2. Probability state is occupied $f(E)$
   electrons obey Fermi-Dirac statistics

$$f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

2 electrons of opposite spins per state

States at lower energy have a higher probability of being occupied

$$n = \# \text{ electrons over a given energy range} = \int_{\text{energy range}} f(E) g(E) \, dE$$

---

Aside

For semiconductors with parabolic energy bands

$$g_c(E) = \text{conduction band density of states} = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}$$

$m^*$ = effective mass of electron

$$g_v(E) = \text{valence band density of states} = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{3/2} (E - E_v)^{1/2}$$

$E_c$ = conduction band energy

$E_v$ = valence band energy
Intrinsic Semiconductor: no impurities

Only source of carriers is excitation from valence \( \Rightarrow \) conduction band

\[
\text{Valence band} \quad \text{highest occupied state at 0 K (w/ electrons)}
\]

\[
\text{Conduction band} \quad \text{Lowest energy band of unoccupied states at 0 K (w/ electrons)}
\]

\[
\# \text{electrons} = \# \text{holes} = \text{Intrinsic concentration}
\]

\[
n = p = n_i
\]

Law of mass action \( np = n_i^2 \) always true at equilibrium

\[
n_i(T) = B \exp\left(\frac{-E_g}{2kT}\right) T^{-3/2}
\]

\[\text{EX} \quad \text{Carrier concentration in silicon at}\]

\[T = 300 \text{ K}\]

\[B = 7.3 \times 10^{15} \text{ cm}^{-3} \text{ K}^{3/2}\]

\[
n = \left( \frac{7.3 \times 10^{15}}{\text{cm}^3 \cdot \text{K}^{3/2}} \right) (300 \text{K})^{3/2} \exp\left(\frac{-1.012 \text{eV}}{1.300 \text{K} \cdot 8.16 \times 10^5 \text{eV}}\right)
\]
This is a small # => silicon has $5 \times 10^{22}$ atoms/cm$^3$ => this means @ 300K one in five atoms is ionized.

** Need to heat intrinsic silicon a lot to get good conductivity.

Doping: Add impurities that contribute extra electrons or holes => extra carriers have a very small binding energy (fraction of thermal energy at room temperature).

See 3 slides

3 regions for doped semiconductors

\[
\frac{1}{T}(\frac{1}{N_i})
\]

Freeze out $n = N_0 + 1$

\[ n = N_D \]

Intrinsic $\quad n = N_i$

\[ T(K) \]

$N_A =$ Density of acceptors (doped) (contribute holes)

$N_D =$ Density of doped donors (contribute electrons)
What happens in each region?

\[ E_c \text{ (conduction band)} \quad T = \text{low} \]

\[ E_v \text{ (valence band)} \quad \text{most donors not ionized} \]

\[ \text{Donor level} \quad \text{any electrons are from ionized donors} \]

\[ \text{Ionized} = \text{gives up electron} \]

Extrinsic

\[ T = \text{room temperature} \]

\[ \text{All dopants ionized} \]

\[ n = N_D \]

Intrinsic

\[ T = \text{high} \]

\[ \text{most electrons from thermal excitation from conduction to valence band} \]

Doped semiconductors are usually in extrinsic region where all dopants ionized

\[ n = N_D = \# \text{Donors/volume} \quad p = N_A = \# \text{acceptors/volume} \]

\[ n = n_i^2 / N_D \]

\[ p = n_i^2 / N_A \]
Physics of p/n Junctions

Outline

1) Formation of p/n Junction
2) Biased p/n Junction
3) Optoelectronic devices based on p/n Junction

p/n Junction: junction between p type (holes) & n type (electrons)

Basis of many semiconductor devices including diode & BJT transistor

After making contact

1) Carriers diffuse leaving ionized donors and acceptors behind

2) Ionized impurities produce electric field that opposes diffusion

3) Two currents balance at equilibrium

See slide

- = electron
+ = hole
⊕ = ionized impurity (donor)
⊙ = ionized impurity (acceptor)
\( V_0 = \text{depletion region - no free charges} \)

Energy band diagram for pn junction (equilibrium)

\( V_0 = \text{Built in voltage also referred to as } V_{bi} \text{ in many texts} \)

\[ V_0 = \text{Built in voltage (uniform doping)} = V_T \ln \left( \frac{N_D N_A}{n_i^2} \right) \]

At equilibrium \( V_{0|_{si}} \approx 0.7 \text{ V} \)

\( Q_{ni} = q A x_n N_D \)

\( Q_{pi} = q A x_p N_A \)

\( \rho = \text{charge density} \)

Charges equal

\[ q_0 A x_n N_D = q_0 A x_p N_A \]
\[ \frac{X_n}{X_P} = \frac{N_A}{N_D} \]  

\[ \Rightarrow \] lower doped region has larger depletion region

Depletion region width solve for \( X_n \) & \( X_P \) in terms of applied and built-in potential 
\( V_r = \) reverse bias (if forward bias, use \(-\) sign in front)

\[ W = X_n + X_P \]

\[ W = \text{depletion region width} = \sqrt{\frac{2 \varepsilon_S}{q_D} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_r)} \]

3.26 and 3.31 (not necessarily equilibrium)

Use Gauss's Law to find field across junction and integrate

Electric field

\[ \frac{d \varepsilon}{dx} = \frac{\rho(x)}{\varepsilon_S} \]

Voltage (V)

\( V_0 = \) built-in voltage

\[ V = \begin{cases} \text{linear} & x < -X_P \\ \text{constant} & -X_P < x < X_n \\ \text{linear} & x > X_n \end{cases} \]
Reverse bias

Energy (E)

Current dominated by drift

Forward Bias

Energy (E)

Reduces internal electric field

Current dominated by diffusion

See slide

Depletion / Junction Capacitance

\[ C = \frac{\varepsilon A}{d} \Rightarrow d = \frac{W}{x_n + x_p} \]

\[ C = \frac{\varepsilon A}{W(W)} \]
\[ C_i = \frac{C_{i0}}{\sqrt{1 + Vr/V_0}} \]

\[ C_{i0} = A \sqrt{\left( \frac{E_{sb}}{2} \right) \left( \frac{N_A N_D}{N_A + N_D} \right) \left( \frac{1}{V_0} \right)} \]

\[ C \]

\[ \text{reverse bias} \quad \downarrow \quad C \]

\[ \text{forward bias} \quad \downarrow \quad \text{current} \]

\[ \text{mainly diffusion: minority carrier injection} \]

**Assumptions**

1. Absent depletion layer
2. Classical statistics
3. Complete dopant ionization
4. Total current through structure

Derive current through P/N junction w/ assumptions #1 - #4 above
Fig 3.12

Minority carrier concentration under forward bias

\[ n_p(x_p) \]

\[ n_p(0) \]

\[ -x_p \]

\[ P_n(x_n) \]

\[ P_n(0_n) \]

\[ \text{excess concentration} \]

\[ x_n \]

\( \Rightarrow \) approach: derive minority carrier concentration as a function of position

\[ \Rightarrow \] calculate resulting current

Forward bias lowers barrier for holes to diffuse into n region

- consider holes injected in n region (minority carriers)

\[ P_n(x_n) = P_{n_0} \exp \left( \frac{V}{V_T} \right) \]

\[ P_{n_0} = \frac{n_i^2}{N_D} \]

\[ P_n = \text{hole concentration in n region} \]
1) Excess concentration of holes in n region result from forward bias

\[
\text{Excess concen} \bigg|_{x = x_n} = P_n(x_n) - P_{n0} = P_{n0} \left( e^{\frac{V}{V_T}} - 1 \right)
\]

2) Solve for steady state concentration ≠ equilibrium

As holes diffuse in material, some recombine with electrons resulting in exponentially decaying profile of holes

\[L_p = \text{diffusion length that characterize decay}\]

\[P_n(x) = P_{n0} + \left\{ \text{excess concen} \right\} \exp \left( -\frac{(x-x_n)}{L_p} \right)\]

\[P_n(x) = P_{n0} + P_{n0} \left( e^{\frac{V}{V_T}} - 1 \right) e^{-\frac{(x-x_n)}{L_p}}\]

\[J_p(x) = -q D_p \frac{dP_n(x)}{dx} \text{ (diffusion current)}\]

\[J_p(x) = q \left( \frac{D_p}{L_p} \right) P_{n0} \left( e^{\frac{V}{V_T}} - 1 \right) e^{-\frac{(x-x_n)}{L_p}}\]
all injected holes recombine with electrons & electrons from external circuit replace combined electrons

max current density at $J_p \max$
this happens at $x = x_n$

$J_{\max} = J_p(x_n) = q\left(\frac{D_p}{L_p}\right) n_p 0 \left(e^{\frac{\nu}{\nu_T}} - 1\right)$

3 same argument for electrons

$J_n(x_p) = q\left(\frac{D_n}{L_n}\right) n_p 0 \left(e^{\frac{\nu}{\nu_T}} - 1\right)$

$J_p(x_n) \neq J_n(x_p)$ values do not change inside depletion region

\[ \text{(Aside)} \]

relevant equation

$\delta p_n = p_n - p_{0} = \text{excess minority carrier concen}$

$D_p \frac{\delta^2 (\delta p_n)}{\delta x^2} - \nu p \leq \frac{\delta (\delta p_n)}{\delta x} + g' - \frac{\delta p_n}{T_{p0}}$

$\delta (\delta p_n) / \delta t$

For $x > x_n$ or $x < x_p \Rightarrow \xi = 0, g \leq 0$

Steady state $\delta / \delta t = 0$

$\frac{\delta^2}{\delta x^2} (\delta p_n) - \frac{\delta p_n}{T_{p0}} = 0$

Total current $I = A (J_p + J_n)$
\[ I = A q_b \left( \frac{D_p}{L_p} P_{no} + \frac{D_n}{L_n} N_{po} \right) (e^{\frac{V}{VT}} - 1) \]

\[ P_{no} = \frac{n_i^2}{N_D} \quad N_{po} = \frac{n_i^2}{N_A} \]

\[ I = A q_b n_i^2 \left[ \frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right] (e^{\frac{V}{VT}} - 1) \]

\[ I = I_s \left[ e^{\frac{V}{VT}} - 1 \right] \]

*See 2 slides on current*