5 \text{mV} \]

c) Find change in output voltage that results from load resistance connected and drawing 1mA.

\[ V_Z = V_{20} + R_Z I_Z \]

\[ \Delta V_0 = R_Z \Delta I_Z = (20 \Omega)(-1 \text{mA}) \]

\[ \Delta V_0 = -20 \text{mV} \]

d) Find \( \Delta V_0 \) when \( R_L = 2 \text{k\Omega} \)
Load current $\approx \frac{0.8V}{2k\Omega} = 3.4mA$

$\Delta I_Z = -3.4mA$

$\Delta V_0 = \Delta I_Z R_Z = (-3.4mA)(0.02k\Omega)$

$\Delta V_0 = -68mV$

* Neglect change in current

**Exact solution**

$I = I_Z + I_L$

$\frac{V^+ - V_0}{R} = \frac{V_0 - V_{Z0}}{R_Z} + \frac{V_0}{R_L}$

Solve for $V_0$

$V_0 = \frac{V^+/R + V_{Z0}/R_Z}{(1/R^+ + 1/R_Z + 1/R_L)}$

$\Delta V_0 = V_0(\text{no load}) - V_0(\text{w/load})$

$\Delta V_0 = -70mV \text{ (exact)}$

E) Find $V_0$ when $R_L = 0.5k\Omega$

$I_L = \frac{0.8V}{0.5k\Omega} = 1.6mA$

Not possible as current through 1st resistor is 0.4mA (means Zener is no longer in breakdown region)
\[ V_o = V_+ \frac{R_L}{R+R_L} = 10V \left( \frac{0.5k\Omega}{0.5k\Omega + 0.5k\Omega} \right) \]

\[ V_o = 5V \]

\[ V_o < V_{ZK} \Rightarrow \text{Zener not operating in breakdown region} \]

f) Min. \( R_L \) that still maintains breakdown

Edge of breakdown \( I_Z = I_{ZK} = 0.2mA \)

\[ V_Z \approx V_{ZK} \approx 6.7V \]

at this point, what is lowest current?

\[ I = \frac{9-6.7V}{0.5V} = 4.6mA \]

\[ I_L = (4.6 - 0.2)mA = 4.4mA \]

\[ R_L = \frac{6.7}{4.4} = 1.5 k\Omega \]

4.5 Rectifier circuit

- Rectifier: essential building block of DC power supply
- Need power supply to deliver constant DC voltage
Power supply has:

1. Transformer
   
   \[ V_s = V_{AC} \frac{N_2}{N_1} \text{ VL rms} \]

   and electrical isolation

2. Diode rectifier: convert AC to unipolar output

3. Filter reduces pulsating output from rectifier

4. Voltage regulator reduces ripples and stabilizes output voltage

4.5.1 Half wave rectifier

   Use positive half of sine input and discard negative side
Fig. 4.21  Half wave rectifier

\[ +V_D - \]
\[ I \]
\[ + \]
\[ V_s \]
\[ R \]
\[ - \]
\[ V_0 \]

Use constant voltage drop model to analyze

Normally \( R \gg R_D \)

\[ \Rightarrow V_0 = V_s - V_D \quad \text{for} \quad V_s > V_D = 0.7V \]

Design parameters to be specified

1) Current handling capability

Largest current diode can conduct

2) Peak inverse voltage (PIV), largest reverse voltage without breakdown

\[ \text{PIV} = V_s \quad \text{for Fig. 4.21} \]

Prudent to require \( V_{\text{breakdown}} = 2 \text{PIV} \)

Notes

1) Can use an exponential model but need to iterate and solve transcendental equation
2) Cannot handle small AC signal

\[ \Rightarrow \text{need different circuit (precursor rectifier)} \]
4.5.2 Full Wave Rectifier

FWR uses both positive and negative pulses to produce polar output.

4.5.3 Bridge Rectifier

Fig 4.2.3

- No center tapped transformer
- Requires 4 diodes instead of 2, but diodes are cheap

\[ V_{out} = V_{in} - 2V_D \]

Uses constant voltage diode model

Positive pulse: D1 & D2 on, D3 & D4 off
Negative pulse: D3 & D4 on, D1 & D2 off

Same current direction & output
PIV: consider positive pulse
\[ V_{D3} \text{ (reverse)} = V_0 + V_{D2} \text{ (forward)} \]
max when \( V_0 = \text{max} \)

\[ PIV = \frac{V_{D3}}{\text{reverse max}} = V_S - 2V_D + V_D \]
\[ = V_S - V_D \]

* Better than 2 diode full wave rectifier
center tapped transformer

\[ V_s = \frac{1}{2} \left( \frac{N_2}{N_1} \right) V_{AC} \]

at any given time, one diode is forward biased and the other is reverse biased.

**Positive pulse**: D1 on, D2 off \( \Rightarrow \) positive \( V_o \)

**Negative pulse**: D1 off, D2 on \( \Rightarrow \) positive \( V_o \)

* Each diode is half wave rectifier

\[ \Rightarrow \] together form full wave rectifier

Transfer characteristics

\[ \text{slope} = -1 \]

\[ V_o \]

\[ \text{slope} = +1 \]

\[ V_s - V_D \]

\[ V_s \]

\[ V_D \]

\[ \text{output} \]

\[ \text{input} \]
Consider positive half of pulse
Reverse bias on D2 = \( V_0 + V_s \)
\( V_0 + V_s \) is max when \( V_0 = V_s - V_D \)
Diode drop of 0.7V

\( V_S = V_s \) peak value of \( V_s \)

\[ P1V = 2V_D - V_s, \text{ 2x that of } \frac{1}{2} \text{ wave} \]