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Design Procedure for Linear Series-Fed Arrays of Microstrip Patches Covered with a Thick Dielectric Layer

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SUMMARY

This report describes a computer-aided procedure developed for the design of linear series-fed arrays of microstrip patches with (or without) a dielectric cover layer.

A Multiport Network Model (MNM) approach for the analysis of radiating microstrip patches has been developed for this project. In this approach, the fields underneath the patch and outside the patch are modeled separately in terms of subnetworks which are characterized in terms of immittance matrices. The segmentation method is used to combine these segments to compute the antenna characteristics such as input impedance and radiation pattern. Analysis techniques developed for multiport network such as sensitivity analysis and optimization procedures can be readily applied to microstrip patches and arrays when MNM is used.

The MNM method has been used for the analysis of multiple-port rectangular patches with (and without) cover layer. Effects of microstrip feed junction reactances and mutual coupling (MC), which are important in the design of such patches, have been included. The method has been also used for the evaluation of MC between rectangular patches and also for the design of multiple-port circular patches. Results obtained by the present approach have been verified by careful experiments.

A computer-aided design CAD package based on MNM has been developed for the design and sensitivity analysis of series-fed arrays of rectangular microstrip patches. The purpose of the CAD software is to eliminate the recourse to try-and-cut experimental iterations used in the present methods of arrays design. Presence of the array losses and MC among array elements, which have substantial effects on array performance, have been included. Results based on this approach have been verified by experiments.
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CHAPTER I

INTRODUCTION

1.1 Concept of microstrip antennas

The concept of creating antennas from printed microstrip lines was first proposed by Deschamps [1] in 1953. Until 1970, only few research efforts on microstrip antennas were reported in the literature [2-4]. The work of Munson [5] and others in early 70's on the use of microstrip antennas as low profile flush mounted antennas on rockets and missiles has started a new antenna industry and a new area of microwave antennas.

As shown in Fig 1.1, a microstrip antenna in its simplest form consists of a thin metal patch of dimensions comparable to a half wavelength on top of a thin dielectric layer (substrate) of thickness h backed by a ground plane. The radiation occurs from the fringing fields between the periphery of the patch and the ground plane. The dielectric constant ($\varepsilon_r$) of the substrate should be low so as to enhance the radiation of the antenna. Various types of printed antennas for use in the microwave and millimeter wave frequency ranges have been developed. These include microstrip patch, printed dipole antenna and stripline slot antenna. The present work reported in this report deals only with microstrip patch antennas. This type of antennas has numerous advantages compared to conventional microwave antennas (such as parabolic dish antenna).
Figure 1.1 Microstrip patch antenna configuration

Some of those advantages are [6]

- Lightweight and low volume.
- Low fabrication cost, suitable for mass production.
- Conformal, can be easily mounted on missiles, rockets, satellites and airplanes.
- Both linear and circular (left hand and right hand) polarizations are obtained easily.
- Dual and multiple frequency operations easily achievable.
- Compatible with integrated circuits; oscillators, amplifiers, phase shifters, etc. which can be fabricated on the same substrate as the patch.

However, microstrip antennas suffer from some disadvantages compared to conventional microwave antennas, including [6]

- Narrow frequency bandwidth.
- Lower antenna gain because of losses (dielectric, conductor and surface wave).
- Poor endfire radiation.
- Radiation from feed network which contributes to cross-polarization radiation.
- Excitation of surface waves along the substrate which puts limitations on the achievable side lobe level of an array.

Special techniques have been devised to overcome some of the disadvantages mentioned above. For a large number of applications, the advantages of microstrip antennas outweigh its disadvantages. These antennas are finding increasing uses in areas such as satellite communication, radars, missile telemetry and biomedical applications.

1.2 Methods of analysis of microstrip patch antennas

One of the interesting features of microstrip antennas is that the patch can be of any planar shape; some of the configurations used in practice [6] are shown in Fig 1.2. The radiation characteristics are different for different shapes.

In this section, we review briefly the various methods used currently for the analysis of microstrip patches. These methods of analysis can be divided into two main categories. In the first type of methods, the radiation is formulated in terms of equivalent magnetic current line sources or slots located at the open edges of the patch. These methods include the one-dimensional transmission line model and the two-dimensional cavity model. Several approximations built in these models (edge admittance characterization modeling of fields underneath the patch) lead to inaccuracies in results, especially when the substrates are not electrically thin (for example when the operation is extended to the millimeter wave frequency). However, for most microstrip applications these methods are found to be adequate, as they yield a general description of the antenna behavior. But when multilayer substrates or when
Figure 1.2 Various microstrip antenna configuration used in practice.
arbitrary shapes are used, the second type of methods making use of the surface current distributions on the patch conductor are used. In the second category of methods, the solution is formulated in terms of an integral equation for the current on the upper conductor. Once the current distribution is computed, antenna characteristics (resonant frequency, far field, input impedance etc.) are calculated.

1.2.1 Transmission line model

A first attempt to model microstrip patch antennas was carried out by the use of an equivalent transmission line [4,7] for rectangular patches fed at one of the radiating edges. This method by far is the simplest method for analysis and design of rectangular patches. For evaluating the radiation characteristics when operating in the (0,n) mode, the antenna is modeled by two parallel slots of magnetic current as shown in Fig 1.3.a. The length of the slot is equal to the width of the patch, and the width of the slot is equal to the height of the substrate. For evaluating the input impedance/admittance, the antenna is modeled by two admittances \((G_r + jB)\) connected to the ends of the equivalent transmission line as shown in Fig. 1.3.b. The slot radiation conductance \((G_r)\) is computed by integrating the real part of the Poynting vector over the hemisphere in the far field. The effects of fringing fields at the two transmission line open ends are accounted for by using an edge susceptance \(B\) or equivalently by extending the antenna length by an appropriate amount. The analysis takes into account the effect of mutual coupling between the two slots, which is significant for the half-wave patch (as discussed in Chapter Four). The mutual coupling conductance \(G_m\) between the two radiating ends is determined by integrating the interference components of the far fields radiated by the two
a) Rectangular microstrip patch represented as two radiating slots for computation of radiation characteristics.

b) Equivalent circuit of a one-port rectangular microstrip patch fed at a radiating edge.

Figure 1.3 Equivalent transmission line model of a rectangular patch.
slots [8]. The effect of the mutual susceptance between the radiating edges and
the mutual conductances between the non-radiating edges have been included
in the transmission line model reported in [9]. Transmission line model for
two-port rectangular patches with ports located at the non-radiating edges is
described in [10]. This model does not account for the variation of the voltages
along the radiating edges and the excitation of higher order modes \((m=0,n)\) of
the rectangular patch when fed by either a coaxial or a microstrip line. The
transmission line model has been extended to the analysis of microstrip patches

1.2.2 Cavity model

This model can be used for geometries in which the two-dimensional
Helmholtz equation permits an analytical solution. The fields underneath the
patch are expanded in terms of the complete set of cavity modes. The radiated
power by the patch is accounted for either by considering the cavity to be filled
by a dielectric with an effective loss tangent [12] or by considering the magnetic
walls to be replaced by impedance walls [13]. This model yields an accurate
determination of the input impedance. The feeds are modeled (for both coaxial
and microstrip feed) by an equivalent \(z\)-directed ribbon of electric current of
rectangular cross section with an effective width. This effective width accounts
for the feed junction reactances and is determined by comparing theoretical and
experimental impedance loci [14]. This model does not account for the effect of
mutual coupling among the edges of the same patch. Extension of the cavity
model to more general shapes is discussed in [36].

1.2.3 Full wave analysis methods
These methods are more accurate, making use of the surface current distribution on the upper conductor. On the other hand, they are mathematically cumbersome and give little insight into the behavior of microstrip antennas. Moreover, extraction of partial effects of different phenomena (like mutual coupling, feed junction reactances) is difficult. One of the most noticeable disadvantages of these methods is the expensive computation time required for computation of the fields, such as inversion of large matrices and the evaluation of Sommerfeld type of integrals.

Spectral domain analysis:

In this analysis the field equations are solved for in the spectral domain (Fourier domain for rectangular patches [15] and Hankel domain for circular patches [16,17]). Algebraic eigenvalue equations for the currents are obtained by using the Galerkin Method. This approach provides an exact mathematical solution, from which the radiation pattern and the resonant frequency are obtained. The imaginary part of the resonant frequency gives the power loss of the patch (the sum of dielectric, conductor, radiation and surface wave power losses). The input impedance of microstrip patches was not evaluated by this method because of the difficulty of modeling the excitation.

Methods based on Green's function:

In this approach a linear relation between the electric field everywhere in space and an elementary electric current dipole source is expressed in terms of a dyadic Green's function [18]. Then the superposition of all current sources on the upper conductor yields the total field radiated by the patch. Using the boundary condition of vanishing tangential electric component on the upper
conductor yields an integral equation for the surface electric current. In the absence of excitation in the formulation, the resonant frequency as well as the current distribution of a mode can be computed. In the presence of excitation (coaxial or microstrip) the input impedance and the radiation pattern are computed. The main difficulty associated with this method is in the computation of the Green’s function for the microstrip configuration (which may consist of one or more substrates and superstrates). The solution of the resulting integral equation has no closed form expression and thus is solved by numerical techniques (like method of moments [19,20]).

Many other numerical methods for the analysis of microstrip patches have been reported, including the wire grid model [21] where the radiating structure is modeled as a fine grid of wire segments. The current in these wire segments are computed by using Richmond’s reaction theorem and then all the antenna characteristics are obtained from the currents. The computation of these currents require considerable computer storage and time. In the finite element method [22], the fields interior to the microstrip antenna cavity are computed by using a finite element approach applied to the inhomogenous wave equation. Microstrip patches have also been analyzed by considering the fields underneath the patch as superposition of TEM waves bouncing of the edges of the patch [23]. This theory is limited to few canonical shapes where ray tracing is possible.

1.3 Analysis of series-fed linear arrays

In most applications, individual microstrip patches are combined together to form linear arrays so that to satisfy the design requirements like gain, side lobe level and beam direction. The simplest form of feed for linear arrays is
the series feeding (shown in Fig 1.4) in which the patches are interconnected by sections of transmission lines [7, 24-27]. This results in a compact arrangement where the feed and the patches are etched on the same side of the board. The series fed array can be terminated in a resistive matched load to increase its bandwidth or to obtain the required array amplitude distribution. When power is fed to the array, a fraction of the power gets radiated by the first element and the remaining power is transmitted to the rest of the array. A similar power distribution takes place at the second element and the subsequent elements. Thus each element in the array acts as a power divider. The amount of power radiated by an element can be controlled by the patch width. When the patches are fed at the non-radiating edges, the power radiated can also be controlled
by the relative locations of the input/output ports.

One commonly used method of analysis of series-fed arrays which has been reported in [24-27] is based on the transmission line model. An equivalent transmission line model for the series fed array is also shown in Fig 1.4. The array is modeled as a cascade of unit cells. Each unit cell consists of the radiation conductance of the slots to which are attached sections of transmission lines. Since the transmission line model does not account for the feed junction reactances, the length of the interconnecting transmission lines are computed by experimentally measuring the transmission phase of the two-port patches 25. The transmission line model is not accurate enough for the design of long series-fed arrays with low side lobe levels. Due to the large size of series fed linear arrays which can be of the order of several wavelengths, the use of full-wave analysis methods like the moment method is impractical due to the need for inversions of large matrices involved in the computations of the fields.

1.4 Mutual coupling effects

The effect of mutual coupling on microstrip array can cause a deterioration in side lobe level and may even cause array blindness [28]. Typical interelement spacing in an array is between 0.5 and 1.0 wavelength. Experimental study of [29] shows that the mutual coupling for the above type of spacing is not negligible. No work dealing with the design of series fed linear array of microstrip patches including the effect of mutual coupling has been reported in the literature. This effect is important for the case of low side lobe level arrays and therefore should be included in the analysis and design of such arrays.
1.5 Microstrip patches with dielectric cover layer(s)

In several applications, microstrip antennas and arrays are coated with dielectric materials for protection against wind, rain etc. Layers of ice can be formed on microstrip patches used outdoors and aboard spacecraft. The presence of such layers affects the performances of the antennas (like the resonant frequency and the input impedance) and causes the excitation of surface waves. These effects become more pronounced when the cover layer thickness increases. The effect of cover layer on the resonant frequency of a rectangular microstrip patch are discussed in [30]. Paper [31] is a full-wave analysis of the input impedance and the mutual coupling among rectangular microstrip patches with a dielectric cover layer. Experimental verifications of the effect of cover layer on the resonant frequencies of rectangular microstrip patches have been reported by [32].

1.6 Need for multiple port microstrip patches

Microstrip patches with multiple number of ports constitute building blocks for linear and planar arrays. Example of an array using multiple port patches [33] are shown in Fig 1.5. The use of multiple port patches results in the design of linear and planar arrays with minimum length of interconnecting transmission lines compared to the use of corporate feed. So the power loss from the feed lines is minimum. The interconnecting lines are usually kept as straight lines so the radiation from those lines is very small. The design of array of Fig 1.5 requires the design of multiple port patches. The design procedure involves the determination of the ports locations so that to obtain a match at the input port and at the same time control the amount of power radiated by the patch as well as the power transmitted to the output port(s). The design
Figure 1.5 Planar array using multiple-port square microstrip patches.
of multiple port rectangular microstrip patches is discussed in *Chapter 4* and
the design of multiple port circular patches is discussed in *Chapter 5*.

### 1.7 Organization of the rest of the report

In the beginning of this chapter, we reviewed briefly the methods of
analysis available for microstrip patches and arrays. The advantages and disad-
vantages of each method have been outlined. The purpose of the work reported
in this report is to present an alternative method of analysis for microstrip
patches and arrays namely the multiport network model (MNM), which is ca-
pable of

(i) Analysis of microstrip patches with arbitrary shapes.

(ii) Analysis of multiple-port microstrip patches.

(iii) Taking into account the excitation of higher order modes inside the patch
and the feed lines.

(iii) Including the effects of mutual coupling when designing microstrip patches
and arrays.

(iv) Taking into account the effect of cover layer.

The MNM was used earlier for the design of wideband rectangular
coupled microstrip patches [34] and for circularly polarized patches of various
shapes [35]. In this approach the patches are analysed by modeling the internal
and external fields in terms of multiport networks. Thus the analysis and design
of patches reduces to the study of an equivalent multiport network problem.
Analysis techniques developed for network problems like optimization and sen-
sitivity analysis can be readily applied to microstrip patches and arrays when
the MNM approach is used.

In Chapter Two, a more detailed description of MNM is presented.
Effects of microstrip feed junction reactances are included in the MNM by modeling sections of feed lines as rectangular segments. Effects of fringing fields are included using both a fringing edge capacitance and a fringing edge inductance. The MNM can be extended to the analysis of microstrip patches of arbitrary shapes using the segmentation and desegmentation methods.

In Chapter Three, the multiport network model is extended to include the effects of mutual coupling on microstrip patches. The mutual coupling can have serious effects on the performance of microstrip patches and arrays. The mutual coupling is modeled in terms of equivalent mutual coupling network (MCN). The elements of the admittance matrix characterizing MCN represent the interactions among the edges of the same patch or among patches of an array. Results for mutual coupling coefficient between two identical rectangular microstrip patches will be compared with available experimental results.

In Chapter Four, the MNM model and the associated network analysis will be used for the analysis, optimization and design of multiple port rectangular microstrip patches. Multiple port rectangular microstrip patches are used extensively for linear and planar arrays. Two-port patches with ports located at the radiating edges as well as along the non-radiating edges are discussed in details. The effect of mutual coupling on a rectangular patch and the relative contributions of different edges are discussed. The performance of a typical two-port rectangular patch using the present model is compared with experimental results.

In Chapter Five, the MNM is used for the analysis and design of circular microstrip patches. The MNM is first employed for the analysis of one-port circular microstrip patches. Comparison for the input impedance using various
models for the external fields will be presented. As in the case of rectangular segments, the Green's function can be expressed as a single summation using a mode-matching technique. The computation of the elements of impedance matrix using the Green's function expressed as single summation, is very efficient. Analysis of two-port circular microstrip patches (which may be considered as elements of a series-fed array) using the MNM model is presented and the results are compared with experiment.

The design of microstrip patches and arrays in the presence of a cover layer is discussed in Chapter Six. The presence of a cover layer affects the patches performance, including the resonant frequency, input impedance and bandwidth. The presence of a cover layer also causes the excitation of surface waves. A method for computing the radiation and surface wave conductances using equivalent magnetic current line sources at the periphery will be given. Variation in the mutual coupling because of a cover layer compared to the case without a cover layer will be discussed.

In Chapter Seven, the MNM is used for the computer-aided design and analysis of linear series-fed arrays of rectangular microstrip patches. A series-fed linear array is analyzed by using the concept of unit cells. Each unit cell consists of a radiating patch with two sections of transmission line connected to it. Once the array specifications are given, the required transmission coefficient of each array unit cell is determined. Analysis procedures developed in Chapters Four and Six are used for the analysis of unit cells with specified transmission coefficients. The dielectric and conductor losses are included in the array design procedure. Sensitivity of the array with respect to different design parameters will be discussed. Results obtained for a 19-element series-fed lin-
ear array with Taylor distribution using this design methodology are compared with experiment.

In Chapter Eight, a summary of the present work is given. Suggestions for further improvements in the model are mentioned.
CHAPTER II

MULTIPORT NETWORK MODEL FOR RADIATING MICROSTRIP PATCHES

In this Chapter, a multiport network model (MNM) approach for the analysis of microstrip patches is discussed. In this approach, the fields underneath the patch, the external fields (radiated, surface wave and fringing fields) and the fields underneath the microstrip feed lines are modeled separately in terms of multiport subnetworks which are characterized in terms of $Z$-matrices or $Y$-matrices. These subnetworks are then combined using segmentation technique to obtain the antenna characteristics such as resonance frequency, scattering parameters, bandwidth and the radiation pattern. The fields on either side of an interface between any two subnetworks are matched at discrete points by subdividing the common interface into a number of sections. The MNM approach can conveniently incorporate the effects of feed junction reactances as well as of the mutual coupling. A detailed description of the subnetworks modeling a microstrip antenna is presented in this chapter. This is followed by an analysis for antenna characteristics based on the segmentation method.

2.1 Separate representation of internal and external fields

The geometry of the microstrip patch antenna along with the coordinates system employed is shown in Fig 2.1. The time dependence $e^{i\omega t}$ is assumed for the fields. The substrate material is assumed to be non-magnetic
(but the presence of a magnetic material can be included in the present analysis). The ground plane and the dielectric substrate are assumed to be infinite in extent.

For electrically thin substrate \( (k_0 h < < 1) \), the fields underneath the patch are analyzed by treating the patch as a two-dimensional planar component. The external fields (radiated power, surface waves and fringing fields) are represented by equivalent edge admittance networks (EAN) connected to the patch periphery at a finite number of points. The following subsections discuss the \( Z \)-matrix characterizing the internal field and the external field networks.
2.1.1 Fields underneath the patch

In most applications, the thickness of the substrate used for microstrip antennas is small compared to the operating wavelength. Fields at the edges of the antenna may vary in the direction perpendicular to the patch, but this $z$-variation decays rapidly as one move inwards and away from the feed and the patch edges, leaving only the fields which are uniform along $z$. So, a solution of the electromagnetic fields in the region between the patch and the ground plane can be obtained by considering the patch as a two-dimensional cavity with magnetic wall boundaries. The $z$-varying fields near the edges are accounted for by edge admittances discussed later.

For patches of regular shapes (rectangles, circles, rings, sectors of circles and rings and three types of triangles), the multiport planar component representing the patch (with ports on the periphery open circuited) can be analyzed by using two-dimensional impedance Green's functions available for these shapes [37,38]. Elements of the $Z$-matrix of the patch are derived from the Green's function as

$$Z_{ij} = \frac{1}{W_i W_j} \int_{W_i} \int_{W_j} G(x_i, y_i | x_j, y_j) (ds_i)(ds_j) \quad (2.1)$$

where $(x_i, y_i)$ and $(x_j, y_j)$ denote the locations of the two ports of widths $W_i$ and $W_j$ respectively (as shown in Fig 2.2). The ports can be located anywhere inside the patch or at the periphery of the patch. A detailed derivation of (2.1) is given in Appendix A. The two line integrals in (2.1) are along the widths of the ports $i$ and $j$. Green's function $G$ is usually a doubly infinite summation with terms corresponding to various modes of the planar resonator (rectangular or circular or triangular) with magnetic walls.
The effect of the dielectric losses is incorporated by considering $\varepsilon_r$ to be a complex quantity. Conductor losses are also included in an approximate manner by defining an equivalent loss tangent $\delta_c$ as [12]:

$$
\delta_c = \frac{P_c}{P_d} \delta_d
$$

where $P_c$ is the power dissipation because of the conductor loss in the patch; $P_d$ is the dielectric loss in the substrate with loss tangent $\delta_d$. In the limit
of magnetic wall boundary, \( \delta_c \) (given by 2.2) is independent of the resonator geometry [38] and is equal to \( \sqrt{(2/\omega \mu \sigma)} / h \), where \( \sigma \) is the conductivity of the patch metalization. In the cavity model [12], the radiation is also considered by defining an equivalent loss tangent \( \delta_r \) as

\[
\delta_r = \frac{P_r}{P_d} \delta_d
\]

where \( P_r \) is the power radiated by the patch.

For patches with composite shapes, an MNM can be constructed by treating the composite shape as a combination of the elementary shapes for which Green’s functions are available. Segmentation and desegmentation methods [37,38] are used for finding the Z-matrix of a composite shape from those of elementary segments. If the patch or one of the segments of a composite patch is of an irregular shape for which the Green’s function is not available, a contour integral method [37,38] can be used to evaluate the elements of the Z-matrix.

2.1.2 Fields outside the patch

In the MNM for radiating microstrip patches, the fields outside the patch (namely; the fringing fields at the edges, the surface wave fields and the radiation fields) are incorporated by adding equivalent edge admittance networks (EAN’s) connected to the edges of the patch (as shown in Fig 2.3). The concept of representing the fringing fields and the radiated power by an edge admittance has been used earlier in conjunction with the transmission line model of rectangular patches [8]. However, in the present case, a multiport edge admittance network (shown in Fig 2.3) is used in place of a single admittance used in the transmission line model (shown in Fig 1.3). When a microstrip
patch has a polygon shape (rectangular and triangular geometries), the periphery of the patch is divided into edges. Each edge may have different voltage distribution. For geometries similar to the circular patch, the whole periphery is taken as a single edge.

The EAN for each edge is a multiport network consisting of combinations of a capacitance $C$, an inductance $L$ (representing the energy stored in the

Figure 2.3 Equivalent edge admittance network for modeling of external fields
fringing electric and magnetic fields respectively) and a conductance $G$ (representing the power carried away by radiation and surface waves). A portion of the typical EAN is shown in Fig 2.4. Similar sections are connected to the ports of the equivalent planar multiport representation of the patch.

![Diagram of equivalent edge admittance network (EAN)](image)

**Figure 2.4 Elements of equivalent edge admittance network (EAN)**

**Edge conductance**

The edge conductance $G$ in an EAN consists of two parts: a radiation conductance $G_r$ and a surface wave conductance $G_s$. The radiation conductance associated with an edge of a microstrip patch is defined as an ohmic conductance (distributed or lumped), which when connected to the edge (continuously or at discrete ports) will dissipate a power equal to that radiated by the patch. If the edge has a width $W$ and the power radiated for a uniform voltage distribution is $P_{rad}$, the radiation conductance of the edge is given by
\(2P_{rad}\), where \(P_{rad}\) is calculated for a unit voltage at the edge. When the voltage amplitude distribution along an edge is given by \(f(s)\), the radiation conductance of the edge is obtained as:

\[
G_r = \frac{2P_{rad}}{\frac{1}{W} \int_0^W f^2(s) ds} \tag{2.4}
\]

where \(s\) denotes the distance along the edge of the patch and \(P_{rad}\) is the power radiated by the edge with a voltage distribution \(f(s)\). A method for computing the radiated power from the edges of a microstrip patch is discussed in Section 2.4.4. The concept of edge conductance can be implemented when \(f(s)\) is known apriori. In most of the cases, microstrip antennas are operated near resonance frequency of the patch and \(f(s)\) is known at least approximately. For more accurate results, iterative computations may be needed. Starting from an approximate \(f(s)\), an analysis based on the MNM model is carried out to evaluate the voltages at the \(n\) ports on the edge. This computed voltage distribution is then used as a modified \(f(s)\) and the computation of \(G_r\) is repeated.

The surface wave conductance \(G_s\) is defined in a similar manner. We write:

\[
G_s = \frac{2P_{sur}}{\frac{1}{W} \int_0^W f^2(s) ds} \tag{2.5}
\]

where \(P_{sur}\) is the power coupled to the surface waves (along the substrate) excited by the voltage distribution \(f(s)\) at the edge. For thin substrates without any cover layer, \(G_s\) is much smaller than \(G_r\) and may be neglected [39].

If we select \(n\) uniformly spaced ports (each representing a section of length \(W/n\)) along the edge, the conductance \(G_p\) connected to each of the ports is taken as \((G_r + G_s)/n\).
Edge capacitance

The edge capacitance $C$ accounts for the energy stored in the fringing electric field at the edge of the patch. The fringing capacitance is found as the excess of total capacitance of the patch over that which would exist if the patch is considered as a two-dimensional capacitor with magnetic walls at the open edges. As in the case of the edge conductance, the edge capacitance is also distributed uniformly over the $n$ ports $(C_p = C/n)$. Formulas for the edge capacitance are available only for simple shapes, namely rectangular [40,41] and circular geometries [43]. For other shapes, the edge capacitance available for a shape closest to the given shape can be used [35]. For example, for a pentagonal patch, Suzuki and Chika [44] have used the results for a circular disk of the same planar area as that of the pentagonal patch. Formula for the edge capacitance of an arbitrary shaped microstrip patch are given in [42].

Edge inductance

The edge inductance $L$ accounts for the energy stored in the fringing magnetic field at the edge of the patch. As in the case of $G$ and $C$, the edge inductance is also distributed uniformly over the $n$ ports $(L_p = L/n)$.

The role of of the edge inductance is illustrated by considering the case of a coaxial-fed rectangular patch (shown in Fig 2.5) operating in the dominant $TM_{10}$ mode. An equivalent MNM of this coaxial-fed rectangular patch is also included in the Figure. The patch has a width $b$ and length $a$ ($\approx \lambda/2$). The substrate has a dielectric constant of $\varepsilon_r$ and thickness $h$. The electromagnetic field distribution for the dominant mode is shown in Fig 2.6. The field distribution in the $y$-$z$ plane is identical to that of a transmission
Figure 2.5 Multiport network model of a coaxial-fed rectangular microstrip patch
line of width $b$. The electromagnetic fields underneath the patch are computed by considering the patch as a two-dimensional segment as discussed in Section 2.1.1.

![Diagram showing E and H-field lines](image)

---

**Figure 2.6** Electromagnetic field distribution for the dominant mode $TM_{10}$ (transverse cross-sectional view)

The presence of fringing magnetic field (which extends beyond the physical boundaries) changes the value of the inductance per unit length of the transmission line and hence the value of propagation velocity and characteristic impedance of the line. The edge inductance per unit length $L_e$ is obtained as the difference between the total inductance $L_T$ per unit length (for the microstrip configuration shown in Fig 2.5) and the inductance $L_p$ associated with the fields underneath the parallel plates of width $b$. For a rectangular patch operating in the dominant mode (when it can be modeled as a transmission line), $L_T$ is given by

$$L_T = \frac{\mu_0 \varepsilon_0}{C_T(\varepsilon_r = 1)}$$

(2.6)
where $C_T(\epsilon_r = 1)$ is the total capacitance per unit length for the microstrip configuration (shown in Fig 2.5) with dielectric replaced by air ($\epsilon_r = 1$). The parallel plate inductance per unit length for a transmission line of width $b$ is given by

$$L_p = \frac{\mu_0 \epsilon_0}{C_p(\epsilon_r = 1)} = \mu_0 \frac{h}{b}$$ (2.7)

where $C_p(\epsilon_r = 1)$ is the parallel plate capacitance with $\epsilon_r = 1$. The total inductance $L_T$ per unit length of the patch is the parallel combination of $L_p$ and $L_c$ on each edge. So, $L_c$ is obtained as

$$L_c = 2 \frac{L_p L_T}{L_p - L_T}$$ (2.8)

Using expressions (2.6-2.8), the fringing inductance per unit length $L_c$ is related to the fringing capacitance $C_e = (C_T - C_p)/2$ by

$$L_c = \frac{\mu_0 \epsilon_0}{C_e(\epsilon_r = 1)}$$ (2.9)

The above discussion shows that the fringing inductance per unit length and the fringing capacitance per unit length (for $\epsilon_r = 1$) at the non-radiating edges of a rectangular patch are related, and therefore $L_c$ can be calculated from the value of the fringing capacitance $C_e$ for the air dielectric. This equivalence can be extended to the case when we have a smooth curved edge (such as a circular shaped patch as will be discussed in Chapter 5).

### 2.1.3 Comparison of various models for fields outside the patch

In the equivalent parallel plate waveguide model for a microstrip line, the fringing fields are accounted for by using an effective width and effective dielectric constant for the line. These two parameters are chosen such that $L_T$
and \( C_T \) per unit length of the original line and the corresponding values for the parallel plate waveguide model are equal. A multiport network representation of the fringing fields in this case, would consist of a parallel plate waveguide of width equal to the physical dimension \( b \) and two edge admittance networks on either side of the parallel plate waveguide. This representation can be extended to two-dimensional resonators also. It needs to be emphasized that the correct representation of edge fields by equivalent networks require the addition of fringing inductance also (as shown in Fig 2.4).

This aspect may be elaborated by considering an example of a probe-fed rectangular microstrip patch with the following parameters: \( a = 1.3128 \) cm, \( b = 0.6 \) cm, \( z_1 = 0.5416 \) cm, \( \varepsilon_r = 2.2 \), \( h = 1/32 \) inch and \( \delta_d = 9 \times 10^{-4} \). The computation of the Z-matrix of the rectangular segment (modeling internal fields) and the Z-matrix characterizing the fringing fields at the radiating edges (segment RE-EAN) are discussed in Chapter Four. For the present discussion, we neglect the value of conductances in the NRE-EAN and consider only the capacitance \( C \) and inductance \( L \).

We compute the input impedance using three different models for the fringing fields along the non-radiating edges. In Model 1, we remove the NRE-EAN and use an effective width \( (b_e = 0.7205 \) cm) and an effective dielectric constant \( (\varepsilon_{re} = 2.066) \) for the PATCH segment. In Model 2, we use an edge admittance network (NRE-EAN) with an edge capacitance as well as an edge inductance \( (L_e = 2\mu_0 h/(b_e - b)) \). In Model 3, we use an edge admittance network (NRE-EAN) consisting of the edge capacitance \( (C_e = \frac{1}{2}\varepsilon_0\varepsilon_r(b_e - b)/h) \) only. These three models are compared by evaluating the input impedance at the coaxial feed as a function of frequency.
The computed values of input impedance using Models 1 and 2 are shown in Table 2.1. The number of ports along each of the non-radiating edges for model 2 is 50. This number was arrived at from iterative computations of the input impedance. The two models give very close results for the input impedance. The computed resonance frequency using Model 1 is 7.5 GHz with the real part of the input impedance equal to 50.05 Ω. On the other hand, the computed resonance frequency using Model 2 is 7.496 GHz with the input impedance $Z_{in} = 52.5$ Ω.

Table 2.1 Variation of input impedance of a coaxial-fed rectangular patch with frequency (using Models 1, 2 and Cavity model)

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.40</td>
<td>8.21 + j24.24</td>
<td>8.67 + j25.17</td>
</tr>
<tr>
<td>7.45</td>
<td>26.65 + j31.00</td>
<td>26.18 + j32.24</td>
</tr>
<tr>
<td>7.49</td>
<td>50.04 + j11.72</td>
<td>53.13 + j7.38</td>
</tr>
<tr>
<td>7.50</td>
<td>50.05 - j0.06</td>
<td>50.85 - j5.10</td>
</tr>
<tr>
<td>7.51</td>
<td>45.13 - j10.06</td>
<td>43.97 - j14.30</td>
</tr>
<tr>
<td>7.55</td>
<td>19.16 - j18.64</td>
<td>17.61 - j19.09</td>
</tr>
<tr>
<td>7.60</td>
<td>7.38 - j11.79</td>
<td>6.85 - j11.75</td>
</tr>
</tbody>
</table>

Computed values of input impedance versus frequency for Model 3 are shown in Table 2.2. The resonance frequency using this model is 6.838 GHz.
Table 2.2 Variation of input impedance of coaxial-fed rectangular patch with frequency (using Model 3)

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.74</td>
<td>7.31 + j23.63</td>
</tr>
<tr>
<td>6.80</td>
<td>30.73 + j33.48</td>
</tr>
<tr>
<td>6.83</td>
<td>57.15 + j11.26</td>
</tr>
<tr>
<td>6.84</td>
<td>56.51 - j4.21</td>
</tr>
<tr>
<td>6.86</td>
<td>39.44 - j22.23</td>
</tr>
<tr>
<td>6.90</td>
<td>14.20 - j20.04</td>
</tr>
<tr>
<td>6.94</td>
<td>6.54 - j13.34</td>
</tr>
</tbody>
</table>

with corresponding input impedance $Z_{in} = 56.7 \, \Omega$. This resonance frequency is very low compared to the resonance frequencies computed using Models 1 and 2. This is due to the fact that the total inductance per unit length in Model 1 is given by $L_T^1 = \mu_0 h/b_e$ and for Model 3 is given by $L_T^2 = \mu_0 h/b$. The ratio (=1.0968) of resonance frequency computed by Model 1 ($f=7.5 \, \text{GHz}$) over that computed by Model 3 ($f=6.838 \, \text{GHz}$) is close to $\sqrt{L_T^2/L_T^1}$ (=1.0958).

When the voltage distribution along an edge is uniform, the voltages at two adjacent ports at the edge are equal and hence no current will flow through the edge inductance. In this case, the edge inductance need not be included and the EAN network simplifies to a parallel pairs of capacitance-conductance only. This situation occurs at the radiating edges of a rectangular patch operating in the dominant mode. Validity of this simplified EAN has been verified by con-
sidering the coaxial-fed rectangular patch of the above example. We compare the results of input impedance as a function of frequency using Model 1 with and without including fringing inductance along the radiating edges. The fringing inductance per unit length (along radiating edges) is related to the fringing capacitance per unit length by expression (2.9). The total fringing capacitance for a radiating edge is given by (4.24). The values of the input impedance using the two models are shown in Table 2.3. As expected, the effect of fringing inductance along the radiating edge is very small.

Table 2.3 Effects of fringing inductance along the radiating edges of a rectangular patch on the input impedance

<table>
<thead>
<tr>
<th>Freq. (GHz)</th>
<th>RE-EAN (no inductance)</th>
<th>RE-EAN (with inductance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.40</td>
<td>8.21 + j24.24</td>
<td>8.19 + j25.23</td>
</tr>
<tr>
<td>7.45</td>
<td>26.65 + j31.00</td>
<td>26.60 + j31.00</td>
</tr>
<tr>
<td>7.49</td>
<td>50.04 + j11.72</td>
<td>50.03 + j11.81</td>
</tr>
<tr>
<td>7.50</td>
<td>50.05 - j0.06</td>
<td>50.08 + j0.02</td>
</tr>
<tr>
<td>7.51</td>
<td>45.13 - j10.06</td>
<td>45.18 - j10.01</td>
</tr>
<tr>
<td>7.55</td>
<td>19.16 - j18.64</td>
<td>19.19 - j18.65</td>
</tr>
<tr>
<td>7.60</td>
<td>7.38 - j11.79</td>
<td>7.39 - j11.80</td>
</tr>
</tbody>
</table>

The common practice for evaluating the resonance frequency of 2-dimensional resonators (incorporating the fringing fields) is to use effective di-
dimensions larger than the physical dimensions. Calculation of these effective dimensions is commonly carried out by finding the total electrostatic capacitance. However, it needs to be pointed out here that the use of effective dimensions modifies the magnetic field distribution, in addition to adding the fringing capacitance at the edges. This yields results consistent with the MNM discussed above since fringing inductance is related to the fringing capacitance as in (2.9). Comparison of the three models discussed above, has shown clearly that the correct resonance frequency will not be obtained if only the fringing capacitance were to be added at the edges.

Comparison with cavity model:

Values of input impedance for the coaxial-fed patch (considered above) have been recomputed using the conventional cavity model [12], and a comparison with Model 1 is shown in Table 2.4. The effects of fringing fields at the non-radiating edges are accounted for in the cavity model by using an effective width and an effective dielectric constant. The effects of fringing fields along the radiating edges are included by using an effective length \(a_e=1.3939 \text{ cm}\) and the effect of radiation from the patch is included by using an effective loss tangent \(\delta_e\) for the substrate. As seen from Table 2.4, the cavity model agrees (within 9.5 % for real part of \(Z_{in}\)) with the MNM model. The agreement in the resonance frequency is 0.12 %.

2.2 Microstrip feed junction reactance

For microstrip-fed patch, the parasitic reactances at the junction between the microstrip feedline and the patch can be incorporated in the multiport network model by considering a small section of the feedline as an equivalent
Table 2.4 Comparison of cavity model with MNM for computation of input impedance of a coaxial-fed patch

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>MNM Model</th>
<th>Cavity Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.40</td>
<td>8.21 + j24.24</td>
<td>8.89 + j25.84</td>
</tr>
<tr>
<td>7.45</td>
<td>26.65 + j31.00</td>
<td>28.54 + j33.26</td>
</tr>
<tr>
<td>7.49</td>
<td>50.04 + j11.72</td>
<td>54.86 + j1.89</td>
</tr>
<tr>
<td>7.50</td>
<td>50.05 - j0.06</td>
<td>49.78 - j10.50</td>
</tr>
<tr>
<td>7.51</td>
<td>45.13 - j10.06</td>
<td>41.12 - j18.13</td>
</tr>
<tr>
<td>7.55</td>
<td>19.16 - j18.64</td>
<td>15.57 - j18.82</td>
</tr>
<tr>
<td>7.60</td>
<td>7.38 - j11.79</td>
<td>6.13 - j11.19</td>
</tr>
</tbody>
</table>

planar circuit connected to the patch at a finite number of (typically five) ports (as shown in Fig 2.7). The exact number of ports depends upon the variation of the fields along the common interface. For example, a larger number of ports is needed when the feedline is along the non-radiating edge of a rectangular patch than when it is along the radiating edge. One port on each side of the FEED segment (near the feed junction plane as shown in Fig 2.7) is used to connect EAN network to the FEED segment. This connection assures the continuity of the current flow tangential to the circumference of the patch.

The effect of fringing fields along the transmission line length are accounted for by using an equivalent effective width for the transmission line. The length of the feedline segment should be long enough (\( \geq \lambda_0/8 \)) so that the
Figure 2.7 Equivalent multiport network for modeling of microstrip feed junction

higher order evanescent modes (excited by the junction) decay and only the dominant quasi-TEM mode is present at the input of the feedline [45]. Thus, only one port is needed at the input end of the line section.

Solution of the field by combining the network models of the patch and of the feedline (by using Kirchhoff's network relations) is equivalent to expansions of the fields (in the feedline as well as in the patch) in series of eigenfunctions, and matching the fields at the interface. A similar procedure has been used previously [46] for the characterization of junction reactances in microstrip circuits. The effect of the feed junction is to add an inductive
reactance in series with the input impedance of the patch at the feed point.

2.3 Modeling of mutual coupling

The flexibility of the multiport network model leads to several advantages when compared with the conventional cavity model, for example, the multiport network model allows us to incorporate the effect of mutual coupling among different edges of a patch or among patches of an array [47] by defining a mutual coupling network (MCN). The edge admittance terms associated with various ports at the edges constitute the diagonal terms of the admittance matrix for MCN. The non-diagonal terms of this matrix are obtained from the ‘reaction’ between the equivalent magnetic current sources at the two corresponding sections of the edges. A detailed analysis of the effect of mutual coupling on rectangular microstrip patches is presented in Chapter III.

2.4 Analysis by segmentation method

Once the $Z$-matrices of different subnetworks modeling the microstrip patch antenna are computed, the segmentation technique is used to combine these $Z$-matrices to obtain the overall $Z$-matrix characterizing the antenna.

2.4.1 Segmentation method

The application of segmentation method to combine two subnetworks is described in Appendix B. The segmentation technique was first proposed by Okoshi and his colleagues [48] for the analysis of microwave circuits using the $S$-matrix formulation. Later, Gupta et al [49] showed that a $Z$-matrix formulation is more suitable to planar circuits and antennas, since the $Z$-matrix follows directly from the Green’s function approach. Another complementary technique for the analysis of planar components is the desegmentation method
[49]. This technique is applicable to shapes which can be considered as obtained from a simple shape by removal of one or more simple shapes.

For the analysis of microstrip patches reported in this report, the Z-matrix formulation is used for all segments. The segmentation is employed by combining two segments at a time. This process is computationally more efficient than when all the segments are combined at the same time, because, combining two segments at a time requires the inversion of a smaller matrix (Eqn. B–3) whose dimensions are proportional to the number of interconnected ports of the two segments. Use of the segmentation method to yield antenna’s characteristics is discussed in next sections.

2.4.2 Matching of fields at interconnecting ports

Matching of the fields inside and outside the patch at the patch periphery as well as along the feed junction is achieved by satisfying equivalent Kirchhoff’s network relations at those interconnecting ports. Equating the voltages at the connected ports is analogous to matching the total tangential E-field, and the continuity of currents ensures the continuity of the total tangential H-field at the edges of the patch. In practice, these boundary conditions are applied at a discrete number of ports along the interconnection. The number of ports is increased until the computed solution converges.

2.4.3 Computation of Z-matrix characterization of the antenna

Each of the subnetwork components of the patch is characterized in terms of Z-matrix. Impedance matrices for the planar components (patch and the transmission line feeds) are obtained by using the Green’s function approach described in Section 2.1.1. The Z-matrix for the EAN network is obtained by
inverting the Y-matrix described in Section 2.1.2. A segmentation formula
[37,38] is used to combine these three Z-matrices, two at a time, to yield the
Z-matrix characterizing the patch. For example, the applications of segmenta-
tion method to a single port antenna yield the input impedance. For a multiple
port patch, the segmentation procedure yields the Z-matrix characterization
with reference to these multiple ports. Elements of S-matrix (reflection and
transmission coefficients) are obtained from the Z-matrix using standard trans-
formation. The resonance frequency of a one-port patch is obtained by finding
the frequency where the reflection coefficient at the input port is zero (or mini-
mum). Also, the impedance bandwidth for a specified VSWR is computed from
the variation of the input impedance with frequency.

2.4.4 Evaluation of radiation characteristics

The segmentation method is used also for evaluating the voltages at
the radiating edges (at the interconnected ports between the PATCH and EAN
segments). These voltage distributions are expressed as equivalent magnetic
current distributions along the edges and used for the evaluation of radiation
characteristics by employing Kirchhoff-Huygen's integration formula.

It may be noted that in the cavity model [12] also, the radiated field
is obtained from the field distribution along the periphery of the patch; the
field distribution in this case being expressed as a sum of the doubly infinite
modes of the patch resonator. In another paper [50] on the planar circuit
analysis, the effect of the radiation loss on the characteristics of planar circuits
has been reported. In this method also, the edge field is expressed as a sum of
modes of the planar patch and the radiated field is evaluated therefrom. In the
MNM method presented in this report, we use the network analysis approach
to find the total field at various discrete points on the patch edges and use this distribution to find the far field.

The far-field electric radiation vector is related to the ports voltage $V_j$ at the patch periphery by [39]

$$\mathbf{L} = -2 \sum_{j}^{NC} \mathbf{a}_t \ W_j \ V_j \ e^{ik_0 \rho_j \sin \theta \cos (\phi - \phi_j)}$$  \hspace{1cm} (2.11)$$

where the factor 2 arises because of the image of the magnetic current element with respect to the ground plane. Other parameters used in (2.11) are as shown in Fig 2.8. The summation in (2.11) is with respect to all the NC ports at the radiating edges of the patch. $V_j$ is the average voltage at the $j^{th}$ port of width $W_j$. The radiation fields are related to the radiation vector $\mathbf{L}$ by [39]:

$$E_\theta = -i \frac{\mathbf{L} \cdot \mathbf{a}_\phi}{2\lambda_0 r} \exp(-ik_0 r)$$ \hspace{1cm} (2.12)$$

$$E_\phi = i \frac{\mathbf{L} \cdot \mathbf{a}_\theta}{2\lambda_0 r} \exp(-ik_0 r)$$ \hspace{1cm} (2.13)$$

and $H_\theta = -E_\phi/\eta_0$, $H_\phi = E_\theta/\eta_0$. Where $\lambda_0$ is the free space wavelength and $\eta_0$ is the free space impedance ($=377$ Ohms). The total power $P_r$ radiated by the patch is given by

$$P_r = \int_{0}^{\pi/2} \int_{0}^{2\pi} \frac{||\mathbf{L} \times \mathbf{a}_r||^2}{8\lambda^2 \eta_0} \sin \theta d\theta d\phi$$  \hspace{1cm} (2.14)$$

where $(\mathbf{a}_\phi, \mathbf{a}_\theta, \mathbf{a}_r)$ are unit vectors in spherical coordinates. The radiated power $P_r$ is used for computation of the radiation conductance $G_r$ (as given by 2.4).
Figure 2.8 Equivalent magnetic current line source for a microstrip patch on thin substrate
2.5 Discussions

A multiport network model has been proposed for modeling and analysis of microstrip patch antennas. Its main advantages are flexibility for incorporating the effect of feedline junction reactances and the effects of mutual coupling. These two effects are important in series-fed array design and need to be considered for ensuring the array performance.

In the MNM discussed in [45], the effect of fringing electric field at an edge of a microstrip patch is accounted for by using an edge capacitance, and power losses by radiation and coupling into surface waves are taken into account by using edge conductances. An additional equivalent edge inductance has been introduced in the MNM to take into account the effects of fringing magnetic fields at the edges of microstrip patch antennas. The effects of this inductance is mainly to cause a shift in the design frequency.

The effect of edge inductance has been illustrated by considering a coaxial-fed rectangular patch. The computed resonance frequencies with and without including the fringing inductance along the non-radiating edges differ by almost 660 MHz. This difference is equivalent to 11 times the bandwidth of this antenna (=60 MHz). This effect is severe because the voltage along the non-radiating edges has a cosine distribution. However, the effect of fringing inductance along radiating edges is very small because the voltage is uniform. For a rectangular patch ($TM_{10}$ mode), the fringing inductances and capacitances along the non-radiating edges may alternatively be taken into account accurately by using an effective width and effective dielectric constant for the patch. This approach will be used for the analysis and the design rectangular microstrip patches discussed in Chapters Four and Six.
For a rectangular patch, the fringing inductance can be evaluated from the knowledge of the fringing capacitance. This equivalence is also expected to hold good for other microstrip antenna shapes. The effects of fringing inductance on the performance of a circular microstrip patch antenna are discussed in Chapter Five.

For electrically thin substrates, the solution of the internal field by modeling the patch as planar component with magnetic walls is accurate. On the other hand accurate formulas are not available for edge admittance values. However, the approach presented in this report allows more accurate expressions of the edge admittance to be incorporated as and when they become available. Also, the model can include the value of the edge capacitance computed from experimental measurement (as discussed in Appendix C).

Using the MNM approach, techniques available for multiport networks [39] such as sensitivity analysis and optimization can be utilized for microstrip antennas and arrays also. Thus, the multiport network modeling approach is ideally suited for the computer-aided design of microstrip antennas and arrays.
CHAPTER III

MNM FOR MUTUAL COUPLING EFFECTS IN MICROSTRIP PATCH ANTENNAS AND ARRAYS

3.1 Introduction

A microstrip patch antenna is usually combined with other patches to form an array to achieve a specified gain, sidelobe level and beam direction (as will be discussed in Chapter 7). The electrical characteristics of a microstrip patch in an array, e.g; input impedance and voltage distribution at the patch periphery, are different from the values for an isolated patch. This is because of the presence of other patches at close proximity (typically at $\lambda/2$ spacing). For better array performance, the effects of the mutual coupling need to be properly accounted for during the design of the array. Effects of mutual coupling are undesirable and may alter the performance of arrays especially for scanned arrays and for arrays with low side lobes.

The role of mutual coupling in the design of microstrip antenna arrays is well recognized. Jedlicka and Carver [29] reported measured mutual coupling between rectangular and between circular microstrip patches. Their measurements show that the mutual coupling is not negligible. Malkomes [53] and Penard and Daniel [52] used the cavity model to study the effect of mutual coupling on rectangular microstrip patches. In their analysis, the interior field of the patch is expanded in resonant modes and the mutual coupling is com-
puted by modeling the patches by magnetic current loops over ground plane using the equivalence theorem. The modeling of patches by magnetic current loops holds only for patches on thin substrates.

When the substrate thickness increases, the coupling due to the surface wave field becomes important. This type of mutual coupling is ignored in the cavity model. An approach which accounts for the presence of the substrate is the moment method [20]. In this method the fields are computed by an integral equation approach using the Green's function for the slab; but this method is very numerically involved, and is not easily extensible to the design of arrays including the effects of mutual coupling. In the improved transmission line model [57] for rectangular patches, the radiation from patches is modeled by four equivalent slots and the mutual coupling is considered by evaluating the interaction among these slots. This model presents an improvement over the earlier transmission line model, because it accounts for the mutual coupling among the edges of the same patch. The transmission line model for mutual coupling is applicable only to rectangular patches.

In this Chapter, the MNM approach developed earlier (Chapter II) is extended to include the effect of mutual coupling on microstrip configurations on thin substrates without cover layer. Effects of mutual coupling on microstrip patches with cover layer will be discussed in Chapter 6. In the proposed approach, the effect of the mutual coupling is incorporated in the analysis of microstrip patch antennas by using equivalent network(s) (called mutual coupling network, MCN) compatible with the MNM. These mutual coupling networks are characterized by admittance matrix representations. The admittance matrix coefficients are evaluated by replacing the fields at the patch periphery by
equivalent magnetic current sources and evaluating the interaction among these sources. The mutual coupling effects computed on the basis of this model are compared with the experimental data on the mutual coupling available in the literature.

3.2 Multiport network modeling of mutual coupling

The MNM for mutual coupling effects on microstrip patches is illustrated by considering rectangular patches. The effects of the radiation and fringing fields along the edges of the patch are included in the MNM by using the EAN segment. This formulation does not yet take into account the effects of external coupling among the edges of the patch. Thus, to account for this external coupling, a mutual coupling network (MCN) needs to be included among the edges of the same patch. Formulations for patches with other geometries can be investigated using procedures similar to that developed for rectangular patches.

3.2.1 Mutual coupling between two edges of the same patch

The multiport network approach for modeling and analysis of microstrip patch antennas has been discussed in Chapter II. When only a single rectangular patch is considered, the mutual coupling between the two radiating edges may be incorporated in the MNM by connecting an additional multiport network between the two edges as shown in Fig 3.1. The MCN network shown in Fig 3.1 is characterized in terms of an admittance matrix \([Y^m]\). Elements of this matrix represent mutual admittances between various sections of the two radiating edges. The number of these sections is decided by the multiport network modeling of the patch. When we consider five sections (corresponding
to 5 ports) at each of the edges, the structure of this matrix is:

\[
[Y^m] = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & \times & \times & \times & \times & \times \\
0 & 0 & 0 & 0 & 0 & \times & \times & \times & \times & \times \\
0 & 0 & 0 & 0 & 0 & \times & \times & \times & \times & \times \\
0 & 0 & 0 & 0 & 0 & \times & \times & \times & \times & \times \\
0 & 0 & 0 & 0 & 0 & \times & \times & \times & \times & \times \\
\times & \times & \times & \times & 0 & 0 & 0 & 0 & 0 & 0 \\
\times & \times & \times & \times & 0 & 0 & 0 & 0 & 0 & 0 \\
\times & \times & \times & \times & 0 & 0 & 0 & 0 & 0 & 0 \\
\times & \times & \times & \times & 0 & 0 & 0 & 0 & 0 & 0 \\
\times & \times & \times & \times & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]  

(3.1)

where the \( \times \) marks denote non-zero mutual admittance terms.

The self admittance terms and the mutual admittances between sections of the same edge are included in the edge admittance network (EAN) associated with each of the radiating edges and therefore corresponding terms in \([Y^m]\) are set zero. Terms marked \( \times \) in the off-diagonal blocks represent the
mutual interaction between two sections of the two edges. These mutual admittance terms $Y^m_{pq}$ ($p=1,..5$, $q=6,..10$) are obtained from the equivalent magnetic current representation of the edge fields. This formulation is similar to the approach followed in [52] and is elaborated in Sec. 3.3

The mutual coupling effects between the non-radiating edges and between any radiating and non-radiating edges are also investigated and incorporated in the MNM in a similar manner.

3.2.2 Mutual coupling between two patches

Mutual coupling between two patches in an array environment is also modeled by an equivalent network. A multiport network model for the mutual coupling between two rectangular patches is shown in Fig 3.2. The 20-port MCN network shown in this figure incorporates the mutual coupling among the four radiating edges (including the coupling between two radiating edges of the same patch as shown in Fig 3.1). As before, the self admittances of various sections and the mutual admittances among sections of the same edge are included in the EAN’s. The EAN’s at the non-radiating edges and the connections among the non-radiating edges and the MCN are not shown in order to keep the figure simple. The admittance matrix $[Y^m]$ describing the MCN in Fig 3.2 has the following structure:

$$
[Y^m] = \begin{pmatrix}
0 & X_{12} & X_{13} & X_{14} \\
X_{21} & 0 & X_{23} & X_{24} \\
X_{31} & X_{32} & 0 & X_{34} \\
X_{41} & X_{42} & X_{43} & 0
\end{pmatrix}
$$

(3.2)

Each of the sub-matrices in Eqn. (3.2) has dimensions of $5 \times 5$ (since the number of ports on each edge has been taken equal to five). The submatrix $[0]$ represents a $5 \times 5$ null matrix, whereas $[X_{nm}]$ represents a $5 \times 5$ mutual
admittance matrix corresponding to the interaction between the \( n^{th} \) and the \( m^{th} \) edges. Because of the reciprocity we have:

\[
[X_{mn}] = [X_{nm}]^t
\]  \hspace{1cm} (3.3)

and from the symmetry \([X_{12}] = [X_{34}], [X_{14}] = [X_{23}],\) and \([X_{13}] = [X_{24}]\). Thus only three submatrices need to be evaluated for constructing the complete \([Y^m]\) matrix in Eqn. (3.2).
Similar MCN may also be introduced between the first patch and the third patch or between any two patches in an array.

3.3 Elements of MCN admittance matrices

For microstrip antennas on electrically thin substrates, the mutual coupling between two edges may be evaluated by representing the edges fields by line sources of equivalent magnetic current and their images with respect to the ground plane. The magnetic current line source at each edge is divided into a number of small sections (typically 5) as discussed in Sec. 3.2. For the evaluation of the mutual coupling, each of these sections is subdivided into smaller elements of length \( d\ell \) so that each sub-section can be considered as a magnetic current element \((d\ell = \lambda_0/100)\). Typically, we consider four elements per section. The amplitude \( M \) of each of the magnetic current elements is equal twice that of the edge voltage \( V_e \) at that location, and the phase of \( M \) is that of the corresponding voltage.

For computation of the MCN matrix, the mutual admittance \( Y_{ij} \) between any two elements (\( i \) and \( j \) belonging to different edges) is evaluated by considering two correspondingly located magnetic current elements as shown in Fig 3.3. An element of magnetic current of length \( d\ell \) located at \((0,0)\) in the \( z=0 \) plane produces magnetic field components \( H_\theta \) and \( H_r \) at a location \((x_j, y_j)\) in the \( z=0 \) plane given by

\[
H_\theta = i \frac{k_0}{4 \pi \eta_0} \frac{M \, d\ell \, \sin \theta}{r} \left(1 + \frac{1}{i \, k_0 \, r} - \frac{1}{(k_0 \, r)^2}\right) \cdot e^{-i \, k_0 \, r} \tag{3.4}
\]

\[
H_r = \frac{M \, d\ell \, \cos \theta}{2 \pi \eta_0 \, r^2} \left(1 + \frac{1}{i \, k_0 \, r}\right) \cdot e^{-i \, k_0 \, r} \tag{3.5}
\]
where \( r = \sqrt{x_j^2 + y_j^2} \) and \( \theta \) is the polar angle measured with respect to the direction of the \( i^{th} \) magnetic current element. \( k_0 \) and \( \eta_0 \) are the propagation constant and the intrinsic impedance respectively for free space.

For the configuration of the two elements shown in Fig. 3.3, the current density induced in the \( j^{th} \) element because of the fields generated by the \( i^{th} \) element is

\[
J_j = (\overline{a}_z \times \overline{H}) \cdot \hat{j} = -H_x \cdot \cos \delta_j - H_y \cdot \sin \delta_j
\]

where the angle \( \delta_j \) is as shown in the Figure and \( \overline{a}_z \) is a unit vector in the \( z \)-direction. The \( x \) and \( y \) components of the magnetic field \( H_x \) and \( H_y \) are related to \( \theta \) and \( r \) by

\[
H_z = H_\theta \cdot \cos \theta + H_r \cdot \sin \theta
\]
\[ H_y = -H_\theta \cdot \sin \theta + H_r \cdot \cos \theta \quad (3.7.b) \]

The mutual admittance \( Y_{ij} \) between the \( i^{th} \) and the \( j^{th} \) elements, with other ports open circuited, is evaluated by computing the current induced at the \( j^{th} \) element due to the fields created by a voltage source of magnitude \( M \) at the \( i^{th} \) element. Since the width of the elements is taken to be very small, the surface current density \( J_j \) is uniform over the element width. So the line current flowing over the top side of the patch in a direction away from the edge is

\[ I_j = J_j \, dl_j \quad (3.8) \]

where \( dl_j \) is the width of subsection \( j \). The \( Y_{ij} \) elements of the \([Y]\)-matrix representing MCN are written as

\[ Y_{ij} = J_j \, dl_j / V_e \quad (3.9) \]

It may be pointed out that, since \( J_j \) is linearly proportional to \( M \), the value of \( M \) (or \( V_e \)) is not needed for the computation of \( Y_{ij} \). Expression (3.9) for \( Y_{ij} \) can also be derived by using the reaction theorem. The size of the matrix \([Y]\) is larger (typically 4 times) than that of the \([Y^m]\) matrix (used in \( Eqns \) (3.1) and (3.2)). So the matrix \([Y^m]\) is reduced to the matrix \([Y]\) by assuming the current flowing in at a port is divided equally into the subsections of that port, and the voltages is the same at all ports. The elements \( Y_{pq}^m \) of the matrix \([Y^m]\) are computed from \( Y_{ij} \) as

\[ Y_{pq}^m = \sum_{j=q_1}^{q_4} \sum_{i=p_1}^{p_4} Y_{ij} \quad (3.10) \]
where $p_1, \ldots, p_4$ and $q_1, \ldots, q_4$ denote elements of sections $p$ and $q$ respectively.

### 3.4 Antenna characteristics in presence of mutual coupling

The multiport network model for the rectangular patch of Fig 3.1 will be discussed in Chapter IV. The network EAN accounts for the power radiated, the power launched as surface waves and the fringing fields. The physical dimensions of the patch are used, because using effective dimensions will shift the location of the equivalent line sources. EANs are combined with MCN to yield a single sub-network. The rectangular segment (PATCH) represents the internal fields of the patch. The impedance matrix of this segment is computed by using the Green's function approach by modeling the antenna as a planar component. The segmentation method is used to combine the Z-matrix representation of the two sub-networks (the PATCH and EAN-MCN combination) to yield the input impedance as well as the voltages at the patch edges.

The configuration of the two patches shown in Fig 3.2 is considered as a two-port network, the external ports being the feed points for the two patches. The EAN and MCN networks shown in Fig 3.2 can be modified to include the contributions of the non-radiating edges to the mutual coupling. These contributions are included in the results reported here. The applications of the segmentation method to this configuration yield the scattering matrix coefficients (the transmission coefficient $S_{21}$ and the reflection coefficient $S_{11}$ at the input of each patch). As before, the analysis also gives the voltage distribution at the patches’ edges. This voltage distribution is used for studying the effect of mutual coupling on the radiation field of the array.

The design procedure for linear arrays of rectangular patches incorporating the effects of mutual coupling will be discussed in Chapter VII.
3.5 Comparison with experimental results

In order to verify the model and the computational procedure outlined above, calculations have been carried out for the E-plane and the H-plane coupling between two identical rectangular microstrip patches. The calculated results are compared to the experimental results of Jedlicka and Carver [29].

Results for the E-plane and the H-plane coupling coefficient $|S_{21}|$ as a function of the normalized spacing $S/\lambda_0$ (where $\lambda_0$ is the wavelength in free space) are shown in Figs. 3.4 and 3.5 respectively. Various parameters for the two patches are: $f=1.405$ GHz, $h=1/16$ inch, $\epsilon_r=2.5$, the patch resonant length $a=6.7$ cm, the patch width $b=10.56$ cm and the distance of the probe feed from the edge $x_1=1.8$ cm.

For the computation of the values of $|S_{21}|$ in Figs. 3.4 and 3.5, the MNM of the antenna was constructed by taking five sections considered along the radiating edges and twelve sections along each of the non-radiating edges. The agreement between the experimental results and the computed results based on the MNM model verifies the validity of the approach presented here. Theoretical results of the mutual coupling based on the transmission line model [9] are also plotted for sake of comparison.

It may be noted that the results computed using the multiport network model agree with the experimental results much better than the transmission line model. This is because the coupling contributed by the non-radiating edges is not incorporated in the results of [9]. Results of mutual coupling, computed using the transmission line model, can be duplicated using MNM by considering mutual coupling because of the radiating edges only and taking one port at each radiating edge in the multiport network of the patch.
Figure 3.4 E-plane mutual coupling between two identical probe-fed rectangular microstrip patches
Figure 3.5 H-plane mutual coupling between two identical probe-fed rectangular microstrip patches
3.6 Discussions

The multiport network modeling approach (discussed in Chapter II) has been extended to include the effect of mutual coupling on rectangular patches. The mutual coupling is modeled as an admittance matrix, whose elements represent the coupling among various edges of the patches. Good agreement between calculated and measured values of coupling coefficient between two identical probe-fed rectangular patches is obtained. The proposed model is compatible with CAD analysis of microstrip arrays discussed in Chapter VII.
CHAPTER IV

TRANSMISSION CHARACTERISTICS OF MULTIPLE-PORT
RECTANGULAR MICROSTRIP PATCH ANTENNAS

4.1 Multiple-port radiating patches

Linear arrays of microstrip patches (which may be elements of two-dimensional planar arrays) often use series-fed configurations (shown in Fig 7.1) in which adjacent radiating elements are connected by microstrip line sections [25,58]. The desired phase distribution along the length of the array is obtained by adjusting the length of the interconnecting lines. For rectangular patches (shown in Fig 4.1), the amplitude distribution along the array is tailored by selecting the appropriate width b for each two-port element. However, for an accurate design of the aperture phase and amplitude distributions, one needs to know the two-port transmission characteristics of these patch radiators used as series-fed linear array elements. Multiple-port rectangular and square microstrip patches have been used in multiple-element broadband microstrip antenna configurations [51] and in two-dimensional arrays [7].

In this Chapter, a method for the design of multiple-port rectangular microstrip patches (operating in the dominant mode) with specified transmission characteristics is discussed. An approximate analysis using the cavity model [55], wherein the radiated power and losses are accounted for by considering an effective loss tangent for the substrate dielectric medium, is discussed
a) ports located along the radiating edges

a) ports located along the non-radiating edges

Figure 4.1 Configurations of two-port rectangular microstrip patches
in Section 4.2. The design of multiple-port rectangular patches for specified transmission characteristics requires the optimization of the port locations and the patch length and width. Starting values of the ports locations, the patch length and width may be obtained by using the dominant mode of the cavity only, which usually is the $TM_{10}$. When this approximation is used, simple relations for the ports locations can be derived. The initial design of patches using this approximate analysis model may be used as the starting point in an optimization scheme using the multiport network model (MNM). Analysis of two-port rectangular patches based on the transmission line model is discussed in Section 4.3. A detailed discussion of MNM for two-port rectangular microstrip patches is given in Section 4.4. Radiation characteristics of two-port rectangular patches are presented in Section 4.6. A comparison between theoretical and experimental results for scattering parameters of two-port rectangular patches with ports located along the non-radiating edges is presented in Section 4.7.

### 4.2 Analysis using cavity model

In the cavity model [55,12], a microstrip patch is considered as a two-dimensional resonator surrounded by perfect magnetic walls. This cavity model, initially proposed by Lo and his colleagues [55,12], has been discussed in Section 1.2.2. We use a slightly modified version of this model as shown in Fig 4.2. An effective width $b_e$ and effective dielectric constant $\epsilon_{re}$ are used to take into account the effects of fringing fields at the non-radiating edges. Expressions for $b_e$ and $\epsilon_{re}$ for a microstrip line are given in [59]. The effects of fringing fields along the radiating edges are included by using an effective length $a_e$ of the
patch which is computed as

\[ a_e = a + 2\Delta l \]  \hspace{1cm} (4.1)

where \( \Delta l \) is related to the edge capacitance \( C \) of an edge by

\[ \frac{\Delta l}{h} = \frac{C}{b_e \varepsilon_0 \varepsilon_{re}} \]  \hspace{1cm} (4.2)

In deriving expression (4.2) it is assumed that the presence of the radiation conductance in parallel with the edge capacitance has no effect on the computation
of the extension $\Delta l$. The result of this approximation on the transmission coefficient and the input impedance is discussed by comparing the cavity model with the multiport network model in Section 4.5.1.

As in Lo's cavity model [55,12], the radiation from the patch is accounted for by considering the effective loss tangent of the dielectric to be larger than the actual value. For a rectangular patch, the effective loss tangent is given by

$$\delta_e = \delta_d + \frac{1.6}{h\sqrt{\mu_0\pi f \sigma_c}} + \frac{16\eta_0 h^2}{3Z(b)\epsilon_{re}\lambda_o^2}$$  \hspace{1cm} (4.3)

where $Z(b)$ is the characteristic impedance of a microstrip line of width $b$, $\lambda_o$ is the free space wavelength and $\sigma_c$ is the conductivity of the patch metalization. The second term in (4.3) is contributed by conductor losses and the factor of 1.6 in the numerator accounts for the effect of surface roughness on the conductor losses. The third term in (4.3) accounts for the radiated power and is based on the radiation conductance formula presented later in (4.26).

The impedance matrix characterization of the rectangular patch can be obtained from the Green's function as discussed in Chapter II. Green's function, expressed as double summation of the rectangular segment modes, is given in Appendix A.

### 4.2.1 Approximate analysis using dominant mode only

As microstrip antennas are usually operated near the resonance frequency of the dominant mode $TM_{10}$, a fairly good approximation is obtained by ignoring other terms of the Green’s function. The simplified Green’s function for the dominant mode may be expressed as

$$G(x_p, x_q) = \frac{i\omega\mu_0 h}{a_e b_e} \frac{2 \cos(\pi x_{ep}/a_e) \cos(\pi x_{eq}/a_e)}{(\pi/a_e)^2 - \omega^2 \mu_0 \epsilon_0 \epsilon_{re}(1 - i\delta_e)}$$  \hspace{1cm} (4.4)
where \( x_{ep} \) and \( x_{eq} \) are the effective locations of ports p and q along the length of the patch, and are related to the actual ports locations \( x_p \) and \( x_q \) (shown in Fig 4.2) by

\[
x_{ep,q} = x_{p,q} + \Delta I
\]  

(4.5)

At resonance, \( a_e \approx \lambda_0 / \sqrt{\varepsilon_{re}} \) and Green's function can be simplified further as

\[
G(x_p, x_q) = K_{1,0} \cos(\pi x_{ep}/a_e) \cos(\pi x_{eq}/a_e)
\]  

(4.6)

where

\[
K_{1,0} = \frac{2Z(b)}{\pi \delta_e}
\]  

(4.7)

The elements \( Z_{pq} \) of the Z-matrix are related to Green's function by equation (2.1).

**4.2.2 S-matrix of two-port patches using dominant mode only**

The approximate analysis, using the dominant mode only, will be employed in this section for the analysis of two-port patches. For two-port patches, the scattering parameters are related to the elements \( Z_{pq} \) of Z-matrix by

\[
S_{11} = \left\{ [Z_{11} - Z_{01}] [Z_{22} + Z_{02}] - Z_{12} Z_{21} \right\} / F
\]  

(4.8 - a)

\[
S_{12} = 2Z_{12} \sqrt{Z_{01} Z_{02}} / F
\]  

(4.8 - b)

\[
S_{21} = 2Z_{21} \sqrt{Z_{01} Z_{02}} / F
\]  

(4.8 - c)

\[
S_{22} = \left\{ [Z_{22} - Z_{02}] [Z_{11} + Z_{01}] - Z_{21} Z_{12} \right\}
\]  

(4.8 - d)

where

\[
F = [Z_{11} + Z_{01}] [Z_{22} + Z_{02}] - Z_{12} Z_{21}
\]  

(4.8 - e)
and $Z_{01}$ and $Z_{02}$ are the characteristic impedances of input and output transmission lines.

The condition for input match at the port 1 is imposed by forcing $S_{11} = 0$. Using (4.8-a), a condition for input match relating elements of $Z$-matrix to $Z_{01}$ and $Z_{02}$ is expressed as

$$Z_{11}Z_{02} - Z_{01}Z_{22} - Z_{01}Z_{02} = 0$$

(4.9)

The corresponding transmission coefficient ($S_{21}$) is given by

$$S_{21} = \frac{Z_{21}}{Z_{11}} \cdot \sqrt{\frac{Z_{01}}{Z_{02}}}$$

(4.10)

In deriving (4.9) and (4.10) we made use of the fact that $Z_{11}Z_{22} = Z_{12}Z_{21}$, which is shown to be true for the case of a two-port rectangular microstrip patch.

Transmission characteristics of two-port rectangular patches, when the ports are located along the radiating edges and when they are located along the non-radiating edges, are discussed separately.

(i) Ports along the radiating edges:

When the two ports are located along the radiating edges (shown as ports 1' and 2' in Fig 4.2), the ports widths are oriented in the $y$-direction and the integration in (2-1) is with respect to the $y$ variable. The values of $x_1$ and $x_2$ are taken as 0 and $a$ respectively and

$$Z_{12} = Z_{21} = -K_{1,0} \cos^2(\pi \Delta l/a_e)$$

(4.11)

$$Z_{11} = Z_{22} = K_{1,0} \cos^2(\pi \Delta l/a_e)$$

(4.12)
where $K_{1,0}$ is given by (4.7). When the widths of the two ports are equal ($Z_{01} = Z_{02} = Z_0$), the condition for the input match (4.9) leads to an impractical constraint stated as

$$Z_0^2 = 0. \quad (4.13)$$

A match at the input port can, however, be obtained when the characteristic impedances ($Z_{01}$ and $Z_{02}$) of the microstrip lines are not equal. In this case, the input match condition can be derived as

$$Z_{02} = \frac{Z_{01}}{1 - Z_0/|K_{1,0} \cos^2(\pi \Delta l/a_c)|}. \quad (4.14)$$

The corresponding value of the transmission coefficient $S_{21}$ is found to be

$$S_{21} = -\sqrt{Z_{01}/Z_{02}}. \quad (4.15)$$

From (4.14) and (4.15) we require that for a match at the input port we need $Z_{02} > Z_{01}$. The negative sign in (4.15) indicates that the phase delay between the input and output is $180^\circ$ at resonance.

As an example, consider a two-port rectangular patch with the following parameters: $\epsilon_r = 2.2$, $b=0.8$ cm, $h=0.381$ mm ($1/64$ inch) and $f=7.5$ GHz. Calculations yield: $\Delta l=0.4154$ mm, $a=1.2999$ cm, $\epsilon_{re}(b)=2.03914$, $Z(b)=10.75$ $\Omega$ and $\delta_e = 1.22 \times 10^{-2}$. Values of $Z_{02}$ needed when $Z_{01}$ varies from 25 $\Omega$ to 150 $\Omega$ for a rectangular patch are given in Table 4.1. Corresponding values of $|S_{21}|$ are also shown in column 3 of Table 4.1. Results shown in the other columns of this table are discussed later in this chapter.

(ii) Ports along the non-radiating edges:

When the two ports are located along the non-radiating edges (shown
Table 4.1  $|S_{21}|$ For a two-port rectangular patch with the ports located along the opposite radiating edges (variation with $Z_{01}$ and $Z_{02}$).

<table>
<thead>
<tr>
<th>$Z_{01}$ (Ω)</th>
<th>$Z_{02}$ (Ω)</th>
<th>Dominant mode</th>
<th>Cavity model</th>
<th>Trans. line model</th>
<th>MNM model</th>
<th>MNM + FJ</th>
<th>MNM + FJ + MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>26.2</td>
<td>0.9772</td>
<td>0.9693</td>
<td>0.9772</td>
<td>0.9667</td>
<td>0.9499</td>
<td>0.9485</td>
</tr>
<tr>
<td>50</td>
<td>54.9</td>
<td>0.9539</td>
<td>0.9462</td>
<td>0.9542</td>
<td>0.9418</td>
<td>0.9256</td>
<td>0.9204</td>
</tr>
<tr>
<td>75</td>
<td>86.7</td>
<td>0.9299</td>
<td>0.9246</td>
<td>0.9311</td>
<td>0.9215</td>
<td>0.9059</td>
<td>0.8969</td>
</tr>
<tr>
<td>100</td>
<td>122.0</td>
<td>0.9054</td>
<td>0.9017</td>
<td>0.9072</td>
<td>0.9001</td>
<td>0.8851</td>
<td>0.8728</td>
</tr>
<tr>
<td>150</td>
<td>205.0</td>
<td>0.8542</td>
<td>0.8521</td>
<td>0.8571</td>
<td>0.8533</td>
<td>0.8396</td>
<td>0.8213</td>
</tr>
</tbody>
</table>
as ports 1 and 2 in Fig 4.2), the integration in (2.1) is with respect to the x variable. For this case, the elements $Z_{pq}$ of the Z-matrix are found to be

$$Z_{11} = K_{1,0}\{\cos(\frac{\pi x_{e1}}{a_e})\text{sinc}(W_1')\}^2 \quad (4.16.a)$$

$$Z_{12} = K_{1,0}\{\cos(\frac{\pi x_{e1}}{a_e})\text{sinc}(W_1')\}\{\cos(\frac{\pi x_{e2}}{a_e})\text{sinc}(W_2')\} \quad (4.16.b)$$

$$Z_{21} = Z_{12} \quad (4.16.c)$$

$$Z_{22} = K_{1,0}\{\cos(\frac{\pi x_{e2}}{a_e})\text{sinc}(W_2')\}^2 \quad (4.16.d)$$

where $\text{sinc}(z) = \sin(z)/z$ and $W_{1,2}' = \pi W_{e1,e2}/(2a_e)$. $W_{e1}$ and $W_{e2}$ are the effective widths of the input and output transmission lines.

For $Z_{01} = Z_{02} = Z_0$ and $x_1 = x_2$ (two ports of equal width located at equal distances from the radiating edge), the condition for input match requires $Z_0 = 0$. However, when $x_1 \neq x_2$ and $Z_{01} = Z_{02} = Z_0$, the input match can be achieved for

$$Z_0 = K_{1,0}'\{\cos^2(\frac{\pi x_{e1}}{a_e}) - \cos^2(\frac{\pi x_{e2}}{a_e})\} \quad (4.17)$$

where

$$K_{1,0}' = K_{1,0}\text{sinc}^2(W') \quad (4.18)$$

and $W' = W_1' = W_2'$. Since $Z_0$ is always positive, we need to denote the port closest to the radiating edge as port 1. When (4.17) is satisfied, the transmission coefficient $S_{21}$ is given by

$$S_{21} = \cos(\frac{\pi x_{e2}}{a_e})/\cos(\frac{\pi x_{e1}}{a_e}) \quad (4.19)$$

The phase of $S_{21}$ in this case is equal to zero (or $\pi$ if either $x_{e2}$ or $x_{e1}$ is greater than $a_e/2$).
For a more general case \((x_1 \neq x_2\) and \(Z_{01} \neq Z_{02}\)), the input match condition is given by

\[
K_{1,0}\{\cos^2\left(\frac{\pi x_1}{a_e}\right)\frac{sinc^2W_1'}{Z_{01}} - \cos^2\left(\frac{\pi x_2}{a_e}\right)\frac{sinc^2W_2'}{Z_{02}}\} = 1
\] (4.20)

The corresponding transmission coefficient is given by

\[
S_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}} \cdot \frac{\frac{\cos(\pi x_2/a_e)sinc(W_2')}{sinc(W_1')}}{\frac{\cos(\pi x_1/a_e)sinc(W_1')}}
\] (4.21)

Let us consider an example of a two-port rectangular patch with the ports located along the non-radiating edges (shown in Fig 4.1). As before, we select \(\epsilon_r = 2.2\), \(f = 7.5\) GHz, \(h = 1/64\) inch and \(b = 0.8\) cm. Characteristic impedances \(Z_{01}\) and \(Z_{02}\) are 50 \(\Omega\) each. Values of \(x_2\) (location of the second port) are varied from 2 mm from the edge to 6 mm (near the middle point of the non-radiating edge). Corresponding values of \(x_1\) to yield a match at the input port \((Z_{in} = 50\ \Omega)\) and the resulting values of the transmission coefficient \(|S_{21}|\) are listed in columns 2 and 3 of Table 4.2 respectively. We notice that for ports located at the non-radiating edges, values of \(|S_{21}|\) ranging from almost 0 to almost 1 may be obtained.

### 4.2.3 Effects of higher order modes under the patch

Values of \(|S_{21}|\) obtained when only the dominant mode is considered (as discussed earlier in this section) are given in columns 3 of Tables 4.1 and 4.2. The effect of higher order modes can be incorporated by using the cavity model with the complete Green’s function rather than using the dominant mode only. When ports are located along the radiating edges, modified values of \(|S_{21}|\) are shown in column 4 of Table 4.1. Values of the input impedance, when
Table 4.2 $|S_{21}|$ for a two-port rectangular patch with both the ports located along the non-radiating edges.

<table>
<thead>
<tr>
<th>$x_2$ (mm)</th>
<th>$x_1$ (mm)</th>
<th>Dominant mode</th>
<th>Cavity model</th>
<th>Trans. line model</th>
<th>MNM model</th>
<th>MNM + FJ</th>
<th>MNM + FJ + MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.519</td>
<td>0.9428</td>
<td>0.8071</td>
<td>0.9450</td>
<td>0.7862</td>
<td>0.7627</td>
<td>0.7484</td>
</tr>
<tr>
<td>3.0</td>
<td>2.596</td>
<td>0.9211</td>
<td>0.8079</td>
<td>0.9241</td>
<td>0.7983</td>
<td>0.7574</td>
<td>0.7388</td>
</tr>
<tr>
<td>4.0</td>
<td>3.575</td>
<td>0.8721</td>
<td>0.7753</td>
<td>0.8767</td>
<td>0.7726</td>
<td>0.6968</td>
<td>0.6698</td>
</tr>
<tr>
<td>5.0</td>
<td>4.444</td>
<td>0.7422</td>
<td>0.6676</td>
<td>0.7496</td>
<td>0.6707</td>
<td>0.5161</td>
<td>0.4758</td>
</tr>
<tr>
<td>6.0</td>
<td>5.055</td>
<td>0.3513</td>
<td>0.3438</td>
<td>0.3591</td>
<td>0.3491</td>
<td>0.1296</td>
<td>0.1162</td>
</tr>
</tbody>
</table>
higher order modes are included, is shown in column 3 of Table 4.3. The input impedance, for the case when higher order modes are ignored, equals the value of $Z_{01}$ in column 1 of this Table. We note that higher order modes contribute mainly to an inductive reactance at the input port and reduce the value of $|S_{21}|$ (because of increased reflection).

When the feed point is located along a non-radiating edge, currents near the input port are transverse to the currents of the dominant mode in the patch. Therefore, the excitation of higher order modes is much more pronounced than that for a feed along the radiating edge. Effects of higher order modes on the input impedance is shown in column 3 of Table 4.4. The effect is mainly inductive and increases for ports located close to the radiating edges (where excitation of higher order modes is more strong). Effects of higher order modes on the transmission coefficient is shown in column 4 of Table 4.2. We note that, in this case, values of $|S_{21}|$ are lower than those obtained by considering the dominant mode only. However, the closed-form results for the dominant mode are useful as initial solutions for the numerical calculations required when the complete Green’s function is used for computations.

### 4.3 Analysis using transmission line model

The analysis of transmission characteristics of two-port rectangular patches using a transmission line model is described in [10]. An equivalent transmission line model of a two-port rectangular patch is shown in Fig 4.3. As discussed in Chapter 1, the edge capacitance $C$ accounts for the fringing fields along the radiating edge of the patch and the conductance $G$ accounts for the radiation and surface wave power losses. Results for $|S_{21}|$ based on this model for ports located at the radiating edges and along the non-radiating
Table 4.3  Input impedance of two-port rectangular patch with both
the ports located along the opposite radiating edges.

<table>
<thead>
<tr>
<th>$Z_{01}$(Ω)</th>
<th>$Z_{02}$(Ω)</th>
<th>Cavity model</th>
<th>MNM</th>
<th>MNM + FJ</th>
<th>MNM + FJ + MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>26.18</td>
<td>25.2 + j4.3</td>
<td>25.1 + j6.2</td>
<td>25.1 + j5.7</td>
<td>25.0 + j5.8</td>
</tr>
<tr>
<td>50</td>
<td>54.95</td>
<td>50.2 + j10.4</td>
<td>50.1 + j14.6</td>
<td>50.1 + j13.9</td>
<td>49.5 + j14.5</td>
</tr>
<tr>
<td>75</td>
<td>86.72</td>
<td>75.2 + j13.1</td>
<td>75.2 + j19.0</td>
<td>75.1 + j18.2</td>
<td>73.5 + j19.8</td>
</tr>
<tr>
<td>100</td>
<td>122.0</td>
<td>100.2 + j14.5</td>
<td>100.2 + j21.6</td>
<td>100.2 + j20.7</td>
<td>97.2 + j23.9</td>
</tr>
<tr>
<td>150</td>
<td>205.5</td>
<td>150.2 + j15.7</td>
<td>150.6 + j24.4</td>
<td>150.6 + j23.2</td>
<td>143.6 + j31.1</td>
</tr>
</tbody>
</table>
Table 4.4 Input impedance of two-port rectangular patch with both the ports located along the opposite non-radiating edges.

<table>
<thead>
<tr>
<th>$x_2$ (mm)</th>
<th>$x_1$ (mm)</th>
<th>Cavity model</th>
<th>MNM</th>
<th>MNM+FJ</th>
<th>MNM+FJ+MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.519</td>
<td>52.1 + j56.6</td>
<td>52.7 + j62.9</td>
<td>50.7 + j62.3</td>
<td>48.5 + j62.0</td>
</tr>
<tr>
<td>3.0</td>
<td>2.596</td>
<td>52.3 + j50.1</td>
<td>52.8 + j53.9</td>
<td>47.2 + j56.1</td>
<td>44.6 + j55.8</td>
</tr>
<tr>
<td>4.0</td>
<td>3.575</td>
<td>52.9 + j44.9</td>
<td>53.5 + j47.4</td>
<td>40.9 + j54.0</td>
<td>37.5 + j53.0</td>
</tr>
<tr>
<td>5.0</td>
<td>4.444</td>
<td>54.0 + j37.4</td>
<td>55.2 + j39.4</td>
<td>28.8 + j51.7</td>
<td>25.3 + j49.1</td>
</tr>
<tr>
<td>6.0</td>
<td>5.055</td>
<td>53.3 + j23.0</td>
<td>55.6 + j24.2</td>
<td>16.3 + j42.8</td>
<td>14.2 + j39.3</td>
</tr>
</tbody>
</table>
Figure 4.3 Transmission line model of a two-port rectangular patch

edges are shown in columns 5 of Tables 4.1 and 4.2 respectively. These results, computed by neglecting feed junctions reactances, are very close to the results based on the dominant mode only. This should be expected because the transmission line model is equivalent to the superposition of all cavity modes which do not vary in the transverse direction (along the non-resonant dimension of the patch) including the dominant mode. Since the antenna is operating in the dominant mode, higher order modes are weakly excited and contribute only to an inductive part in the input impedance. Values of input impedance for ports located along the radiating edges and for ports located along the non-radiating edges are very close to those computed using the dominant mode only.

4.4 Analysis using MNM model

Multiport network model for the analysis of microstrip patches on thin substrates has been discussed in Chapter II. MNM for rectangular patches is investigated in this section by considering the case of a two-port rectangular microstrip patch. The method of analysis can be easily extended for the analysis of rectangular patches with more than two ports. Figure 4.4 shows a two-
port rectangular microstrip patch on thin substrate. An equivalent multiport
network model of the patch is also shown in Fig 4.4. In the MNM, the elec-
tromagnetic fields underneath and outside the patch are modeled separately. A
detailed description of each network shown in the figure is carried out in the
following sections.

4.4.1 Fields underneath the patch

The fields underneath the patch are computed by modeling the patch
as a two-dimensional planar network (denoted by PATCH in Fig 4.4), with a
number of discrete ports located along the edges (where EAN and FEED seg-
ments are connected to the PATCH segment). Each port represents a small
section (of length \(W_p\)) of the edge of the patch. \(W_p\) is chosen so small that
the fields over this length may be assumed to be uniform. The elements \(Z_{pq}\)
of the \(Z\)-matrix of the rectangular segment PATCH are derived from the Green’s
function using (2.1). Green’s function \(G\), derived using an expansion into eigen-
functions of the rectangular segment, is given in Appendix A. For the rectan-
gular resonator, the double summation is reduced to a single summation [60]
by evaluating one of the summations involved analytically. Expressions for \(Z_{pq}\)
for different locations of ports \(p\) and \(q\) are given below.

Case 1: Both ports (\(p\) and \(q\)) are oriented along same direction (\(x\) or \(y\))

When both ports \(p\) and \(q\) are oriented along the \(x\) direction we have

\[
Z_{pq} = -A \sum_{n=0}^{L} \sigma_n F_n(y) \cos(\gamma_n(x - a)) \cos(\gamma_n x) \frac{sinc(\frac{k_n W_p}{2})sinc(\frac{k_n W_q}{2})}{\gamma_n \sin(\gamma_n a)}
\]

\[
- iA \sum_{n=L+1}^{\infty} F_n(y) \frac{sinc(\frac{k_n W_p}{2})sinc(\frac{k_n W_q}{2})}{\gamma_n} \exp[-i\gamma_n(x - x)]
\]

(4.22 - a)
Figure 4.4 Equivalent multiport network model of a two-port rectangular microstrip patch
where,
\[ F_n(y) = \cos(k_n y_p) \cos(k_n y_q) \]  \hspace{1cm} (4.22 - b)

and \( A = i \omega \mu_0 \hbar / b, x > = \max(x_p, x_q), x < = \min(x_p, x_q) \) and \( \sigma_n \) equals 1 for \( n = 0 \) and 2 otherwise. \( \gamma_n = \pm \sqrt{k^2 - k_n^2} \) where, \( k^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_r (1 - i \delta) \) and \( k_n = n \pi / b \). The sign of \( \gamma_n \) is chosen such that its imaginary part is negative. The choice of the value of \( L \) (for which the trigonometric functions are replaced by their large arguments) is selected such that the imaginary part of \( (\gamma_n a) \) is less than or equal to 50.

When ports \( p \) and \( q \) are oriented in the \( y \) direction, \( Z_{pq} \) is computed from expression (4.22-a) by interchanging \( n \) and \( m \), \( x \) and \( y \), \( a \) and \( b \).

**Case II:** Ports \( p \) and \( q \) are oriented in different directions (\( x \) and \( y \))

Let us denote the port oriented along the \( x \) direction by \( p \) and the port oriented along the \( y \) direction by \( q \). If \((x > - x <)(y > - y <)\) then \( Z_{pq} \) is given by

\[
Z_{pq} = -A \sum_{n=0}^{L} \sigma_n F_n(y) \cos(\gamma_n (x > - a)) \cos(\gamma_n x <) \frac{\text{sinc}(\frac{k_n W_q}{2}) \text{sinc}(\frac{\gamma_n W_p}{2})}{\gamma_n \sin(\gamma_n a)} \\
- A \sum_{n=L-1}^{\infty} F_n(y) \text{sinc}(\frac{k_n W_q}{2}) \exp[-i \gamma_n (x > - x < - W_p / 2)] \frac{\gamma_n^2 W_p}{\gamma_n^2 W_p} 
\]  \hspace{1cm} (4.22 - c)

However, if \((y > - y <)(x > - x <)\) the outer summation is with respect to \( m \) and \( Z_{pq} \) is obtained by interchanging \( n \) and \( m \), \( a \) and \( b \), \( p \) and \( q \), \( x \) and \( y \).

The effect of dielectric losses is accounted for by taking \( \varepsilon_{re} \) to be complex. The conductor losses for the patch and the ground plane are included by using an effective loss tangent \( \delta_c = 1/(\hbar \sqrt{\mu_0 / \pi f \sigma_c}) \).
4.4.2 Description of EAN network

In the MNM modeling of radiating microstrip patches, the fields outside the patch (the fringing fields at the edges, the surface wave fields and the radiation fields) are incorporated by adding equivalent edge admittance networks (EAN's) to the various edges of the patch (as shown in Fig 4.4). As discussed in Chapter II, EAN's at the radiating edges are multiport networks consisting of the parallel combination of the capacitance $C$ (representing the energy stored in the fringing fields) and the conductances $G$ (representing the power carried away by radiation and surface waves). A capacitance-conductance pair is connected to each port of the planar equivalent circuit of the patch. Each edge capacitance and conductance are uniformly divided among the NC/2 ports along each radiating edge. The Y-matrix (characterizing the EAN network) is a diagonal matrix with identical elements $Y_{ee}$ given by

$$Y_{ee} = \frac{2(G_r + G_s + j\omega C)}{N C}$$ \hspace{1cm} (4.23)

For the radiating edges of a rectangular patch, an expression for the capacitance $C$ is given by [61]:

$$C = \frac{b}{2} \left\{ \frac{\epsilon_{re}(f)}{c_0 Z_0(air)} - \frac{\epsilon_0 \epsilon_r a}{h} \right\}$$ \hspace{1cm} (4.24)

where $\epsilon_{re}(f)$ is the effective dielectric constant taking dispersion into account, and $Z_0(air)$ is the characteristic impedance of an equivalent transmission line of width $a$ and $\epsilon_r = 1$. An empirical expression using half dispersion is found to give better agreement between theoretical and measured resonant frequencies. This expression is given by:

$$C = \frac{|C(\epsilon_{re}(0)) + C(\epsilon_{re}(f))|}{2}$$ \hspace{1cm} (4.25)
Another approach is to deduce value of $C$ from experimental measurements. Determination of the edge capacitance of rectangular patches from the measured resonance frequency of unloaded patches is given in Appendix C.

The radiation conductance $G_r$ (for each radiating edge) may be computed by using an equivalent magnetic line source and is given by [39]:

$$G_r = \begin{cases} 
\frac{b_e^2}{90\lambda_0^2}, & b_e \leq 0.35\lambda_0; \\
\frac{b_e}{120\lambda_0} - \frac{1}{60\pi^2}, & 0.35\lambda \leq b_e \leq 2\lambda_0; \\
\frac{b_e}{120\lambda_0}, & 2\lambda_0 \leq b_e.
\end{cases} \quad (4.26)$$

The surface wave conductance $G_s$ is related to the radiation conductance by [39, p.56]:

$$\frac{G_r}{G_s} = \frac{\cos^2(k_{y1}h)\epsilon_r(k_{y1}h)^2}{\cos^2(k_{y1}h)\epsilon_r(k_{y2}h)^2 + (k_{y1}h)^2(k_{y2}h)} \quad (4.27)$$

where $(k_{y2}h) = \sqrt{(k_0h)(\epsilon_r - 1) - (k_{y1}h)^2}$,

$$(k_{y1}h)^2 = \frac{(k_0h)^2\epsilon_r^2(\epsilon_r - 1)}{\epsilon_r^2 + \tan^2(k_{y1}h)} \quad (4.28)$$

and $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. Expression (4.28) is a transcendental equation for $(k_{y1}h)$ and is solved numerically. For thin substrates, $G_s$ is much smaller than $G_r$ (1% or less) and can be neglected in the computations.

The radiation from the non-radiating edges is small (about 17 dB below) compared to that from the radiating edges (as shown in Section 4.6.2) and may be neglected. Consequently, the network EAN for the non-radiating edges consists of edge inductance and capacitance only. As discussed in Chapter two, these fringing inductance and capacitance are incorporated in the solution of internal fields by using an effective width and effective dielectric constant for the PATCH segment.

4.4.3 Microstrip feed junction reactances
The parasitic reactances at the junction between the microstrip feedlines and the patch are incorporated in the multiport network model by considering sections of the feedlines as rectangular segments (denoted by FLN in Fig 4.4) connected to the patch at a finite ND number of ports. The $Z$-matrices characterizing the feed line networks FLN are computed from the Green's function of a rectangular segment. $Z_{pq}$ are computed by using an effective width and effective dielectric constant $\epsilon_{re}$ for the transmission lines. The dielectric and conductor losses are accounted for in the same manner as for the patch. The transmission line length $l$ has to be taken long enough (at least $\lambda_0/8$ or greater than the line width) so to ensure that the higher order evanescent modes decay out and only the dominant quasi-TEM mode is present at the other end of the line. When the patch is fed along the non-radiating edges, an effective length $l_e$ for the feedline is used because the physical width of the patch is replaced by an effective width. The effective length is given by

$$l_e = l - (b_e - b)/2$$  \hspace{1cm} (4.29)

where $b_e$ is the effective width of the patch of width $b$.

4.4.4 Modeling of MCN network

The multiport network model discussed above can be extended to incorporate the effect of mutual coupling between the two radiating edges as discussed in Chapter III. This is done by inserting a mutual coupling network MCN as shown in Fig 4.4. The edge admittance terms associated with various ports at the edges constitute the diagonal terms of the admittance matrix for MCN. The non-diagonal terms of this matrix are obtained from the 'reaction' between the equivalent magnetic current sources at the two corresponding sec-
tions of the edges. The mutual coupling admittance \( Y_{pq} \) between two ports \( p \) and \( q \) describes the current induced at port \( q \) due to a unit voltage at port \( p \). \( Y_{pq} \) is given by Equation (3.8). The surface current density \( J_q \) induced at the \( q \)th port is given by

\[
J_q = -H_x \cos \delta_q + H_y \sin \delta_q
\]  
(4.30)

where \( H_x \) and \( H_y \) are related to \( H_\theta \) and \( H_r \) by

\[
\begin{align*}
H_x &= H_\theta \cos \theta + H_r \sin \theta \\
H_y &= -H_\theta \sin \theta + H_r \cos \theta
\end{align*}
\]  
(4.31)

The magnetic field components \( H_\theta \) and \( H_r \) are given by

\[
\begin{align*}
H_\theta(x_q,y_q) &= i \frac{k_0 dl_p \sin(\theta - \psi)}{2\pi \eta_0 r} \left\{ 1 + \frac{1}{ik_0 r} - \frac{1}{(k_0 r)^2} \right\} e^{-ik_0 r} \\
H_r(x_q,y_q) &= \frac{dl_p \cos(\theta - \psi)}{\pi \eta_0 r} \left\{ 1 + \frac{1}{ik_0 r} \right\} e^{-ik_0 r}
\end{align*}
\]  
(4.32)

where \( dl_p \) is the width of the \( p \)th port. \( \psi \) is the angle between the unit vector tangential to the periphery (\( \vec{a}_t \) shown in Fig 4.5) and the \( y \) axis. \( \delta \) is the angle between the outward normal to the periphery (\( \vec{a}_n \) shown in Fig 4.5) and the \( y \) axis. For the rectangular patch, values of the angles \( \psi \) and \( \delta \) for segments located at the four edges are also shown in Fig 4.5.

### 4.5 Computational details for two-port patches

The most significant advantage of the multiport network model discussed above is that the planar network analysis techniques can be extended for the analysis, design and optimization of microstrip antennas and arrays. Once each of the multiport components shown in the antenna model of Fig 4.4, (namely the PATCH, 2 FLN's, EAN's and MCN) has been characterized in terms of \( Y \)- or \( Z \)-matrices, the segmentation method (Appendix B) is used to
evaluate the characteristics of the overall network.

Impedance matrices for the three planar components (PATCH and the two FLN's) are obtained by using (4.22). The segmentation technique is used to combine, two segments at a time, these three Z-matrices into a single Z-matrix characterization of the combination. This yields a \( (2 + NC) \times (2 + NC) \) matrix with respect to 2 external ports and NC ports along the radiating edges. The segmentation formula is used again to combine MCN (which includes EAN's) with the \( (2 + NC) \times (2 + NC) \) Z-matrix representation of the rest of the model as evaluated earlier. Results of the later segmentation step consist of two parts: (i) a \( 2 \times 2 \) Z-matrix characterization with respect to the two ports 1 and 2, and (ii) values of voltages at the NC interconnecting ports between the patch and the two EAN's at the radiating edges.

The Z-matrix characterization with respect to the two external ports
(1 and 2 shown in Fig 4.4) is converted to an S-matrix representation using
(4.8) and yields the input reflection coefficient and the transmission coefficient for
the two-port patch. From the frequency response of the antenna, the resonant
frequency as well as the antenna bandwidth are evaluated. From the voltage at
the patch edges, the radiation characteristics of the antenna are computed.

4.5.1 Comparison with other approaches

Results of $|S_{21}|$ for the typical examples discussed earlier for ports
located at the radiating edges (shown in Table 4.1) and for ports located at
the non-radiating edges (shown in Table 4.2) were recomputed using the above
formulation. The results based on MNM are obtained by combining only the
two EAN's with the PATCH segment (with ND=1 and NC=8). The difference
between the MNM and the cavity model is mainly due to the difference in mod-
eling of the external fields. In the cavity model, the fringing fields are accounted
for by using an equivalent extension $\Delta l$ at the radiating edges (as given by 4.2),
and the radiation is accounted for by using an effective loss tangent for the
substrate (given by the third term of 4.3). At the other hand, in the MNM
we connect to the radiating edge of the patch an equivalent network (EAN)
as discussed in section 4.4.2. The effects of feed junctions at the input/output
ports and the effects of mutual coupling are discussed in sections 4.5.2 and 4.5.3
respectively.

4.5.2 Effects of feed junction reactances

Effect of input/output microstrip feed junctions reactances on the
transmission coefficient of a two-port rectangular patch are shown in columns
7 of Tables 4.1 and 4.2. Computed values of input impedance, including the
effects of feed junction, are given in columns 4 of Tables 4.3 and 4.4. The effect of feed junctions are included by combining the two FLN's with the PATCH and EAN networks. When ND=1, the effect of microstrip feed junction reactances are not present in the analysis. In the present calculations ND=5 and NC=8 have been used. For ports located at the radiating edges the effects on \( Z_{in} \) and \(|S_{21}|\) is small. However for ports located at the non-radiating edges, junctions reactances affect both \( Z_{in} \) and \(|S_{21}|\). These effects are larger for ports located near the center of the non-radiating edge, where the dominant mode is not strongly excited.

4.5.3 Effects of mutual coupling

Effects of the mutual coupling between the radiating edges is included by combining the MCN network with the other networks (PATCH + FLNs + EANs). This procedure has been discussed in details in Chapter three. The effects on \( S_{21} \) are also included in columns 8 of Tables 4.1 and 4.2. Effects of mutual coupling on the input impedance is shown in columns 6 of Tables 4.3 and 4.4. For ports located along the radiating edges, the effect of mutual coupling is to slightly decrease the real part and to increase the imaginary part of the input impedance. This effect increases for increasing values \( Z_{01} \) and \( Z_{02} \). For ports located along the non-radiating edges, the effect of mutual coupling is to decrease both the real and imaginary part of the input impedance and decrease slightly \(|S_{21}|\).

4.6 Radiation characteristics

When microstrip patches are used as radiators, we need to know their radiation characteristics. In this section the radiation characteristics of rectan-
gular patches have been studied using MNM approach.

4.6.1 Computation of radiation pattern

As discussed in Chapter II, the radiation characteristics are obtained by first calculating the aperture field in the plane of the antenna and then evaluating the far-zone field by Kirchhoff-Huygen's formula. Making use of the equivalence principle, the electric field distribution at the periphery of the rectangular patch is replaced by equivalent magnetic current line sources as shown in Fig 4.6. The magnetic current (in Volts) of the line source is related to the voltage at the edge by

\[
M = \begin{cases} 
+2V(x,y)a_y, & x = 0, \\
+2V(x,y)a_z, & y = b, \\
-2V(x,y)a_y, & x = a, \\
-2V(x,y)a_z, & y = 0.
\end{cases} \quad (4.33)
\]

The factor 2 accounts for the image of the magnetic current into the ground plane. Each magnetic current line source at the edges is divided into small sections over which the fields are assumed to be uniform. These sections correspond to various ports on the edges as considered in the segmentation method. The port voltages V is computed by using the MNM model. Expressions for far-field components of a magnetic field distribution were given earlier in Chapter II.

For rectangular microstrip patches operating in the dominant mode \( TM_{10} \), the radiation is mainly due to the two radiating edges of width \( b \) separated by a distance \( a \approx \lambda_c/2 \). For the dominant mode the voltages at the two radiating edges are uniform and \( 180^\circ \) out of phase. So the radiation fields from those two edges add up in broadside. As an example, we consider a two-port rectangular patch with \( a=1.3403 \text{ cm}, b=0.5 \text{ cm}, x_1=0.4038 \text{ cm}, x_2=0.4409 \text{ cm}, \)
Figure 4.6 Equivalent magnetic current line source for a rectangular patch

\( \epsilon_r = 2.2, \ h = 1/64 \text{ inch}, \ f = 7.5 \text{ GHz} \) and characteristic impedances of input and output feed lines equal 50 \( \Omega \).

Figure 4.7 shows the E-plane \((\phi = 0)\) radiation pattern, computed by considering only the radiating edges. Figure 4.8 shows the H-plane far field pattern \((\phi = 90^\circ)\). The computed beamwidth is 97° in the E-plane and 88°
Figure 4.7 E-plane far field pattern of a two-port rectangular microstrip patch
Figure 4.8 H-plane far field pattern of a two-port rectangular microstrip patch
in the H-plane. Dotted curves show cross-polarization patterns contributed by the non-radiating edges (as discussed below) in the two cases.

4.6.2 Effects of non-radiating edges

The effects of the non-radiating edges (on the radiation characteristics) are included by considering the same two-port rectangular patch as in Section 4.6.1 with 40 ports along each of the non-radiating edges. The radiation fields from these edges are perpendicular to the radiation fields from the radiating edges, and contribute to the cross-polarization. The radiation pattern of the field radiated from the non-radiating edges is also included in Figs 4.7 and 4.8 as the cross-polarization patterns. The cross-polarization level is almost of the order of -25 dB or less in the E-plane. However, the cross-polarization in the H-plane is -37 dB at $\theta = -90^\circ$ and increases to -20 dB at $\theta = +90^\circ$ and is -25 dB in broadside direction.

Table 4.5 shows the amount of the power radiated from the non-radiating edges of a rectangular patch operating in the dominant mode $TM_{10}$ compared to that from the radiating edges as a function of the patch width $b$. When the antenna is operating in the dominant mode, the voltage at a non-radiating edge has approximately a cosine distribution. Thus, the radiation from this edge is expected to be very small because the radiation from one half of the edge cancels the radiation from the other half. The radiated powers from an isolated radiating edge and a non-radiating edge are shown in columns 3 and 4 of Table 4.5 respectively. For the dominant mode, the radiated fields from the two radiating edges are in phase and add in the broadside direction. Thus, the radiated power from the two radiating edges, as shown in column 5 of the Table, is larger than that from an isolated edge. However, the radiated fields
Table 4.5  Radiation power from the edges of a rectangular patch operating in the dominant mode.

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>a (mm)</th>
<th>Power radia. from 1 RE</th>
<th>Power radia. from 1 NRE</th>
<th>Power radia. from 2 RE's</th>
<th>Power radia. from 2 NRE's</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.4683</td>
<td>3.471 $10^{-6}$</td>
<td>2.934 $10^{-5}$</td>
<td>8.345 $10^{-6}$</td>
<td>2.115 $10^{-7}$</td>
</tr>
<tr>
<td>2.0</td>
<td>1.4358</td>
<td>1.386 $10^{-5}$</td>
<td>2.696 $10^{-5}$</td>
<td>3.403 $10^{-5}$</td>
<td>7.740 $10^{-7}$</td>
</tr>
<tr>
<td>4.0</td>
<td>1.4060</td>
<td>5.520 $10^{-5}$</td>
<td>2.489 $10^{-5}$</td>
<td>1.379 $10^{-4}$</td>
<td>2.822 $10^{-6}$</td>
</tr>
<tr>
<td>6.0</td>
<td>1.3916</td>
<td>1.232 $10^{-4}$</td>
<td>2.393 $10^{-5}$</td>
<td>3.103 $10^{-4}$</td>
<td>5.975 $10^{-6}$</td>
</tr>
<tr>
<td>8.0</td>
<td>1.3829</td>
<td>2.165 $10^{-4}$</td>
<td>2.337 $10^{-5}$</td>
<td>5.475 $10^{-4}$</td>
<td>1.007 $10^{-5}$</td>
</tr>
<tr>
<td>10.0</td>
<td>1.3771</td>
<td>3.335 $10^{-4}$</td>
<td>2.300 $10^{-5}$</td>
<td>8.445 $10^{-4}$</td>
<td>1.491 $10^{-5}$</td>
</tr>
<tr>
<td>12.0</td>
<td>1.3729</td>
<td>4.721 $10^{-4}$</td>
<td>2.273 $10^{-5}$</td>
<td>1.196 $10^{-3}$</td>
<td>2.027 $10^{-5}$</td>
</tr>
</tbody>
</table>
from the non-radiating edges are 180° out of phase and cancel in broadside direction. Hence, the radiated power from the two non-radiating edges, as shown in column 6 of the Table is very small compared to the radiated power from an isolated non-radiating edge. For all values of b, the radiated power from the two non-radiating edges is at least -17 dB down compared to that from the two radiating edges. This contribution has been neglected in the computation of results reported in this report.

4.6.3 Effects of radiation from feed lines

The effect of the radiation from the feed lines on the radiation characteristics is investigated by considering the feed lines as rectangular segments with ports along the transmission line length. As an example, we consider the two-port rectangular patch used in previous sections. The radiated fields from the transmission lines has the same polarization as the field from the radiating edges. Figures 4.9 and 4.10 show the radiation from the two microstrip lines compared to that from the two radiating edges in the E-plane and H-plane respectively. The fields radiated by the feed lines are 35 dB down in the broadside direction both in the E-plane and H-plane. Away from broadside, the radiation from the feed lines increases, in the E-plane, to -20 dB at 90° from broadside. The radiation from the feed lines is mainly due to small sections of the lines nearer to the feed junction plane.

4.7 Experimental results and comparison with theory

The design and analysis procedure of the two-port rectangular microstrip patch presented above has been verified by an experiment. A two-port patch antenna was fabricated on a 1/32 inch substrate with \( \varepsilon_r = 2.2 \). The patch
is fed by 50 Ω feedlines located along the non-radiating edges. A sketch of the antenna fabricated and various dimensions are included in Fig 4.11. Measurements of the input reflection coefficient and the transmission coefficient were carried out using the HP 8510 automatic network analyzer. Measured VSWR values are plotted in Fig 4.11 and are compared with the theoretical results based on the multiport network model. The comparison is encouraging. The slight downward shift of the experimental curve can be attributed to the ac-

Figure 4.9 Parasitic radiation from the input and output feed lines of a two-port rectangular microstrip patch (E-plane)
Figure 4.10 Parasitic radiation from the input and the output feed lines of a two-port rectangular microstrip patch (H-plane)

The actual value of the dielectric constant being slightly higher than 2.2 or the edge capacitance given by expression (4.25) being lower than the actual value.

Experimental and theoretical values of $|S_{21}|$ are compared in Fig 4.12. The very good agreement observed verifies the validity of the theoretical results reported in this report. Another more confirmatory verification of the theory is obtained from the values of the phase delay of the signal being transmitted from port 1 to port 2. The experimental values were obtained by phase measurements using HP 8510 and deembedding the phase delays introduced by
Figure 4.11 Comparison of theoretical and experimental results for input VSWR of two-port rectangular patch
Figure 4.12 Comparison of theoretical and experimental results for the magnitude of $S_{21}$.
connecting microstrip lines and connectors. The agreement between theoretical and experimental values shown in Fig. 4.13 (with an accuracy better than 4°) is a very definite indication of the validity of the MNM model.

4.8 Concluding remarks

The multiport network model has been applied to the analysis of two-port rectangular patches. Such two-port patches form elements of series-fed microstrip arrays (discussed in Chapter 7). Results presented in Section 4.3 show that it is possible to match a two-port patch at the input port and, at the same time, achieve the required transmission coefficient to the second port in order to taper the amplitude distribution of the array appropriately.

Usually the design of two-port rectangular patches (used as elements of a series-fed array) for specified transmission characteristics require the optimization of the design parameters. A very good starting point in the optimization process is the use of the results based on the dominant mode analysis (discussed in Section 4.2).

Radiation from feedlines have also been investigated using MNM. Results obtained show that the radiation from the feed lines is very small compared to that from the patch. Power radiated from the non-radiating edges is 17 dB less than that from the radiating edges. Thus, the effects of radiations from feed lines and from the non-radiating edges are small and have been neglected in the analysis.

Details analysis has been presented for two-port patches when the ports are located either along the two radiating edges or along the two non-radiating edges. This formulation, can be extended to other configurations such as: (i) when one of the ports is along a radiating edge and the other port
Figure 4.13 Comparison of theoretical and experimental results for the phase of $S_{21}$
is along a non-radiating edge, or (ii) when one of the ports is a coaxial feed from underneath the patch and the second port is along any one of the four edges. The basic approach is also applicable when one wants to design patches with more than two ports, such as in the case of multiresonator broad-band microstrip antennas discussed in [51] or for the dual polarized, doubly series fed array discussed in [7].
CHAPTER V

MULTIPOWER NETWORK MODEL FOR CIRCULAR MICROSTRIP PATCH ANTENNAS

5.1 Introduction

Besides the rectangular patch, the circular geometry has been one of the widely investigated microstrip configurations. Earlier work on circular resonators has dealt with the computation of the resonance frequency [62]. A discussion of different formulas used for the computation of the resonance frequency of circular resonators is included in [42]. A formula for the computation of the edge capacitance of a large microstrip patch is also given in [42]. A recent paper [63] presents a comparison of different formulas used for the evaluation of the resonance frequency of circular patches on thick substrates with experiment. Characteristics of circular patches used as radiators have been studied in detail using the cavity model [12,13,64,65]. Circular patches have also been studied using a moment method analysis of the wire grid model [21], and a generalized transmission line model [66]. A more accurate analysis of circular patches using a full wave analysis is reported in [16].

Variations of the circular patch include the addition of two radial microstrip stubs at the circumference [67] and double coaxial stubs [68] for dual frequency operation and for controlling the radiation pattern. These types of patches can be considered as multiple port patches. For the design of linear mi-

* Work on the circular microstrip patch antenna was not a part of the research project sponsored by the Navy, but is included here (for sake of completeness) as another example where the MNM model has been applied.
microstrip antenna arrays, adjacent circular patches are connected by microstrip line sections. For an accurate design of aperture phase and amplitude distributions in those type of arrays, one needs to know two-port transmission characteristics of circular patches.

The purpose of the present investigation is to extend the multiport network model (MNM) described in Chapter II to the analysis of one-port and two-port circular microstrip patches. The internal fields underneath the patch are computed by modeling the antenna as a two-dimensional resonator. An approximate analysis [69] is reported wherein only the dominant mode of the cavity is considered, fringing fields being accounted for using effective radius, and the radiated power and losses are accounted for by considering an effective loss tangent for the substrate dielectric medium. A more accurate analysis is obtained by modeling the external fields as a multiport network and incorporating the effect of microstrip feed junctions reactances by modeling sections of feed lines as rectangular planar segments. Results computed by this approach are verified by an experiment. The direction polarization of the radiated wave of a circular patch is set by the feed location. To make the polarization independent of the feed location, a variation of the circular patch, with a perfect thin conductor wall located radially ands used to short the patch to the ground plane, has been analyzed.

5.2 Method of analysis

Configurations of one port coaxial-fed and microstrip-fed patches are shown in Figs 5.1 and 5.2 respectively. Equivalent multiport networks for the two configurations are also shown in these figures. For microstrip patches on thin grounded dielectric substrates (of thickness h), the fields underneath the
Figure 5.1 Multiport network model of a coaxial-fed circular microstrip patch
Figure 5.2 Multiport network model of microstrip-fed circular microstrip patch
patch do not vary in the vertical direction (z-direction normal to the patch) except at the feed junction and the patch periphery. The fields underneath the patch are TM to z, and are computed by considering the patch as a two-dimensional circular resonator surrounded by a magnetic wall. The Z-matrix characterization of this circular segment (denoted by 'PATCH' in Figs 5.1 and 5.2), can be obtained by using the Green's function formulation as discussed in Chapter II. The dielectric and conductor losses associated with the fields underneath the patch are taken into account by using an effective loss tangent for the substrate. Computations of the elements of the Z-matrix of a circular segment, including dielectric and conductor losses, are discussed in Section 5.3.

The configuration of the microstrip-fed patch (shown in Fig 5.2) is divided into a circular segment (denoted by PATCH) and a feed line segment (denoted by FEED). As seen in the figure, the FEED segment is not a rectangular segment because of the curved boundary of the circular patch. However, the width of typical microstrip feed lines (with characteristic impedances 50 Ω or higher) is small compared to the radius of the patch. Therefore, the elements of the Z-matrix of the FEED segment are computed (to a good approximation) by considering this segment to be a rectangle. An effective width ($W_e$) and an effective dielectric constant ($\epsilon_{re}$) are used for the rectangular segment modeling the feed line. Elements of the Z-matrix of a rectangular segment have been given in Chapter 4.

The power radiated from the patch and the effect of fringing fields at the edges are included in the MNM analysis by connecting an equivalent edge admittance network (EAN) to the antenna's periphery at a finite number of ports NC (as shown in Figs 5.1 and 5.2). Elements of the Y-matrix (character-
izing the EAN network) for both the coaxial-fed and the microstrip-fed patches will be given in Section 5.4. For thin substrates, the power launched as surface waves is small [39] and has been neglected in the analysis reported here.

Once the Z-matrices for all segments are computed, the segmentation technique is employed to obtain the Z-matrix of the patch antenna (input impedance for the case of one-port patch) and the voltage distribution at the patch periphery. From the Z-matrix, the scattering matrix (input reflection coefficient for the case of one-port patch) is computed and the transmission characteristics of the patch are determined. From the voltage at the radiating edges, the radiation characteristics of the antenna are computed.

5.3 Computation of Z-matrix of a circular segment

The elements of the Z-matrix of a circular segment, are obtained from the Green's function as described in Chapter II. The elements of the Z-matrix are related to Green's function by

\[
Z_{ij} = \frac{1}{W_i W_j} \int \int G(\rho_i, \phi_i | \rho_j, \phi_j)(ds_i)(ds_j)
\]

The integration in (5.1) is along the widths of ports i and j. \(W_{i,j}\) and \((\rho_{i,j}, \phi_{i,j})\) are the widths and the locations respectively of the ith and jth ports.

When the port i is located at the periphery, \((ds_i) = a(d\phi_i)\) and \(W_i = 2a\Delta_i\), where \(2\Delta_i\) is the angular width (in radians) of the port i. When the port i is located inside the patch (as in the case of the port modeling the coaxial feed shown in Fig 5.1), the port width is assumed to be oriented along the \(\phi\)-direction and hence \((ds_i) = \rho_i(d\phi_i)\) and \(W_i = 2\rho_i\Delta_i\), where \(W_i\) is the equivalent width of the coaxial central conductor [14]. For the coaxial-fed patch of Fig
5.1, the whole periphery is divided into NC number of equal width ports and hence \(2\Delta_i = 2\pi/NC\). However for the microstrip-fed patch of Fig 5.2, the open section of the periphery is \(2\pi - \psi\), where \(\psi\) is the angular width of the feed junction \((\psi = \sin^{-1}(W_e/a))\). In this case, the angular width of ports at the open periphery (where EAN is connected) is \(2\Delta = (2\pi - \psi)/NC\). When ND ports are taken along the microstrip feed junction, the angular width of these ports is \(2\Delta_i = \psi/ND\).

The Green's function of a circular resonator with magnetic wall at the periphery can be computed using two different approaches. In the first approach, the Green's function is expanded into eigenfunctions of the circular resonator. Expression for the Green's function based on this approach and the corresponding \(Z_{ij}\) are given in Section 5.3.1. In the second approach, the Green's function is derived by use of the mode-matching technique. The Green's function and the elements of the \(Z\)-matrix based on this approach are given in Section 5.3.2. Comparison between these two formulations is presented in Section 5.3.3. Expressions for Green's function of a circular patch with a shorted radial wall are derived in Section 5.3.4 by using the two techniques mentioned above.

### 5.3.1 Expansion of Green's function into eigenfunctions

In this approach, the Green's function is expanded as a double summation over the eigenfunctions of the circular geometry with magnetic walls [37]. The expansion coefficients are determined by using the orthogonality of the eigenfunctions. The computation of the Green's function for a circular patch with magnetic wall at the periphery is carried out in Appendix A. The elements \(Z_{ij}\) of the \(Z\)-matrix with respect to ports i and j are computed using
(5.1) and are given by

\[ Z_{ij} = \frac{i\omega \mu h}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \text{sinc}(n\Delta_i) \text{sinc}(n\Delta_j) \cos(n\phi_{ij}) J_n(k_{nm}\rho_i) J_n(k_{nm}\rho_j)}{(a^2 - n^2 / k_{nm}^2)(k_{nm}^2 - k^2)} \frac{J_n^2(k_{nm}a)}{J_n^2(k_{nm}a)} \]  

(5.2)

where \( \phi_{ij} = (\phi_i - \phi_j) \) and \( k_{nm} \) is obtained from the mth zero of the derivative of the nth order Bessel function \( J_n \)

\[ \frac{\partial}{\partial \rho} J_n(k_{nm}\rho) \big|_{\rho=a} = 0 \]  

(5.3)

and \( \text{sinc}(x) = \sin(x)/x \). The first zero of (5.3) for \( n=0 \) is equal to zero and \( \sigma_n \) is given by

\[ \sigma_n = \begin{cases} 1, & \text{if } n=0, \\ 2, & \text{otherwise}. \end{cases} \]  

(5.4)

\( a \) is the radius of the circular segment and \( k^2 = \omega^2 \mu \varepsilon_0 \varepsilon_r (1 - i\delta_e) \), where \( \delta_e = \delta_d + \delta_c \). \( \delta_d \) is the loss tangent of the dielectric substrate, and \( \delta_c \) is the effective conductor loss tangent and is given approximately [38] by

\[ \delta_c = \frac{1}{h(\pi f \mu \sigma)^{1/2}} \]  

(5.5)

where \( \sigma \) is the conductivity of the patch and the ground plane. When the ports are located at the periphery, \( \rho_i = \rho_j = a \) and expression (5.2) reduces to

\[ Z_{ij} = \frac{i\omega \mu h}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \text{sinc}(n\Delta_i) \text{sinc}(n\Delta_j) \cos(n\phi_{ij})}{(a^2 - n^2 / k_{nm}^2)(k_{nm}^2 - k^2)} \]  

(5.6)

### 5.3.2 Green’s function using mode-matching technique

An alternative method for the computation of Green’s function \( G \) uses solutions of Helmholtz equation (which \( G \) has to satisfy) in the region I \( (\rho > \rho_0) \) and region II \( (\rho < \rho_0) \) and continuity of the fields at the common circular interface \( (\rho = \rho_0) \) as shown in Fig 5.3. Helmholtz equation for \( G \) in cylindrical coordinates is
Figure 5.3 Configuration of a circular segment for computation of Green's function

\[
\left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k^2 \right) G = -\frac{i \omega \mu h}{\rho} \delta(\rho - \rho_0) \delta(\phi - \phi_0) \tag{5.7}
\]

Since there exists a periodicity of $2\pi$ in the $\phi$-direction, $G$ can be expanded into an exponential Fourier series as

\[
G(\rho, \phi|\rho_0, \phi_0) = \sum_{-\infty}^{+\infty} e^{in(\phi - \phi_0)} G_n(\rho, \rho_0) \tag{5.8}
\]

where $G_n(\rho, \rho_0)$ is a solution of

\[
\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \left( k^2 - \frac{n^2}{\rho^2} \right) \right] G_n(\rho, \rho_0) = -\frac{i \omega \mu h}{2\pi \rho} \delta(\rho - \rho_0) \tag{5.9}
\]
The boundary conditions to be satisfied by $G_n(\rho, \rho_0)$ are:

(i) at the magnetic wall

$$\left. \frac{\partial}{\partial \rho} G_n(\rho, \rho_0) \right|_{\rho=a} = 0 \quad (5.10)$$

(ii) $G_n(\rho, \rho_0)$ is continuous at $\rho = \rho_0$, and

(iii) at the source location, $G_n$ satisfies the following boundary condition:

$$\lim_{\epsilon \to 0} \left( \rho \left. \frac{\partial}{\partial \rho} G_n \right|_{\rho=\rho_0+\epsilon} - \rho \left. \frac{\partial}{\partial \rho} G_n \right|_{\rho=\rho_0-\epsilon} \right) = -\frac{i\omega \mu h}{2\pi} \quad (5.11)$$

Using the above boundary conditions, the Green’s function $G$ (given by 5.8) is obtained as

$$G(\rho, \phi|\rho_0, \phi_0) = \frac{i\omega \mu h}{4} \sum_{n=0}^{\infty} \sigma_n \cos(n(\phi - \phi_0)) \frac{J_n(k\rho_\rho)}{J_n'(ka)} \begin{bmatrix} J_n(k\rho_>)Y_n'(ka) - Y_n(k\rho_>)J_n'(ka) \end{bmatrix} \quad (5.12)$$

where $Y_n$ is Bessel function of the second kind of order $n$, and $\rho>$ and $\rho<$ are defined by

$$\begin{cases} 
\rho_> = \text{Max}(\rho, \rho_0) \\
\rho_< = \text{Min}(\rho, \rho_0) 
\end{cases} \quad (5.13)$$

The elements $Z_{ij}$ of the $Z$-matrix are derived from Green’s function using (5.2) and are given by

$$Z_{ij} = \frac{i\omega \mu h}{4} \sum_{n=0}^{\infty} \sigma_n \cos(n\phi_{ij}) \text{sinc}(n\Delta_i) \text{sinc}(n\Delta_j) \frac{J_n(k\rho_<)}{J_n'(ka)} \begin{bmatrix} J_n(k\rho_>)Y_n'(ka) - Y_n(k\rho_>)J_n'(ka) \end{bmatrix} \quad (5.14)$$

where $\rho>$ and $\rho<$ are given by (5.13) by replacing $(\rho, \rho_0)$ by $(\rho_i, \rho_j)$. When both ports $i$ and $j$ are located at the periphery, $\rho_i = \rho_j = a$ and (5.14) reduces
\[ Z_{ij} = \frac{i\eta_0 h}{2\pi a} \sum_{n=0}^{\infty} \sigma_n \text{sinc}(n\Delta_i) \text{sinc}(n\Delta_j) \cos(n\phi_{ij}) \frac{J_n(ka)}{J'_n(ka)} \]  

Expression (5.15) for ports located at the periphery is identical to the result given in [70].

5.3.3 Comparison of the two formulations

In the previous sections we discussed two different approaches for the computation of the Greens' function of a circular segment. These two formulations are compared by evaluating the input impedance of a circular segment with: \( a=0.8324 \) cm, \( f=7.5 \) GHz, \( \epsilon_r=2.2 \), and \( h=1/32 \) inch. The loss tangent of the dielectric is taken to be equal to 0.028 (this value is obtained from expression (5.35) and includes the radiation loss). These computations have been carried out on VAX 8550. Cases when the two ports are located at the periphery and when they are located inside the patch are discussed separately.

a) Ports at the patch periphery

For ports located at the periphery, expression for elements of \( Z \)-matrix based on expansion of Green's function in eigenfunctions is given by (5.6). This expression involves a double summation. In this expression, the \( m \)th zeros of the derivative of Bessel function of order \( n \) need to be evaluated. Methods for computation of these zeros are given in [71]. These zeros are computed only once and tabulated. Expression of \( Z_{ij} \) derived in Sec. 5.3.2 using the mode matching technique is given by (5.15). In contrast to (5.6), the series in (5.15) consists of only a single summation. In the expression (5.15) we need to compute Bessel functions and their derivatives of different orders and for the complex argument \( ka \). This is carried out by using a backward recurrence formula for Bessel
functions [71]. For very large orders, Bessel functions and their derivatives are approximated using Debey's asymptotic formulas [71]. These asymptotic approximations are very accurate for Bessel functions and their derivatives of order 30 or higher for $|ka| \simeq 1.84$.

Figure 5.4 shows the relative error in $|Z_{11}|$ as a function of the number of terms in the summations. For the double summation, $n$ and $m$ are equal to the number of terms shown in the figure. Also shown on the two plots are the values of the computation time. For a relative error of 0.2%, the number of terms needed in the single summation (5.15) is 80 with a corresponding time of 0.07 CPU seconds, whereas the number of terms needed for the double summation (5.6) is $n=m=200$ and the corresponding computation time is 1.67 CPU seconds.

For the results reported in this Chapter, the summation (5.15) is chosen with 100 terms. This number of terms assures an error in the computation less than 0.1%.

b) Ports inside the patch

For ports located inside the patch, expressions for computation of $Z_{ij}$ are given by (5.2) and (5.14). Expression (5.2), computed using expansion of Green's function into eigenfunctions, involves a double summation over indices $m$ and $n$. Unlike the case where ports are located at the periphery, here we need to evaluate Bessel functions using a backward recurrence formula for every $n$ and $m$. This makes the computation very slow. $k_{nm}$ are computed from the tabulated zeros of derivatives of Bessel functions. On the other hand, expression (5.14) involves only a single summation. In this formula, we need to evaluate Bessel functions of first and 2nd kind and their derivatives for different orders, but for only three distinct values of arguments. The computations of
Figure 5.4 Comparison of formulas for computation of Z-matrix of a circular segment
Bessel functions $J$ of the first kind for every $n$ are carried out using a backward recurrence formula, whereas Bessel functions $Y$ of the 2nd kind are computed using a forward recurrence formula [71].

Table 5.1 Comparison of results for $|Z_{11}|$ of a circular patch using expression (5.2) and (5.12)

<table>
<thead>
<tr>
<th>Number of terms</th>
<th>Expression (5.6)</th>
<th>Expression (5.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>Z_{11}</td>
</tr>
<tr>
<td>10</td>
<td>42.24</td>
<td>1.96</td>
</tr>
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<td>20</td>
<td>38.17</td>
<td>9.74</td>
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<tr>
<td>60</td>
<td>35.42</td>
<td>207.0</td>
</tr>
<tr>
<td>200</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

* Computation time in CPU seconds on VAX5880

The two formulations are compared by evaluating the input impedance of the circular segment (with parameters as given in case a) and with the input port located at $\rho_0 = a/2$. Table 5.1 shows the variation of $|Z_{11}|$ as a function of the number of terms used in the summations. Computation of $|Z_{11}|$ based on expression (5.2) is indeed very slow. For example; for $n=m=60$, the relative error in the value obtained is 3% of the correct value 34.338 and the corre-
sponding computation time is 207 CPU seconds. On the other hand, numerical computation of $|Z_{11}|$ based on expression (5.12) is very efficient, e.g., the value computed using $n=60$ has a relative error of only $0.4\%$ with a computation time of only $0.2$ CPU seconds.

### 5.3.4 Green's function of circular patch with a shorted radial wall

A circular patch with a thin metallic radial wall inserted vertically between the patch and the ground plane (as shown in Fig 5.5) at the location $\phi = 0$ (extending from the center to the periphery) is considered in this Section. The Green's function for this configuration can be determined using the two approaches as discussed for the circular patch in previous sections. The boundary condition which Green's function has to satisfy at the location of the shorted wall is
\[ G(\rho, \phi|\rho_0, \phi_0) \bigg|_{\phi=0} = 0 \] (5.16)

This boundary condition can be satisfied by the part of \( G \) which depends on \( \phi \) only. Thus the dependence of \( G \) on \( \rho \) will remain the same as for the circular patch discussed in previous Sections. Following the same procedure discussed in Appendix A and Section 5.3.2, expression for Green's function using eigenfunctions expansion technique is found as

\[ G(\rho, \phi|\rho_0, \phi_0) = \frac{i\omega \mu h}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \sin(n\phi) \sin(n\phi_0) J_n(k_{nm}\rho) J_n(k_{nm}\rho_0)}{(a^2 - n^2 / k_{nm}^2)(k_{nm}^2 - k^2) J_n^2(k_{nm}a)} \] (5.17)

For ports \( i \) and \( j \) located at the periphery, \( Z_{ij} \) is obtained as

\[ Z_{ij} = \frac{i\omega \mu h}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \sin(n\Delta_i) \sin(n\Delta_j) \sin(n\phi_i) \sin(n\phi_j)}{(a^2 - n^2 / k_{nm}^2)(k_{nm}^2 - k^2)} \] (5.18)

On the other hand, using the mode matching technique, Green's function is obtained as

\[ G(\rho, \phi|\rho_0, \phi_0) = \frac{i\omega \mu h}{4} \sum_{n=0}^{\infty} \sigma_n \sin(n\phi) \sin(n\phi_0) \frac{J_n(k\rho_0 <)}{J_n'(ka)}. \]

\[ \left[ J_n(k\rho_0 >) Y_n'(ka) - Y_n(k\rho_0 >) J_n'(ka) \right] \] (5.19)

For ports \( i \) and \( j \) located at the periphery, \( Z_{ij} \) is given by

\[ Z_{ij} = \frac{i\eta_0 h}{2\pi a} \sum_{n=0}^{\infty} \sigma_n \sin(n\Delta_i) \sin(n\Delta_j) \sin(n\phi_i) \sin(n\phi_j) \frac{J_n(ka)}{J_n'(ka)} \] (5.20)

### 5.4 Description of edge admittance network
In the multiport network model, the effects of fringing fields are modeled by adding an edge admittance network EAN connected to the patch periphery at NC number of ports (as shown in Figs 5.1 and 5.2). The EAN consists of an inductance L, a capacitance C and a conductance G which are divided uniformly among the NC ports. For the coaxial-fed patch, the Y-matrix characterizing the EAN network has dimensions of \((NC \times NC)\). This Y-matrix relates the currents and voltages at the NC ports at the periphery. The numbering of ports for the EAN network for a coaxial-fed patch are also shown in Figure 5.1. In terms of the equivalent circuit of the EAN network (shown in Fig 5.1), the elements of the Y-matrix are expressed as:

\[
\begin{align*}
Y(i,i) &= G_P + j\omega C_P + \frac{1}{j\omega L_P}, & 1 \leq i \leq NC, \\
Y(i,i-1) &= -\frac{1}{j\omega L_P}, & 2 \leq i \leq NC, \\
Y(i,i+1) &= -\frac{1}{j\omega L_P}, & 1 \leq i \leq NC - 1, \\
Y(1,NC) &= Y(NC,1) = -\frac{1}{j\omega L_P}.
\end{align*}
\]

(5.21.a)

where \(G_P = G/NC\), \(C_P = C/NC\) and \(L_P = L/NC\). Other elements of the Y-matrix are equal to zero. G, C and L correspond to the complete circumference of the patch.

For the microstrip-fed patch, the EAN network is connected to the NC ports along the periphery and to the 2 ports along the side of the feed line (near the feed junction plane as shown in Fig 5.2). The connection of the EAN to the FEED segment assures the continuity of the current flow tangential to the circumference. The numbering of ports for the EAN network is also shown in Fig 5.2. In terms of the equivalent circuit of the EAN network (shown in Fig 5.2), the elements of the Y-matrix are expressed as
\[
\begin{align*}
Y(i,i) &= G_P + j\omega C_P + \frac{1}{j\omega L_P}, & 4 \leq i \leq NC + 1, \\
Y(i,i-1) &= Y(i,i+1) = -\frac{1}{j\omega L_P}, & 4 \leq i \leq NC + 1, \\
Y(1,1) &= Y(2,2) = \frac{2}{j\omega L_P}, \\
Y(1,NC+2) &= Y(2,3) = \frac{2}{j\omega L_P}, \\
Y(3,3) &= Y(NC+2,NC+2) = G_P + j\omega C_P + \frac{2}{j\omega L_P}.
\end{align*}
\]

(5.21.b)

where \( G_P = G(1 - \psi/2\pi)/NC, \ C_P = C(1 - \psi/2\pi)/NC \) and \( L_P = L(1 - \psi/2\pi)/NC \). Other elements of the Y-matrix are equal to zero.

a) Edge capacitance

The effects of fringing electric fields at the patch periphery are accounted for by including an edge capacitance connected to each of the ports at the open edge in parallel with the radiation conductance \( G \). The edge capacitance is divided uniformly over the NC ports. A formula for the static fringing edge capacitance of a circular resonator is derived from [42] as

\[
C = 2\varepsilon_0 a \left\{ \ln\left(\frac{4a}{h}\right) - 1 + \varepsilon_r \ln \frac{2\pi}{2Q_0(\frac{1-\varepsilon_r}{1+\varepsilon_r})} \right\}
\]

(5.22)

where

\[
Q_0(x) = \sum_{m=1}^{\infty} x^m \ln(m)
\]

Another expression commonly used for the design of circular microstrip patches is derived from Kirchhoff's formulas [72] as

\[
C = 2\varepsilon_0 a \left\{ \ln\left(\frac{\pi a}{2h}\right) + 1.7726 \right\}
\]

(5.23)

Another method to account for the fringing fields (both electric and magnetic) at the patch periphery is the use of an effective radius \( a_e \) for the circular segment instead of the actual width. The effective radius \( a_e \) is related
to the actual radius $a$ and the fringing capacitance $C$ by

$$a_e = a \left(1 + \frac{hC}{\pi \varepsilon_r \varepsilon_0 a^2}\right)^{1/2} \tag{5.24}$$

This effective radius approach is used in the cavity model [65] and has been used in the approximate analysis of two-port patches using the dominant mode (as discussed in Section 5.6).

b) Edge inductance

The edge inductance $L$ accounts for the energy stored in the fringing magnetic fields. These fringing magnetic fields are associated with the tangential current flowing at the edge of the patch. Since the width of the NC ports along the edge are small, the fringing inductance per unit length $L_e$ along the edge is approximately related to the edge capacitance per unit length as:

$$L_e = \frac{\mu_0 \varepsilon_0}{C_e(\varepsilon_r = 1)} \tag{5.25}$$

where $C_e(\varepsilon_r = 1)$ is the edge capacitance per unit length when dielectric is replaced by air. This formulation can be justified by using similarity between current distribution of a circular patch and that of a rectangular patch operating in the dominant mode.

The fringing inductance was not needed for EAN's at a radiating edge of a rectangular patch (as discussed in Chapter II). This is because of the fact that, for the dominant mode, the voltage is uniform along the radiating edge and no current flows through the inductance if connected. Fringing inductances were required for a rectangular patch when EAN is connected along a non-radiating edge where the voltage has a cosine variation. Thus, fringing inductances are
needed also for a circular patch where the voltage has a cosine distribution along the circumference.

c) Edge conductance

The total radiation conductance of the edge is divided uniformly over the NC ports of the EAN network. The total conductance of the circular patch is related to the power radiated from the circular patch and to the voltage distribution at the edge by

\[
G_r = \frac{2P_r}{\frac{1}{2\pi} \int_0^{2\pi} V^2(\phi) d\phi}
\]  

(5.26)

where the integration in (5.26) is along the patch periphery. For a circular patch operating in the dominant mode (1,1), the voltage distribution at the periphery is \( V(\phi) = V_m \cos(\phi - \phi_m) \), where \( \phi_m \) is the angle at which the voltage is maximum. Therefore, the integration in the denominator of equation (5.26) is equal \( \frac{1}{2} V_m^2 \).

The radiated power \( P_r \) (in 5.26) is computed by using the equivalence theorem. This is done by replacing the fringing field at the periphery by an equivalent magnetic current loop of radius \( a_e \). For the dominant mode (1,1), the radiated power is found as

\[
P_r = \frac{(V_m k_0 a_e)^2}{8\eta_0} \int_0^{\pi/2} \int_0^{2\pi} [F_{\theta}^2(\theta, \phi) + F_{\phi}^2(\theta, \phi)] \sin \theta d\theta d\phi
\]  

(5.27)

where \( F_\theta \) and \( F_\phi \) describe the far field pattern of the patch and are given by

\[
F_\theta(\theta, \phi) = \cos(\phi - \phi_m)[J_2(k_0 a_e \sin \theta) - J_0(k_0 a_e \sin \theta)]
\]  

(5.28)

\[
F_\phi(\theta, \phi) = \cos \theta \sin(\phi - \phi_m)[J_2(k_0 a_e \sin \theta) + J_0(k_0 a_e \sin \theta)]
\]  

(5.29)
$P_r$ can be computed as an infinite summation by using a series expansion for the Bessel functions $J_0$ and $J_2$ [39]. The radiation conductance of the edge is then found as

$$G_r = \pi \frac{[k_0 a_e]^2}{2\eta_0} I_1(k_0 a_e) \quad (5.30)$$

where $I_1$ is given by

$$I_1(x) = \frac{4}{3} - \frac{8}{15} x^2 + \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+4}}{(k + 4)!((k + 2))!} \frac{4k^3 + 34k^2 + 94k + 88}{4k^2 + 24k + 35} \quad (5.31)$$

In the cavity model [73], the effect of the radiated power on the circular patch is included in the solution of the internal field by using an effective loss tangent. The effective radiation loss tangent $\delta_r$ is computed as

$$\delta_r = \frac{h\mu f(k_0 a_e)^2 I_1(k_0 a_e)}{120(k_0 a_e \sqrt{\varepsilon_r} - 1)} \quad (5.32)$$

Expression (5.32) for $\delta_r$ has been used in the approximate analysis of two-port patches using the dominant mode (as will be discussed in Section 5.6).

5.5 Analysis of one-port circular patch

In this section, an analysis of one-port circular patches is carried out using the MNM approach described earlier. For the case of a coaxial-fed patch, the PATCH segment is combined with the EAN segment using the segmentation method. This step yields the voltages at the edges of the patch and the value of input impedance at the coaxial feed. For a microstrip-fed patch, the segmentation method is first applied to combine the PATCH segment with the FEED segment. The results of this step is the Z-matrix with respect to the input port of the feed line, one port on each side of the feed line and NC ports at the periphery. Then, a segmentation method is used to combine the com-
pound segment (PATCH+FEED) with the EAN network. This yields the port voltages at the patch periphery as well as the input impedance of the patch. The input impedance can then be transferred to the location of the feed junction plane using simple transmission line analysis. We start the analysis by comparing various models for the fields outside the patch. Then the MNM is verified by comparing the computed results with available experimental results in [73].

5.5.1 Comparison of various models for external fields

For this study, we consider a coaxial-fed circular microstrip patch with following parameters: \( a = 0.78284 \) cm, \( h = 1/32 \) inch, \( \epsilon_r = 2.2 \) and location of feed \( \rho_0 = 0.212 \) cm. As discussed in Section 5.4, the effect of fringing fields is accounted for in the MNM by adding an EAN network. The fringing capacitance has been computed using expression (5.23) and the inductance is computed from (5.25). In the cavity model, the effect of fringing fields is accounted for by using an effective radius, and the radiation is incorporated by using an effective loss tangent. The two models are compared by looking at the input impedance \( Z_{in} \). Table 5.2 shows the values of \( Z_{in} \) as a function of frequency. As seen from this table, the two models yield almost the same results for the resonance frequency. However the computed value of input impedance (at resonance) using the cavity model is lower than that computed using MNM. This difference stems from the inherent approximation in the computation of radiation loss using an effective loss tangent in the cavity model.

The effects of fringing inductance are investigated by computing the input impedance using capacitance and conductance only in the EAN network. The results for \( Z_{in} \) are shown in Table 5.3. The computed resonance frequency
Table 5.2 Comparison of results for input impedance of a circular patch using MNM and cavity model

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>MNM Model</th>
<th>Cavity Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.40</td>
<td>63.34 + j47.38</td>
<td>62.26 + j43.64</td>
</tr>
<tr>
<td>7.42</td>
<td>67.62 + j36.56</td>
<td>65.81 + j33.22</td>
</tr>
<tr>
<td>7.44</td>
<td>67.89 + j24.61</td>
<td>65.04 + j22.11</td>
</tr>
<tr>
<td>7.46</td>
<td>64.12 + j13.68</td>
<td>61.03 + j12.19</td>
</tr>
<tr>
<td>7.48</td>
<td>57.59 + j5.36</td>
<td>54.69 + j4.72</td>
</tr>
<tr>
<td>7.50</td>
<td>49.98 + j0.06</td>
<td>47.48 - j0.00</td>
</tr>
<tr>
<td>7.52</td>
<td>42.54 - j2.65</td>
<td>40.47 - j2.40</td>
</tr>
<tr>
<td>7.54</td>
<td>35.91 - j3.51</td>
<td>34.22 - j3.12</td>
</tr>
<tr>
<td>7.56</td>
<td>30.31 - j3.21</td>
<td>28.92 - j2.78</td>
</tr>
<tr>
<td>7.58</td>
<td>25.69 - j2.23</td>
<td>24.53 - j1.81</td>
</tr>
<tr>
<td>7.60</td>
<td>21.92 - j0.89</td>
<td>20.93 - j0.50</td>
</tr>
</tbody>
</table>

is \( f=7.208 \) GHz, which is 3.9 % lower than that computed using EAN (\( f=7.5 \) GHz) with inductance included. Therefore, an edge inductance needs to be added to the EAN network to correctly model the fringing fields.

5.5.2 Comparison with experiment

The MNM for circular patches is verified by comparing the computed
Table 5.3 Input impedance of circular patch using EAN without edge inductance

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Input Impedance (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.10</td>
<td>57.33 + j51.45</td>
</tr>
<tr>
<td>7.12</td>
<td>63.69 + j42.47</td>
</tr>
<tr>
<td>7.14</td>
<td>66.79 + j31.12</td>
</tr>
<tr>
<td>7.16</td>
<td>65.66 + j19.31</td>
</tr>
<tr>
<td>7.18</td>
<td>60.77 + j9.21</td>
</tr>
<tr>
<td>7.20</td>
<td>53.69 + j2.02</td>
</tr>
<tr>
<td>7.208</td>
<td>50.63 + j0.01</td>
</tr>
<tr>
<td>7.22</td>
<td>46.06 - j2.19</td>
</tr>
<tr>
<td>7.24</td>
<td>38.92 - j4.08</td>
</tr>
<tr>
<td>7.26</td>
<td>32.74 - j4.39</td>
</tr>
<tr>
<td>7.28</td>
<td>27.59 - j3.74</td>
</tr>
<tr>
<td>7.30</td>
<td>23.39 - j2.58</td>
</tr>
</tbody>
</table>

Results of input impedance of a microstrip-fed patch with the measured results reported in [73]. The patch has the following parameters: \( a = 6.75 \) cm, \( h = 1.59 \) mm, \( \varepsilon_r = 2.62 \) and the width of the microstrip feed is \( w = 4.45 \) cm (corresponding to a 50 \( \Omega \) microstrip line). Computed results using formulas (5.22) and (5.23) for the edge capacitance are shown in Fig 5.6. Results obtained using formula
(5.23) for edge capacitance are in better agreement with the experimental results than those calculated using expression (5.22). Also shown in the figure are results computed using the generalized edge boundary condition (GEBC) method [74].

5.6 Analysis of two-port patch using dominant mode only

Transmission characteristics of the two-port circular microstrip patch (shown in Fig 5.7) may be approximated by considering only the dominant mode to be present. The effects of external fields are included in the analysis by using an effective loss tangent \( \delta_e \) and effective radius \( a_e \) for the PATCH segment and removing the EAN network. Also, the feed junction reactances are not included. This procedure is identical with the cavity model for circular patches as discussed in [73] but with only one mode. When the circular microstrip patch antenna is operating near the resonance frequency of the mode \((n,m)\), usually the dominant \((1,1)\) mode, approximate results can be obtained by considering this resonant mode only [69]. The contribution of higher order modes will be discussed in Section 5.7 using the complete expression for Green’s function.

At the resonance of the \((n,m)\) mode (imaginary part of the input impedance equals zero), \( k_{nm} = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r} \) and \( Z_{ij} \) given in (5.2) can be simplified as

\[
Z_{ij} = \frac{h \sigma_n \text{sinc}(n \Delta_i) \text{sinc}(n \Delta_j) \cos(n \phi_{ij})}{\pi \omega \delta_e \epsilon_0 \epsilon_r (a_e^2 - n^2 / k_{nm}^2)} \frac{J_n(k_{nm} \rho_i)J_n(k_{nm} \rho_j)}{J_n^2(k_{nm} a_e)} \tag{5.33}
\]

where \( \phi_{ij} \) is the angular separation between the \(i\)th and \(j\)th ports. For the dominant \((1,1)\) mode \((k_{11} a_e = 1.84118)\) and also approximating \( \text{sinc}(x) \) by 1, \( Z_{ij} \) is given by

\[
Z_{ij} = 1.674 \frac{h \delta_e}{\lambda_0 \eta_0} \frac{J_1(k_{11} \rho_i)J_1(k_{11} \rho_j)}{J_1^2(k_{11} a_e) \cos(\phi_{ij})} \tag{5.34}
\]
Reference at the edge (10 MHz clockwise increment)

- Measurement [73]
- GEBC [74]
- Δ Edge capacitance (5.22)
- ○ Edge capacitance (5.23)

MNM Model

Figure 5.6 Input impedance locus of one-port microstrip-fed circular patch
Figure 5.7 Multiport network model of a two-port circular microstrip patch
where \( \eta_0 \) and \( \lambda_0 \) are the free space impedance and wavelength respectively. \( \delta_e \) is the effective loss tangent of the patch and is given by

\[
\delta_e = \delta_d + \delta_c + \delta_r
\]  

where \( \delta_d \) is the loss tangent of the dielectric, \( \delta_c \) accounts for ohmic conductor losses of the patch and ground plane and is given by (5.5), and \( \delta_r \) accounts for the power radiated from the patch and is given by (5.32).

For a two-port circular patch (fed at the periphery), elements of the Z-matrix are given by

\[
Z_{11} = Z_{22} = 1.674 \frac{h \eta_0}{\lambda_0 \delta_e}
\]  

and

\[
Z_{12} = Z_{21} = Z_{11} \cos(\phi_{12})
\]  

A scattering matrix representation may be derived from the Z-matrix obtained above (using expression 4.8). Condition for a match at the input port \( S_{11} = 0 \) yields for a two-port patch

\[
Z_0 = Z_{11} \sin(\phi_{12})
\]  

where \( Z_0 \) is the characteristic impedance of the input and output transmission lines. Condition (5.38) also yields a match at the output port. When (5.38) is satisfied, the corresponding transmission coefficient \( S_{21} \) is given by

\[
S_{21} = \frac{\cos(\phi_{12})}{[1 + \sin(\phi_{12})]}
\]  

Equation (5.39) indicates that, for a two-port circular patch, the transmission coefficient \( S_{21} \) varies from 1 to 0 as the angular separation \( \phi_{12} \) is changed from
0° to 90°. When \( \phi_{12} \) increases from 90° to 180°, \( S_{21} \) changes from 0 to -1.

As an example, a two-port circular microstrip patch with following parameters: \( a_e = 0.7902 \text{ cm}, \ h = 1/32 \text{ inch and } \epsilon_r = 2.2 \) has been considered. The computed resonance frequency is \( f = 7.5 \text{ GHz} \) and the corresponding effective loss tangent \( \delta_e = 3.15 \times 10^{-2} \). Table 5.4 shows the variation of \( S_{21} \) with respect to \( \phi_{12} \) and the corresponding values of \( Z_0 \) which yield a match at the input port. We notice from equation (5.38) that \( Z_0 \) is a function of \( \phi_{12} \). For values of \( S_{21} \) close to unity as shown in Table 5.4, the required \( Z_0 \) for input match becomes very small and makes the design impractical.

### Table 5.4 Variation of two-port characteristics of a circular patch with \( \phi_{12} \) (using equations 5.38 and 5.39)

<table>
<thead>
<tr>
<th>( \phi_1 ) (degrees)</th>
<th>( Z_0 = Z_{in}(\Omega) ) (for input match)</th>
<th>Trans. Coeff. ( S_{21} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>197.4</td>
<td>+0.577</td>
</tr>
<tr>
<td>60</td>
<td>341.9</td>
<td>+0.268</td>
</tr>
<tr>
<td>90</td>
<td>394.9</td>
<td>0.000</td>
</tr>
<tr>
<td>120</td>
<td>341.9</td>
<td>-0.268</td>
</tr>
<tr>
<td>150</td>
<td>197.4</td>
<td>-0.577</td>
</tr>
<tr>
<td>175</td>
<td>34.41</td>
<td>-0.916</td>
</tr>
</tbody>
</table>

Circular patch with shorted radial wall

For an input match and a given \( S_{21} \) of a two-port circular patch,
the required characteristic impedance of the feed line varies with the angular separation between the locations of the input and output ports. For values of $S_{21}$ close to unity (typical values needed in design of series fed arrays), the width of the input transmission line becomes very large. This feature is not desirable in array design where the width of the feed lines have to be small. A design method for partially overcoming this problem is the use of circular patches with a shorted radial wall (Fig 5.5) introduced in section 5.3.4. Using the dominant $(1,1)$ mode approximation, the elements of the $Z$-matrix for the two-port patch are given by

$$Z_{ij} = Z_b \sin(\phi_i) \sin(\phi_j)$$ (5.40)

where $Z_b$ is equal to $Z_{11}$ given in (5.34).

A scattering matrix representation may be derived from the $Z$-matrix. When the input and the output transmission line impedances are chosen to be identical, a condition for a match at the input port ($S_{11} = 0$) yields

$$Z_0 = Z_b [\sin^2(\phi_1) - \sin^2(\phi_2)]$$ (5.41)

where $Z_0$ is the characteristic impedance of the feed lines. The corresponding transmission coefficient $S_{21}$ is obtained as

$$S_{21} = \sin \phi_2 / \sin \phi_1$$ (5.42)

From equations (5.41) and (5.42), a match at the input port is obtained if $\sin \phi_1$ is larger than $\sin \phi_2$. For a fixed value of $Z_0$, the transmission coefficient is given in terms of $\phi_2$ as

$$S_{21} = \sin \phi_2 / \sqrt{Z_b / Z_0 + \sin^2 \phi_2}$$ (5.43.a)
Corresponding values of $\phi_1$ are obtained from (5.40). Equivalently we can express $S_{21}$ in terms of $\phi_1$ as

$$S_{21} = \sqrt{1 - (Z_0/Z_b)/\sin^2 \phi_1} \quad (5.43.b)$$

Equation (5.43.a) shows that $S_{21}$ is zero for $\phi_2 = 180^\circ$ (the value $\phi_2 = 0$ is not used because of the location of the shorted wall). The maximum value of $S_{21}$ is obtained from (5.43.b) when $\phi_1 = \pi/2$ (and $\phi_2 = \cos^{-1} \sqrt{Z_0/Z_b}$) and is given by

$$S_{21}^{Max} = \sqrt{1 - Z_0/Z_b} \quad (5.44)$$

Table 5.5 shows the variation of the magnitude of the transmission coefficient for the two-port patch with shorting radial wall (with parameters as that in the previous example) as a function of feed locations. Although a wide range of $S_{21}$ values can be obtained for a fixed value of $Z_0$, as seen from (5.44) the maximum value of $S_{21}$ which can be obtained is still a function of $Z_0$. This limitation on the possible maximum value of $S_{21}$, makes the two-port circular patch not as suitable for a large series-fed array (where values of $S_{21}$ close to unity may be required) as the rectangular elements discussed in Chapter 4.

5.7 Detailed analysis using MNM

An accurate analysis of the transmission characteristics of a two-port microstrip-fed circular patch will be carried out using the MNM approach. This approach, as discussed in Chapter II, accurately takes into account the effect of feed junction reactances. Once each of the components shown in the multiport network model of Fig 5.7 has been characterized in terms of impedance or admittance matrix, the segmentation method of Appendix B is used to find the
Table 5.5 $|S_{21}|$ of two-port circular patch with radial wall as a function of feed locations

| $\phi_1$ (deg.) | $\phi_2$ (deg.) | $|S_{21}|$ |
|-----------------|-----------------|----------|
| 20.8            | 0.00            | 0.000    |
| 30.0            | 20.6            | 0.702    |
| 40.0            | 32.4            | 0.832    |
| 50.0            | 42.7            | 0.885    |
| 60.0            | 52.1            | 0.912    |
| 70.0            | 60.4            | 0.925    |
| 80.0            | 66.7            | 0.932    |
| 90.0            | 69.1            | 0.934    |

two-port characteristics of the patch.

5.7.1 Computational details

The impedance matrices for the two feedline segments are computed by using 100 terms in the single summation of (4.22). The number of ports ND at the feed junctions, where the feedline segments are connected to the patch, is decided by computing the effect of increasing the value of ND on the antenna characteristics iteratively. The impedance matrix of the patch is computed using expression (5.6). The number of terms used in the summation is N=50. The number of ports NC (at the patch periphery where EAN is connected) is chosen such that the width of each port is $\leq 8^\circ$. As in the
case of one-port microstrip-fed patch (shown in Fig 5.2), the EAN network is connected to the periphery and to the feed line segments. The elements of the 
\((NC + 4) \times (NC + 4)\) Y-matrix characterizing the EAN network is evaluated from its equivalent circuit as discussed in Section 5.4.

The segmentation method is first applied to combine the two FEED segments with the PATCH segment. The result of this step is the Z-matrix with respect to the two external ports (1 and 2 as shown in Fig 5.7), 2 ports on each side of the feed lines and the NC ports at the periphery. This network is combined with the EAN network to yield the voltage at the NC ports on the periphery. Also obtained is the Z-matrix relating voltages and currents at the two external ports. From this Z-matrix, the two-port transmission characteristics are obtained.

5.7.2 Effects of higher order modes under the patch

Approximate results for two-port transmission characteristics of circular patch using the dominant mode only has been discussed in Section 5.6. In this section we study the contribution of higher order modes to the characteristics of two-port circular patch by evaluating the input impedance \(Z_{in}\) and the magnitude and the phase of \(S_{21}\). Table 5.6 shows the value of \(Z_{in}\) using the dominant mode only and the variation of \(Z_{in}\) as a function of number of terms taken in the single summation of (5.15). Also shown in Table 5.6, are the amplitude and the phase of the transmission coefficient. Excitation of higher order modes inside the patch affects both real and imaginary part of \(Z_{in}\) as well as the phase and the magnitude of \(S_{21}\). Accurate values are obtained by using \(N=50\) or larger in (5.15).
Table 5.6 Effects of higher order modes on the characteristics of two-port circular microstrip patch

<table>
<thead>
<tr>
<th>Number of terms</th>
<th>$Z_{in}$ ($\Omega$)</th>
<th>Magnitude of $S_{21}$</th>
<th>Phase of $S_{21}$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1) mode</td>
<td>41.2 - j33.8</td>
<td>0.775</td>
<td>-179.2</td>
</tr>
<tr>
<td>5</td>
<td>49.3 - j8.18</td>
<td>0.826</td>
<td>-200.4</td>
</tr>
<tr>
<td>10</td>
<td>50.2 - j1.72</td>
<td>0.842</td>
<td>-205.7</td>
</tr>
<tr>
<td>20</td>
<td>50.8 + j0.37</td>
<td>0.843</td>
<td>-206.7</td>
</tr>
<tr>
<td>50</td>
<td>50.8 + j0.42</td>
<td>0.844</td>
<td>-207.2</td>
</tr>
<tr>
<td>100</td>
<td>50.8 + j0.42</td>
<td>0.844</td>
<td>-207.2</td>
</tr>
</tbody>
</table>

5.7.3 Effects of feed junction reactances

The effects of microstrip feed junction reactances are investigated by studying the variation of two-port antenna characteristics with the number of ports taken at the feed junction. When there is only one port at the feed junction, the effects of feed junction reactances are not present in the analysis. Table 5.7 shows the variation of the input impedance as a function of ND. Also shown in the Table is the magnitude and phase of $S_{21}$. The variation in the input impedance is mostly in the imaginary part. Accurate results are obtained by using ND equal to or greater than 5.

5.8 Comparison with experiment

The numerical results for two-port transmission characteristics of cir-
Table 5.7 Effects of feed junction reactances on the characteristics of a two-port circular microstrip patch

<table>
<thead>
<tr>
<th>Number of terms (ND)</th>
<th>$Z_{in}$ (Ω)</th>
<th>Magnitude of $S_{21}$</th>
<th>Phase of $S_{21}$ (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51.45 + j5.56</td>
<td>0.862</td>
<td>-210.3</td>
</tr>
<tr>
<td>3</td>
<td>51.30 + j2.10</td>
<td>0.847</td>
<td>-208.3</td>
</tr>
<tr>
<td>5</td>
<td>50.81 + j0.42</td>
<td>0.844</td>
<td>-207.2</td>
</tr>
<tr>
<td>8</td>
<td>50.71 + j0.00</td>
<td>0.844</td>
<td>-207.0</td>
</tr>
<tr>
<td>10</td>
<td>50.68 - j0.14</td>
<td>0.843</td>
<td>-206.8</td>
</tr>
</tbody>
</table>

A two-port circular microstrip patch, fed with microstrip lines located at the periphery with $a=0.7825$ cm, $\varepsilon_r=2.2$, $h=1/32$ inch, $\phi_{12} = 156.5^\circ$, has been considered. The characteristic impedance of the input and output lines is chosen to be equal to 50 Ω and the corresponding widths of the transmission lines are $W_1 = W_2=2.426$ mm. The measurements were done using HP8510 Automatic Network Analyzer.

Figure 5.8 shows the variation of theoretical and experimental results for the input VSWR versus frequency from 7 GHz to 8 GHz. The theoretical results have been computed by using formula (5.23) for the edge capacitance. The computed resonance frequency using MNM is 7.6 GHz and the measured resonance frequency is approximately 7.65 GHz. The bandwidth of the two-port patch for $VSWR \leq 1.5$ is of the order of 10%.
Figure 5.8 Comparison between theory and experiment for input VSWR of a two-port circular microstrip patch.
Figure 5.9 shows a comparison between theoretical and measured magnitude of the transmission coefficient of the two-port patch. Good agreement between theory and experiment is obtained over the whole range of frequency considered. Theoretical values of $|S_{21}|$ are slightly smaller than the measured values. This may be due to the extra losses in the connectors, which are not accounted for in the theory.

Variation of the phase of the transmission coefficient with frequency is shown in Fig 5.10. Theoretical curve is shifted from the measured curve by almost 7°. A phase delay of 7° is approximately the delay caused by 0.5 mm length of 50 Ω microstrip line. Thus, the difference between theoretical and measured results can be attributed to an error in the calibration used to extract the phase delay of the input and output transmission lines.

5.9 Concluding remarks

The MNM approach has been extended for the analysis of circular microstrip patches. The external fields are modeled by a network consisting of inductances, capacitances and conductances. The resonance frequency computed without including the edge inductance was 3.9 % lower than that computed with the edge inductance included. Effects of fringing inductances and capacitances can alternatively be taken into account by using an equivalent effective radius for the patch.

Two different formulations for the computation of the elements of the Z-matrix for a circular patch have been investigated and compared. The use of single summation for computation of the elements of the Z-matrix of a circular segment makes the method more efficient.

The MNM also has been extended to the analysis of two-port circular
Figure 5.9 Comparison between theory and experiment for magnitude of transmission coefficient of a two-port circular microstrip patch
Figure 5.10 Comparison of theory and experiment for phase of transmission coefficient of two-port circular patch.
patches. The results obtained from this study, shows that is possible to control the transmission coefficient by varying the angular separation between input and output ports. However, to achieve a match at the input port, the characteristic impedance $Z_0$ of the input and output transmission lines also must be selected appropriately for every angular separation. For two-port circular patches with a shorted radial wall, a wide range of values of $S_{21}$ can be obtained for a specified value of the characteristic impedance $Z_0$. Usually the design of two-port patches (used as element of series-fed array) for specified transmission characteristics requires the optimization of the design parameters. A very good starting point in the optimization process is the use of the results based on the dominant mode analysis. This method of analysis can be easily extended to circular patches with 3 or more ports, if needed.

The MNM for circular microstrip patches has been verified by the experiment of Lo et al [73]. Very good agreement is observed for the input impedance locus. Also good agreement with experimental results have also been obtained for the scattering parameters of a two-port patch (designed and fabricated as a part of this thesis work).

The MNM for circular patches, presented in this chapter, can be used for the evaluation of mutual coupling between circular microstrip patches. The MNM approach for circular patches discussed in this chapter can be modified to include the effects of mutual coupling among sections of the patch periphery on the input impedance and resonance frequency. The analysis can also be used for the computation of the parasitic radiation from the microstrip feed lines (as was done for the rectangular patch in Chapter 4). The method can also be used for the analysis of the circular patch [67] with two radial microstrip stubs at the circumference and the circular patch [68] with double coaxial stubs.
CHAPTER VI

MNM FOR MICROSTRIP PATCHES COVERED WITH A DIELECTRIC LAYER

6.1 Patches with a dielectric cover layer

In many applications, microstrip antennas and arrays require a dielectric cover layer to provide protection from heat, physical damage and the environment. When microstrip antennas are used outdoors, they are exposed to snow and icing. Because of the inherent narrow bandwidth of microstrip antennas, the presence of dielectric layer(s) can seriously alter the performances of these antennas (e.g; resonance frequency, input impedance, Q-factor and the radiation pattern) and can cause the excitation of surface waves.

The design of arrays, for given specifications, requires the accurate determination of the amplitude and phase distributions. The presence of a dielectric layer over the array affects both the amplitude and the phase distributions which are otherwise controlled by the transmission lines and the radiating elements. These effects may lead to the operation of the array outside the band, the increase in side lobe level and/or a change in the main beam direction; therefore, the presence of a dielectric cover (which may consists of one or more layers) over microstrip antennas and arrays should be taken into account in the initial design itself.

Microstrip transmission lines with a cover layer have been investigated
in [75,76]. A useful formula relating the effective dielectric constant $\varepsilon_{re}$ to the line parameters is given in [75]. Basic properties of microstrip patch antennas with a cover layer are discussed in [77], by solving the problem of a Hertzian electric dipole over a ground plane with a dielectric cover layer. For rectangular microstrip patches, Bahl and others [30] describe the fractional change in the resonance frequency resulting from the presence of a cover layer by computing $\varepsilon_{re}$ of the patch (modeled by an equivalent transmission line). These authors also present experimental data for the effects of a dielectric loading on the characteristics of microstrip antennas (e.g; resonance frequency, input reflection coefficient and bandwidth). Experimental study of the resonance frequency of rectangular patches for various cover layer thicknesses have been carried out in [32]. An empirical expression relating the resonance frequency to the cover layer thickness is given in [32]. The cavity model has been used to obtain an equation relating the input impedance and the feed locations with and without cover layer. Hansen and Patzold [31] presented a full-wave moment method analysis of the input impedance and the mutual coupling among rectangular patches with a dielectric cover layer. The effects of a dielectric cover layer on a serpent antenna have been reported in [78]. The effects of lossy and lossless cover layer(s) on the radiation characteristics and excitation of surface waves of a microstrip patch on thin substrates are discussed in [79]. Analysis of rectangular microstrip patches based on Wiener-Hopf technique has been investigated in [80]. Cover layer may be used to improve the performance of microstrip patches. Criteria for obtaining nearly omnidirectional radiation pattern are reported in [81], and a gain enhancement method for microstrip antennas covered with a dielectric layer are presented in [82].
The purpose of the work reported in this Chapter is to extend the multiport network model (discussed in Chapter 2) to the analysis and design of microstrip patch antennas on thin substrates covered with a dielectric layer. Designs of one-port and two-port rectangular patches with a cover layer have been carried out and the results are compared with experiment. The mutual coupling between two identical one-port microstrip-fed rectangular patches is investigated and the results obtained are compared with experiment for typical values of spacing. Effects of surface waves, which are excited because of the presence of the cover layer, are also discussed.

6.2 Multiport network model

The MNM method for microstrip patches with cover layer is illustrated in this chapter by considering the case of rectangular patches. Figure 6.1 shows a two-port rectangular microstrip patch over ground plane separated by a thin substrate of thickness h and dielectric constant $\varepsilon_{r2}$ and covered by a thick dielectric cover of thickness d and dielectric constant $\varepsilon_{r1}$. In the MNM analysis of microstrip patches, as discussed in Chapter 2, the solution of the internal and the external fields are formulated separately in terms of networks. The multiport network representation of the two-port rectangular patch with cover layer is also shown in Fig 6.1. Each network is characterized in terms of an impedance/admittance matrix and connected to other networks at a finite number of ports (denoted by NC and ND in the Figure). The number of ports at each interconnection is decided by the fields variation along that interface. The segmentation technique is used to combine these networks together for computing the antenna characteristics (e.g; the input impedance and the voltages at the radiating edges of the patch). A method for obtaining the elements
Figure 6.1 Equivalent multiport network of a rectangular microstrip patch with a cover layer
of the Z-matrix of each network shown in Fig 6.1, when a cover layer is present, is discussed in the following sections.

6.2.1 Modeling of patch and feed line segments

The rectangular patch segment (denoted by PATCH in Fig 6.1) accounts for the internal fields inside the patch and the fringing fields at the non-radiating edges of the patch (as discussed in Chapter II). Since the substrate used is taken to be thin, the fields underneath the patch have no variation in the z-direction. Therefore these fields (underneath the patch) can be computed by modeling the antenna as a two-dimensional planar network with magnetic wall located at the edges of the patch. The fringing (electric and magnetic) fields at the non-radiating edges, which are affected by the presence of the cover layer, are accounted for by moving the locations of the magnetic walls by an appropriate amount. This is done by using an effective dielectric constant \( \varepsilon_{re} \) and an effective width \( b_e \) for the planar segment. A method for evaluating \( \varepsilon_{re} \) of a microstrip line with a cover layer is given in [75]. The elements of the Z-matrix of the PATCH segment (derived from the Green’s function) are evaluated from (4.12) by using \( \varepsilon_{re} \) and \( b_e \) computed with a cover layer present.

The effects of microstrip feed line junction reactances are included in the MNM by considering short sections of the feed lines as rectangular planar segments (denoted by FLN in Fig 6.1). The fields underneath the feed lines are also expanded as a sum of the rectangular cavity modes using the Green’s function technique. The effect of the dielectric cover layer is included by using an effective dielectric constant \( \varepsilon_{re} \) and an effective width \( W_e \) of the microstrip lines with the cover layer. The contribution of the feed junctions to the radiation and the excitation of surface waves can also be included in the analysis by taking
ports along the width of the feed line segments and along the patch segment near the locations of the feed junctions (as done in Chapter Four). Those effects are assumed to be of a second order and are not included in the analysis reported in this Chapter.

6.2.2 Modeling of EAN and MCN networks

The edge admittance network EAN (shown in Fig 6.1), accounts for the effects of fringing fields, the power radiated and the power coupled to surface waves at the radiating edges. The radiated power is represented by an edge conductance $G_r$. Similarly, the power coupled to the surface waves is represented by an edge conductance $G_s$. The fringing field at the radiating edges of the patch, which causes an accumulation of electric charges, is represented in terms of an edge capacitance $C$. Each edge of the patch is divided into a number of small segments, and the conductances $G_r$ and $G_s$ and the capacitance $C$ are distributed uniformly over those segments.

The edge conductances $G_r$ and $G_s$ are determined by using equivalent magnetic current line sources at the edges of the patch (as discussed in Chapter Two). The electromagnetic fields of a magnetic dipole over ground plane with a dielectric cover layer are derived in Appendix D. Using the results of Appendix D, the free space and surface wave far fields are obtained and hence $G_r$ and $G_s$ can be evaluated.

The edge capacitance $C$ at the radiating edges is determined from the measured resonance frequencies of unloaded rectangular microstrip patches. This procedure is discussed in Section 6.4. Analytical computation of edge capacitance requires the precise determination of the fringing field distribution. Thus the approximation of fringing aperture fields by an equivalent magnetic
current line source, used for the computation of edge conductances and mutual coupling, cannot be used for the computation of \( C \).

The network MCN (shown in Fig 6.1) accounts for the external interaction between the radiating edges of the patch. A similar network may be used to represent the coupling among all edges of the patch or between two patches of an array. The elements of the \( Y \)-matrix characterizing MCN network are also obtained by modeling the field at the aperture by an equivalent magnetic current line source. This procedure is discussed in Section 6.5.

### 6.3 Edge conductances with cover layer

As discussed in Section 6.2, the edge conductances \( G_r \) and \( G_s \) account for the power radiated and the power launched into surface waves respectively. The conductances \( G_r \) and \( G_s \) of an edge are related to the voltage distribution at the edge by

\[
G_{r,s} = \frac{2P_{r,s}}{W \int V^2(s)ds} \tag{6.1}
\]

where the integration is along the patch edge of length \( W \), and \( V(s) \) is the voltage distribution along the edge. \( P_{r,s} \) is computed for the edge voltage distribution \( V(s) \). For a microstrip patch operating in the dominant mode, the voltage is uniform \( (=V_0) \) along the radiating edges and the denominator in (6.1) reduces to \( V_0^2 \).

The radiated power \( P_r \) and the surface wave power \( P_s \) are computed by using the equivalence principle. We consider the case of an arbitrary shaped patch shown in Fig 6.2 (rectangular geometry being a special case). Assuming the substrate to be thin, the edge aperture field is replaced by an equivalent
Figure 6.2 Equivalent magnetic current model of an arbitrary shaped patch
magnetic current line source over a ground plane (as shown in Fig 6.2). This line source is then divided into a number of small segments. The EM fields generated by the line source is the superposition of the fields due to each magnetic current element of moment $P_m^i = V_i W_i$, where $V_i$ is the value of the voltage at the location of the $i$th element of width $W_i$. In order to compute $P_r$ and $P_s$, we need to evaluate the free-space and surface-wave far field components of the electromagnetic fields of a magnetic current element arbitrary located in the $z=0$ plane.

6.3.1 Free-space far field of a magnetic current element

The free space far field of the magnetic current element (considered in Appendix D) can be evaluated by using the steepest descent method [83]; but it can be readily computed by using the following identity

$$I = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{f}(\alpha, \beta, u_0) e^{-u_0 k_0 z + ik_0 (\alpha z + \beta y)} \frac{d\alpha d\beta}{u_0} \bigg|_{r \to \infty}$$

$$\approx 2\pi f(\alpha \to \nu_z, \beta \to \nu_y, u_0 \to i\nu_z) \frac{e^{-ik_0 r}}{k_0 r}$$

(6.2)

where $\nu_z = \sin \theta \cos \phi$, $\nu_y = \sin \theta \sin \phi$, $\nu_z = \cos \theta$ and $u_0$ is given by (D.19). The spherical coordinates are as shown in Fig D.1 with the magnetic current element oriented along the x-axis. Using (6.2), (D.17) and (D.18), the free space far field approximations for $\Pi_{\epsilon z}$ and $\Pi_{m z}$ are

$$\Pi_{\epsilon z}(\phi, r, \theta) = -\frac{\epsilon_r P_m}{2\pi} \frac{\sin \phi \cot \theta}{\sqrt{\sin(\sqrt{k_0 d}) - i \cos \theta \epsilon_r}} \frac{e^{-ik_0 (r - d \cos \theta)}}{k_0 r}$$

(6.3)
\[ \Pi_{mz}(\phi, r, \theta) = \frac{P_m}{2\pi \eta_0 \cos \theta} \frac{\sqrt{\varepsilon} \cos \phi \cot \theta}{\cos(\sqrt{k_0 d}) - i \sqrt{\varepsilon} \cos(\sqrt{k_0 d})} \frac{e^{-ik_0(r \cdot d \cos \theta)}}{k_0 r} \]  

(6.4)

where \( \sqrt{\varepsilon} = \sqrt{\varepsilon_{r1} - \sin^2 \theta} \). In spherical coordinates, and approximated to the order of \( 1/k_0 r \), we have \( \nabla \rightarrow -ik_0 r \). Then using equation (D.3), the electric field \( \overline{E}(r, \phi, \theta) \) (which defines the polarization of the radiation) is given by

\[ \overline{E}(r, \phi, \theta) \bigg|_{r \to \infty} = -k_0^2 \Pi_{e_z} \sin \theta \overline{a}_\theta + k_0^2 \eta_0 \Pi_{m_z} \sin \theta \overline{a}_\phi \]  

(6.5)

Therefore, \( E_\phi \) and \( E_\theta \) are obtained as

\[ \begin{align*}
E_\phi &= \frac{k_0 P_m}{2\pi} e^{-ik_0 r} F_\phi(\phi, \theta) \\
E_\theta &= \frac{\varepsilon_{r1} k_0 P_m}{2\pi} e^{-ik_0 r} F_\theta(\phi, \theta)
\end{align*} \]  

(6.6)

where \( F_\phi \) and \( F_\theta \) describe the H-plane (\( \phi = 0^\circ \)) and the E-plane (\( \phi = 90^\circ \)) far field patterns of the dipole and are given by

\[ \begin{align*}
F_\phi &= \frac{\sqrt{\varepsilon} \cos \theta \cos \phi e^{ik_0 d \cos \theta}}{\cos \theta \sin(\sqrt{k_0 d}) - i \sqrt{\varepsilon} \cos(k_0 d)} \\
F_\theta &= \frac{\cos \theta \sin \phi e^{ik_0 d \cos \theta}}{\sqrt{\varepsilon} \sin(\sqrt{k_0 d}) - i \varepsilon_{r1} \cos \theta \cos(k_0 d)}
\end{align*} \]  

(6.7)

The radiation pattern as well as the radiation conductance of a patch can be obtained from equations (6.6) and (6.7).

### 6.3.2 Computation of radiated power

For the arbitrarily shaped patch of Fig 6.2, the radiation conductance is computed from the power radiated from the edges of the patch using equation (6.1). The power radiated from the patch is found by integrating Poynting
vector over the half hemisphere and is given by

\[ P_r = \frac{1}{2} \int_0^{\pi/2} \int_0^{2\pi} (|E_\theta|^2 + |E_\phi|^2) \frac{r^2 \sin \theta d\theta d\phi}{\eta_0} \]  

(6.8)

For a magnetic current element, the contribution of \( E_\theta \) to the radiated power (denoted by \( P_r^{TM} \)) corresponds to the TM-to-z mode, whereas the contribution of \( E_\phi \) (denoted by \( P_r^{TE} \)) corresponds to the TE-to-z mode. Using equations (6.6) and (6.7), \( P_r^{TE} \) and \( P_r^{TM} \) are given respectively by

\[ P_r^{TE} = \frac{(k_0 P_m)^2}{8\pi \eta_0} \int_0^{\pi/2} \frac{(\varepsilon_{r1} - \sin^2 \theta) \sin \theta \cos^2 \theta}{\cos^2 \theta \sin^2 (\sqrt{k_0 d}) + (\varepsilon_{r1} - \sin^2 \theta) \cos^2 (\sqrt{k_0 d})} d\theta \]  

(6.9)

\[ P_r^{TM} = \frac{(\varepsilon_{r1} k_0 P_m)^2}{8\pi \eta_0} \int_0^{\pi/2} \frac{\sin \theta \cos^2 \theta}{\varepsilon_{r1} \cos^2 \theta \cos^2 (\sqrt{k_0 d}) + (\varepsilon_{r1} - \sin^2 \theta) \sin^2 (\sqrt{k_0 d})} d\theta \]  

(6.10)

when \( d \to 0 \) and \( \varepsilon_{r1} \to 1 \), the radiated power computed using equations (6.9) and (6.10) is identical to that derived for a magnetic current element over ground plane without a cover layer.

For the arbitrary shaped patch shown in Fig 6.2, the far field is given by

\[ \overline{E^j} = \sum_{i=1}^{N} \overline{E^i} \]  

(6.11)

where the summation is over the N segments along the periphery. Each segment (shown in Fig 6.2 is characterized by its location \((\phi^i_0, \rho^i_0)\) and the orientation angle \(\psi^i_0\) of the magnetic current with respect to the \(x_0\)-axis. The far field of the ith dipole expressed in the \((x_0, y_0, z_0)\) coordinates system is given by
\[
\begin{align*}
E_{\phi_0}^i &= \frac{k_0 P_m^i e^{-ik_0 r_0}}{2\pi r_0} F_{\phi_0}^i(\phi_0, \theta_0) \\
E_{\theta_0}^i &= \frac{\epsilon_{\epsilon_1} k_0 P_m^i e^{-ik_0 r_0}}{2\pi r_0} F_{\theta_0}^i(\phi_0, \theta_0)
\end{align*}
\] (6.12)

where \( F_{\phi_0}^i \) and \( F_{\theta_0}^i \) for the ith dipole are given by

\[
\begin{align*}
F_{\phi_0}^i &= \frac{\sqrt{\cos \theta_0 \cos(\phi_0 - \psi_0^i)} e^{ik_0 \rho_0^i} \cos(\phi_0 - \phi_0^i) \sin \theta_0}{\cos \theta_0 \sin(\sqrt{k_0 d}) - i \sqrt{k_0 d}} \\
F_{\theta_0}^i &= \frac{\cos \theta_0 \sin(\phi_0 - \psi_0^i) e^{ik_0 \rho_0^i} \cos(\phi_0 - \phi_0^i) \sin \theta_0}{\sin(\sqrt{k_0 d}) - i \epsilon_{\epsilon_1} \cos \theta_0 \cos(\sqrt{k_0 d})}
\end{align*}
\] (6.13)

Using equations (6.11), (6.12) and (6.13) the far field pattern and the radiated power from the patch can be computed. Thus the radiation conductance can be obtained for an arbitrarily shaped patch. For a rectangular patch (shown in Fig 6.1), \( \psi_0^i = -\pi/2 \) for segments located at the radiating edge defined by \( x=0 \) and is equal to \( \pi/2 \) for segments located at the radiating edge defined by \( x=a \).

### 6.3.3 Surface wave field of a magnetic current element

To compute the surface wave fields, we need to extend the path of integration in equations (D.29) from \(-\infty\) to \(+\infty\). This is done by using the following integral relationship [83]

\[
\int_{-\infty}^{+\infty} J_n(\lambda \rho) f(\lambda^2) \lambda^{n+1} d\lambda = \frac{1}{2} \int_{-\infty}^{+\infty} H_n^{(2)}(\lambda \rho) f(\lambda^2) \lambda^{n+1} d\lambda
\] (6.14)

So the new expressions for \( \Pi_{ez} \) and \( \Pi_{mx} \) are
\[
\begin{align*}
\Pi_{\varepsilon z}(\phi, \rho, z) &= -i\pi \sin \phi \int_{-\infty}^{\infty} \tilde{F}_e(\lambda^2, z) H_1^{(2)}(k_0 \lambda \rho) d\lambda \\
\Pi_{\mu z}(\phi, \rho, z) &= -i\pi \cos \phi \int_{-\infty}^{\infty} \tilde{F}_m(\lambda^2, z) H_1^{(2)}(k_0 \lambda \rho) d\lambda
\end{align*}
\tag{6.15}
\]

![Diagram](image)

Figure 6.3 Integration path in the complex $\lambda$-plane

The complex $\lambda$-plane for the integrations in equations (6.15) is shown in Fig 6.3. Closing the contour of integration as shown in the figure (by adding
the integration paths \( C_2 \) and \( C_3 \), the integration in (6.15) is equal to the integration along the branch cut (path \( C_3 \) in clockwise direction) plus the residue contribution at the poles \( \lambda_p \) (\( p=1,2,3... \)). The integration along the branch cut yields the continuous spectrum, while the residue calculations at the surface wave poles give the surface wave contributions to the integral. The integration along the path \( C_2 \), which is taken at \( \infty \), is zero because the fields become zero as \( \lambda \to \infty \).

The TM and TE surface wave poles \( \lambda_p \) are the zeros of \( D_{TM} \) and \( D_{TE} \) respectively

\[
\begin{align*}
    D_{TM}(\lambda_p) &= \varepsilon_{r1} u_{0p} \cos(k_0 du_{1p}) - u_{1p} \sin(k_0 du_{1p}) = 0 \\
    D_{TE}(\lambda_p) &= iu_{0p} \sin(k_0 du_{1p}) + iu_{1p} \cos(k_0 du_{1p}) = 0
\end{align*}
\]

(6.16)

where \( u_{0p} = \sqrt{\lambda_p^2 - 1} \) and \( u_{1p} = \sqrt{\varepsilon_{r1} - \lambda_p^2} \). The maximum number of poles for the TM and TE surface wave modes are given by [84]

\[
\begin{align*}
    N_{TM}^{\text{max}} &= \text{Intg} \left( \frac{\Delta}{\pi} + 1 \right) \\
    N_{TE}^{\text{max}} &= \text{Intg} \left( \frac{\Delta}{\pi} + \frac{1}{2} \right)
\end{align*}
\]

(6.17)

where \( \Delta = k_0 d \sqrt{\varepsilon_{r1} - 1} \) and \( \text{Intg}(x) \) denotes the integer part of \( x \). Equation (6.17) shows that there is always a fundamental TM mode which has no cutoff frequency. The cutoff frequency for the first TE surface wave mode is given by

\[
f_{TE}^c = \frac{3 \times 10^8}{4d \sqrt{\varepsilon_{r1} - 1}}
\]

(6.18)

The cutoff frequency \( f_{TM}^c \) for the first higher order TM surface wave mode is twice \( f_{TE}^c \). It should be noticed that if \( \lambda_p \) is a zero of (6.16), so is \(-\lambda_p \). Thus
the zeros are symmetrically located along the real axis. For the cover layer parameters considered in this study, only the fundamental TM surface wave mode \( p=1 \) is excited.

The surface wave contribution to the integrals due to the fundamental mode is computed from the residue at the pole \( \lambda_p \) and is given by

\[
\Pi^s_{cz} = -\frac{i\mathcal{P}_m}{2} \left\{ \frac{\varepsilon_{r1} e^{-k_0 u_{0p}(z-d)}}{D'_{TM}(\lambda_p)} H^{(2)}_1(k_0 \lambda_p \rho) \sin \phi; \quad z \geq d, \right. \\
\left. \frac{u_{1p} \cos(\ldots) + \varepsilon_{r1} u_{0p} \sin(\ldots)}{u_{1p} D'_{TM}(\lambda_p)} \cos(k_0 u_{1p} z) H^{(2)}_1(k_0 \lambda_p \rho) \sin \phi, \quad 0 \leq z \leq d. \right. 
\] (6.19)

where \( \ldots = k_0 d u_{1p} \) and

\[
D'_{TM}(\lambda_p) = k_0 d \lambda_p \left\{ \frac{\varepsilon_{r1}}{k_0 d u_{0p}} + \frac{\tan(k_0 d u_{1p})}{k_0 d u_{1p}} + \frac{1}{\cos^2(k_0 d u_{1p})} \right\} 
\] (6.20)

The components of the electromagnetic fields of a magnetic current element are computed from \( \Pi^s_{cz} \) using equations (D.4) and (D.5). In the computation of the mutual coupling and surface wave power (discussed later) we need to know \( E_z, H_\rho \) and \( H_\phi \) only. Using equation (D.14), \( E_z \) is found as

\[
E_z(\rho, \phi, z) = k_0^2 \lambda_p^2 \Pi^s_{cz}(\rho, \phi, z) 
\] (6.21)

The magnetic field components \( H_\phi \) and \( H_\rho \) are computed by using equation (D.5) and are given by

\[
\left\{ \begin{array}{l}
H_\rho = \frac{i k_0 \varepsilon_{r1}}{\eta_0 \rho} \frac{\partial}{\partial \rho} \Pi^s_{cz} \\
H_\phi = -\frac{i k_0 \varepsilon_{r1}}{\eta_0} \frac{\partial}{\partial \rho} \Pi^s_{cz}
\end{array} \right.
\] (6.22)
Expressions (6.21) and (6.22) are used for computing the power coupled surface wave and for investigating the contribution of surface wave fields to the mutual coupling.

6.3.4 Computation of surface wave power

The surface wave conductance $G_s$ of the arbitrary shaped patch of Fig 6.2 is computed from the power launched into surface wave as given by equation (6.1). For a microstrip patch with a cover layer, the surface wave power is divided into two parts. The first part $P_s^f$ represents the power carried by the fields which exist outside the dielectric cover layer and which decay exponentially as we move away from the air-dielectric interface. The second part of the power $P_s^c$ is due to the fields inside the dielectric cover layer. Both of these components decay approximately as $1/\sqrt{\rho}$ in the radial direction. The surface wave power is computed by integrating the Poynting vector over the surface of a cylinder of radius $\rho$ ($\to \infty$) and height $z$ ($\to \infty$) with its base on the ground plane. The contribution to the integral from the bottom surface is zero since the tangential electric field is zero at the surface of a perfect conductor. The contribution of the surface area of the top of the cylinder is zero since the fields are zero when $z \to \infty$ as given in equation (6.19). The only contribution to the integral comes from the periphery of the cylinder and is given by

$$
\begin{align*}
T_s^c &= -\frac{1}{2} \text{Re} \int_0^d \int_0^{2\pi} E_z H_{\phi}^* \rho d\phi dz \bigg|_{\rho \to \infty} \\
T_s^f &= -\frac{1}{2} \text{Re} \int_d^\infty \int_0^{2\pi} E_z H_{\phi}^* \rho d\phi dz \bigg|_{\rho \to \infty}
\end{align*}
$$

(6.23)

The field components $E_z$ and $H_{\phi}$ are given by equations (6.21) and (6.22)
respectively. Since the integrations in (6.23) are evaluated at \( \rho \to \infty \), Hankel function \( H_1^{(2)} \) can be replaced by its large argument approximation [71] as

\[
H_1^{(2)}(k_0 \lambda_p \rho) \approx \sqrt{\frac{2}{\pi k_0 \lambda_p \rho}} e^{-i(k_0 \lambda_p \rho - \frac{\pi}{4})}
\]  

(6.24)

For a magnetic current element, \( P_s^f \) and \( P_s^c \) are obtained as

\[
P_s^f = \frac{(P_m \epsilon_r \lambda_p)^2}{8 \eta_0 \eta_0 u_{0p}(D'_{TM})^2}
\]  

(6.25)

and

\[
P_s^c = \frac{d(P_m \epsilon_r \lambda_p)^2}{8 \eta_0 (u_{1p} D'_{TM})^2} \left[ 1 + \frac{\sin(2k_0 du_{1p})}{2k_0 du_{1p}} \right] 
\cdot \left[ u_{1p} \cos(k_0 du_{1p}) + \epsilon_r \eta_0 u_{0p} \sin(k_0 du_{1p}) \right]^2
\]  

(6.26)

The surface wave far field of the \( i \)th segment (shown in Fig 6.2) is obtained from the previously computed results for a dipole located at the origin and oriented along the x-axis by replacing \( \rho \) by \( \rho_0 - \rho_0^i \cos(\phi_0 - \phi_0^i) \) in the exponent of Hankel function (6.24) and replacing \( \phi \) by \( \phi_0 - \psi_0^i \) in (6.19). The surface wave power given by (6.23) (and hence the surface wave conductance) for the arbitrary shaped patch (shown in Fig 6.2) is computed by superposing the fields due to all segments at the periphery.

**6.3.5 Numerical results for \( G_r \) and \( G_s \)**

Expressions for edge conductances based on radiated and surface wave powers are given by (6.1). The computation of radiated power, as given by (6.8), requires the evaluation of double integrations (with respect to \( \theta \) and \( \phi \)
variables). On the other hand, the computation of surface wave power, as given by (6.23), requires the evaluation of only a single integration (with respect to $\phi$ variable) because the integration with respect to the $z$ variable can be evaluated analytically. The integrations with respect to $\theta$ and $\phi$ variables are evaluated by using an adaptive Newton-Cote integration routine [85] for each variable of integration. The computation of the surface wave pole (zero of $D_{TM}$ given by 6.16) is carried out using a zero-finding routine [85].

Figure 6.4 shows the variation of normalized conductances $g_r$ and $g_s$ of a magnetic dipole over ground plane with the normalized thickness ($k_0d$) of a lossless dielectric layer with $\epsilon_{r1}=2.2$. The operating frequency is taken to be 7.5 GHz. The conductances are normalized with respect to the radiation conductance of a magnetic dipole over ground plane without a dielectric cover layer. For zero thickness, $g_r$ equals 1 and $g_s$ is zero as expected. When the normalized thickness of the dielectric cover layer increases, $g_s$ increases monotonically (only the fundamental TM mode is assumed to be excited). The first TE surface wave mode will be excited for $k_0d = 1.434$. As seen from the figure, $g_r$ decreases with increasing value of $k_0d$ to a minimum value then starts to increase.

Figure 6.5 shows the radiation efficiency of a magnetic dipole over ground plane as a function of the normalized cover layer thickness. The efficiency is defined as $G_r/(G_r + G_s)$. When $k_0d=0$, the efficiency is 1 (no surface wave loss). Then the efficiency decreases with increase in the cover layer thickness. It becomes almost constant for $k_0d \geq 0.8$. Results computed are for the case when only the fundamental mode is excited. The efficiency curve shown in Fig 6.5 is a good approximation to that of a rectangular microstrip patch.
Figure 6.4 Variation of normalized radiation and surface wave conductances of a magnetic current element with normalized cover layer thickness \((k_0 d)\)
Figure 6.5 Radiation efficiency of a magnetic current element over a ground plane versus the normalized cover layer thickness ($k_0d$)
Figure 6.6 shows the variation of the radiation conductance of a radiating edge of a rectangular microstrip patch (uniform distribution) versus the width for following cover layer thicknesses: \( d = 0, \frac{1}{10} \) and \( \frac{1}{8} \) inch and \( \epsilon_r = 2.2 \). The variation of \( G_r \) with the edge width follows approximately a square law behavior over the range of patch width considered.

Figure 6.7 shows the variation of the surface wave conductance of the radiating edge of a rectangular patches with \( \frac{1}{10} \) inch and \( \frac{1}{8} \) inch thick cover layers respectively. The variation of \( G_s \) with the edge width follows approximately a square law behavior also.

6.4 Experimental determination of edge capacitance

The edge capacitance accounts for the fringing fields at the radiating edge of the rectangular microstrip patch. Since no closed form expression for edge capacitance of rectangular patches with cover layer is available, a technique using the measured resonance frequency of the patch is used (Appendix C).

*Table* 6.1 shows the measured resonance frequency of 4 rectangular patches with \( \frac{1}{10} \) inch thick cover layer \( (\epsilon_r = 2.2) \). From the measured resonance frequency, the edge capacitance \( C \) is computed by making the imaginary part of equation (C.1) equal to zero. The computed values of edge capacitance are also shown in *Table* 6.1. The variation of the edge capacitance with frequency is assumed to be very small around the measured frequency. From the four computed values of \( C \) and the value \( C = 0 \) for a patch of zero width, an expression for \( C \) as a function of the patch width is derived. This expression for \( C \) is based on a straight line least square fit [86] of the values determined from measurement, and is given by
Figure 6.6 Radiation conductance of an edge with uniform distribution versus edge width.
Figure 6.7 Surface wave conductance of an edge with uniform distribution versus edge width
Table 6.1 Computation of edge capacitance of rectangular patches with 1/10 inch cover layer

<table>
<thead>
<tr>
<th>Width b (mm)</th>
<th>Length a (cm)</th>
<th>Res. Freq. $f_r$ (GHz)</th>
<th>Edge Capac. C (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.017</td>
<td>1.3372</td>
<td>7.1300</td>
<td>0.06717</td>
</tr>
<tr>
<td>4.008</td>
<td>1.3359</td>
<td>7.0475</td>
<td>0.14141</td>
</tr>
<tr>
<td>6.030</td>
<td>1.3411</td>
<td>7.0450</td>
<td>0.19320</td>
</tr>
<tr>
<td>8.043</td>
<td>1.3376</td>
<td>7.0200</td>
<td>0.27115</td>
</tr>
</tbody>
</table>

$$C(b) = 0.033259 \times b + 9.2055 \times 10^{-4} \quad (6.27)$$

where $C(b)$ is the capacitance in pF and $b$ is the patch width in mm.

Figure 6.8 shows a plot of expression (6.27) for $C$ as function of the patch width. Also shown in the same figure are values of $C$ obtained from measurement.

An approximate method for the computation of the edge capacitance $C$ is the use of the following formula due to [61]:

$$C(b) = \frac{b}{2} \left[ \frac{\varepsilon_{re}(a)}{c_0 Z_0(air)} - \varepsilon_0 \varepsilon_{r2} \frac{a}{h} \right] \quad (6.28)$$

where $c_0 = 3 \times 10^8$ (m/s) and $\varepsilon_{re}$ is the effective dielectric constant of an equivalent transmission line with cover layer of width $a$. $Z_0(air)$ is the characteristic impedance of a microstrip line of width $a$ with $\varepsilon_{r1} = \varepsilon_{r2} = 1$ and $h$ is the sub-
Figure 6.8: Edge capacitance of a rectangular microstrip patch with 1/10 inch thick cover layer versus patch width.
strate thickness. Results based on equation (6.28) are also shown in Fig 6.8. These results are in very good agreement with the measured results. The difference between values based on (6.27) and those based on (6.28) is less than 5.5 % for the range of values of width considered ($b \leq 10$ mm).

### 6.5 Admittance matrix of MCN network

The mutual coupling network MCN accounts for the external interaction between the edges of the rectangular patch. An example of an MCN network between the radiating edges is shown in Fig 6.1. A similar network may be used to represent the mutual coupling between two elements of an array as shown in Fig 6.9. As discussed in Chapter 3, for the computation of the elements of Y-matrix characterizing the MCN network, we need to evaluate the mutual coupling between two magnetic current elements.

The mutual coupling admittance between the two segments $i$ and $j$ (shown in Fig 6.2) is expressed as

$$Y_{ij} = J_j W_j / V_i$$  \hspace{1cm} (6.29)

where $W_j$ is the width of the $j$th segment, $V_i$ is the value of the voltage of the $i$th segment. $J_j$ is the current density induced in the $j$th segment because of the fields created by the $i$th segment. The current density $J_j$ is given by

$$J_j = H_{y_0}^i \sin \psi_0^j - H_{z_0}^i \cos \psi_0^j$$  \hspace{1cm} (6.30)

where $\psi^j$ is the direction of the magnetic current $j$ with respect to the $x$-axis. $H_{z_0}^i$ and $H_{y_0}^i$ are given by

$$\begin{cases} H_{z_0}^i = H_{\rho}^i \cos(\phi + \psi_0^i) - H_\phi^i \sin(\phi + \psi_0^i) \\ H_{y_0}^i = H_{\rho}^i \sin(\phi + \psi_0^i) + H_\phi^i \cos(\phi + \psi_0^i) \end{cases}$$  \hspace{1cm} (6.31)
Figure 6.9 Equivalent multiport network model for mutual coupling between two one-port rectangular patches with cover layer.
where $\rho = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ and $\tan(\phi - \psi_i^0) = (y_i - y_j)/(x_i - x_j)$. $H_\rho$ and $H_\phi$ are evaluated at the plane $z=0$ and are derived from $\Pi_{ez}$ and $\Pi_{mz}$ as

$$\begin{cases}
H_\rho^i = H_\rho^{TM} + H_\rho^{TE} \\
H_\phi^i = H_\phi^{TM} + H_\phi^{TE}
\end{cases}$$

(6.32)

where

$$\begin{cases}
H_\rho^{TM} = \frac{ik_0 P_m \epsilon_r e_{r1}}{2\pi \eta_0} \cos \phi \int_0^\infty \frac{\Gamma_e J_1(k_0 \lambda \rho)}{u_1 \lambda} d\lambda \\
H_\rho^{TE} = \frac{-ik_0 P_m}{2\pi \eta_0} \cos \phi \int_0^\infty \frac{k_0 \lambda u_1}{\Gamma_m} \left[ J_0(k_0 \lambda \rho) - \frac{J_1(k_0 \lambda \rho)}{k_0 \lambda \rho} \right] d\lambda
\end{cases}$$

(6.33)

and

$$\begin{cases}
H_\phi^{TM} = \frac{-ik_0 P_m \epsilon_r e_{r1}}{2\pi \eta_0} \sin \phi \int_0^\infty \frac{\Gamma_e k_0 \lambda}{u_1} \left[ J_0(k_0 \lambda \rho) - \frac{J_1(k_0 \lambda \rho)}{k_0 \lambda \rho} \right] d\lambda \\
H_\phi^{TE} = \frac{ik_0 P_m}{2\pi \eta_0} \sin \phi \int_0^\infty \frac{u_1 J_1(k_0 \lambda \rho)}{\Gamma_m \lambda} d\lambda
\end{cases}$$

(6.34)

where $P_m = V_i W_i$ and $W_i$ is the width of the ith segment. The subscripts TM and TE denotes the contribution of TM and TE modes to the mutual coupling respectively.

**Numerical results**

The effects of mutual coupling on microstrip patches with cover layer can be investigated by first looking at the mutual coupling admittance between two magnetic dipoles. The integrations in equations (6.33) and (6.34) involves Bessel functions which are oscillating functions. A technique for an accurate
evaluation of these type of integrals is described in [83], which uses pole extraction technique and the method of averages. These integrals are evaluated for discrete values of spacing \( \rho \). Typically, 40 values by wavelength have been used. The computed results are tabulated and a cubic spline interpolation routine [85] is used to compute these integrals for arbitrary values of \( \rho \).

Using the above techniques, the E-plane normalized mutual coupling admittance \( Y_m \) (which includes the surface wave contribution) between two dipoles is shown in Figs 6.10 and 6.11. Figure 6.10 shows the normalized mutual conductance \( g_m \) (real part of \( Y_m \)) for 1/10 inch cover layer with \( \epsilon_r=2.2 \) and \( f=7.5 \) GHz. The mutual conductance is normalized to the self radiation conductance of a reference dipole on a ground plane without a cover layer. For comparison, also shown in the figure is the normalized mutual conductance between two magnetic dipoles on a ground plane without a dielectric cover layer. For small values of normalized spacing, \( g_m \) for \( d=1/10 \) inch is 1.5 times larger than the corresponding value for the case without the cover layer. This ratio increases when the spacing becomes larger. This is due to the surface wave contribution whose fields decay approximately as \( 1/\sqrt{\rho} \). The same conclusion can be drawn for the normalized mutual susceptance shown in Fig 6.11. The mutual susceptance is also normalized to the self radiation conductance of a dipole without cover layer.

The H-plane mutual conductance and susceptance between two magnetic dipoles are also shown in Figs 6.12 and 6.13 respectively. The mutual coupling in the H-plane is much smaller than the mutual coupling in the E-plane. Also shown in these figures are the results without cover layer.
Figure 6.10 Mutual coupling conductance between two magnetic dipoles in the E-plane
Figure 6.11 Mutual coupling susceptance between two magnetic dipoles in the E-plane
Figure 6.12 Mutual coupling conductance between two magnetic dipoles in the H-plane
Figure 6.13 Mutual coupling susceptance between two magnetic dipoles in the H-plane
The effect of mutual coupling on rectangular microstrip patches with cover layer is discussed by considering the mutual coupling between two identical one-port patches. For this purpose, three microstrip patches with the following cover layer thicknesses; \(d=0\) inch, \(1/10\) inch and \(1/8\) inch have been considered. The patches are chosen to have the same width and the dimensions of these patches are given in Table 6.2.

Table 6.2 Design dimensions of one-port patches

<table>
<thead>
<tr>
<th>Patch Dimensions</th>
<th>No cover layer</th>
<th>1/10 inch cover</th>
<th>1/8 inch cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (mm)</td>
<td>1.3223</td>
<td>1.2639</td>
<td>1.2569</td>
</tr>
<tr>
<td>(b) (mm)</td>
<td>6.06</td>
<td>6.06</td>
<td>6.00</td>
</tr>
<tr>
<td>(x_1) (mm)</td>
<td>5.416</td>
<td>5.065</td>
<td>4.908</td>
</tr>
<tr>
<td>(W_{TL}) (mm)</td>
<td>1.155</td>
<td>1.040</td>
<td>1.040</td>
</tr>
</tbody>
</table>

The effect of a cover layer on the \(H\)-plane coupling coefficient between these one-port patches is shown in Fig 6.14. As shown in this figure, the mutual coupling in the presence of \(1/10\) inch cover layer is about 0.5 dB higher at \(\lambda_0 /2\) spacing and about 2 dB higher at \(\lambda_0\) spacing compared to the case without a cover layer. For the design parameters considered, the mutual coupling between patches with \(1/8\) inch cover layer is smaller than between patches with \(1/10\) inch cover layer.
Figure 6.14 Variation of mutual coupling coefficient $|S_{21}|$ between two one-port patches as a function of the normalized spacing $S/\lambda_0$. 
Figure 6.15 shows a comparison between the E-plane and the H-plane coupling coefficients between one-port rectangular microstrip patches with 1/10 inch thick dielectric cover layer. The H-plane coupling is stronger than the E-plane coupling up to a spacing of $0.7\lambda_0$, then it becomes smaller and at a spacing of $2\lambda_0$ the H-plane coupling is 10 dB lower.

From the results presented in this section, the effect of mutual coupling on rectangular patches in presence of cover layer is of the same order as that without cover layer.

6.6 Experimental results and comparison with theory

The design procedure for microstrip patches in presence of cover layer using the MNM method have been discussed above. This procedure has been verified by comparing the results obtained for rectangular patches with experiments. The substrate is taken to be 1/64 inch thick with a dielectric constant of 2.2. The dielectric cover layer is taken to be 1/10 inch thick with a dielectric constant of 2.2. The dielectric constants for the substrate and the cover layer were measured using a microstrip transmission line ring resonator and found to be very close to the value 2.2. All experimental measurements have been carried out on HP 8510 Network Analyzer.

A comparison between theoretical and measured values for the reflection coefficient of one-port rectangular microstrip patch is shown in Fig 6.16. The design dimensions of the one-port patch are: $a = 1.2622$ cm, $b = 0.6013$ cm and $x_1 = 0.5055$ cm. The characteristic impedance of the input microstrip transmission line is 50 Ω (width = 1.04 mm). The measured resonance frequency is 7.475 GHz compared to the theoretical value of 7.485 GHz. This represents a discrepancy of 0.134%. Also, we note from the figure that the measured
Figure 6.15 Comparison of E-plane and H-plane mutual coupling between two one-port rectangular patches with 1/10 inch thick dielectric cover layer
Figure 6.16 Comparison of experiment and theory for $|S_{11}|$ of one-port rectangular patch with 1/10 inch thick cover layer
variation of $|S_{11}|$ with frequency is similar to the theoretical variation. The bandwidth of the one-port patch for (VSWR $\leq$ 2) is 80 MHz or 1.06%.

The design procedure has also been verified by comparing the computed results with experiment for a two-port rectangular microstrip patch (which constitutes element of series-fed array). Figure 6.17 shows a good agreement between theoretical and measured variations of input reflection coefficient $|S_{11}|$ with frequency. The measured resonant frequency is 7.485 GHz, the corresponding theoretical value is also 7.485 GHz. However, the measured $|S_{11}|$ at the resonance frequency is -25.6 dB, whereas the theoretical value is -33.5 dB. The dimensions of the two-port rectangular patch is: $a=1.278$ cm, $b=0.6012$ cm, $x_1=0.45$ cm and $x_2=0.5$ cm. The characteristic impedance of the input and the output microstrip transmission lines is 50 $\Omega$ (width = 1.04 mm).

Figure 6.18 shows a good agreement between theoretical and measured variations of transmission coefficient $|S_{21}|$ with frequency. At the resonance frequency $f=7.485$ GHz, the measured $|S_{21}|$ is -2.8 dB and the theoretical value is -2.64 dB. The discrepancy between measured and theoretical values of $|S_{21}|$ may be due to the losses in the connectors which are not taken into account in the theoretical results.

The effects of mutual coupling on microstrip patches with cover layer have been investigated experimentally by measuring the magnitude of the coupling coefficient $|S_{21}|$ between two identical one-port rectangular patches with 1/10 inch thick cover layer. The coupling coefficient was measured for spacing values $S (= 1, 2, 3, 5$ and $8$ cm) between the patches (as shown in Fig 6.9).

Figure 6.19 shows the theoretical and measured values of $|S_{21}|$ in the H-plane as a function of the normalized spacing $s = S/\lambda_0$. The dimensions
Figure 6.17 Comparison of experiment and theory for $|S_{11}|$ of two-port rectangular patch with 1/10 inch thick cover layer.
Figure 6.18 Comparison of experiment and theory for $|S_{21}|$ of two-port rectangular patch with 1/10 inch thick cover layer.
of the two identical one-port patches are: \( a = 1.259 \) cm, \( b = 0.6 \) cm and \( x_1 = 0.499 \) cm. The theoretical results have been obtained by including the mutual coupling among all edges of the two patches. The agreement between experiment and theory is good and this provides a verification of the procedure used for modeling the mutual coupling effects in microstrip patches with a dielectric cover layer.

6.7 Discussions

The multiport network model MNM approach for microstrip patches have been extended to the analysis of patches covered with a dielectric layer. The equivalence principle is used to replace the aperture fields at the edges by an equivalent magnetic current line source. This approximation holds for microstrip patches on thin substrate. The key step in the analysis is the determination of the electromagnetic fields due to a magnetic dipole over ground plane with a dielectric cover layer.

The power launched as surface wave (described in terms of surface wave conductance) decreases the efficiency of microstrip patches with cover layer. For most applications the ground plane is truncated at a distance of the order of a wavelength from the patch edge. Surface waves get diffracted at this truncation and cause unwanted radiation which contributes to the increase in the side lobe level and to the cross-polarization. Thus, special care should be taken to reduce these effects by absorbing the surface waves using lossy materials outside the patch region.

The mutual coupling in presence of cover layer has been investigated. The results obtained show that the mutual coupling in this case is of the same order as when no cover layer is present.
Figure 6.19: H-plane mutual coupling coefficient between two identical one-port rectangular patches with 1/10 inch thick cover layer.
Results based on MNM model for microstrip patches with cover layer have been compared with experiments. The agreement is very good for both the input reflection and the resonance frequency of one-port rectangular patch. The agreement is equally good for the H-plane coupling between two identical one-port patches and also for the input reflection, resonance frequency and transmission coefficient of a two-port rectangular microstrip patch.
CHAPTER VII

DESIGN AND SENSITIVITY ANALYSIS OF SERIES-FED LINEAR ARRAYS

7.1 Series-fed arrays

Single microstrip patches with or without a dielectric cover layer (discussed in previous chapters using MNM approach) have low antenna gain. They need to be used in an array configuration in order to obtain the antenna gain, beamwidth and side lobe level needed in most applications. When the feed network is etched on the same side of the substrate, two different configurations (corporate and series feeding arrangements) are commonly used for exciting array elements. When the corporate (parallel) feed shown in Fig 7.1 is used, the input power is distributed to each array element via power splitters. A specified array amplitude distribution is achieved by changing the impedance of the transmission lines in the power dividers. The space required to implement this type of feed increases for increasing number of array elements and so the losses incurred in the feed becomes large.

Series-fed arrangements (shown in Fig 7.2), are compact because the feed lines lengths are inherently minimized. This reduces the spurious radiation from the feed lines. Also the dissipation losses (which decreases the array efficiency) are decreased. A comparison between the corporate and the series feed arrangements is presented in [39]. For the design example discussed in this
Chapter, the series-fed linear array with ports located along the non-radiating edges (shown in Fig 7.2.a) has been selected due to the following considerations: (i) Need for polarization to be perpendicular to the array axis, (ii) desirability of lower feed line losses (high efficiency) and (iii) restriction on the space available in the direction transverse to the array axis.

Earlier attempts to analyze series-fed arrays of microstrip patches with ports located along the radiating edges (E-plane containing the array axis as shown in Fig 7.2.b) make use of the transmission line model [7]. In this model, the array is modeled as cascaded elements interconnected by microstrip line sections (as discussed in Section 1.3). Use of this model for designing long arrays, exhibited anomalies in the measured pattern [25] such as increased side lobe levels and unpredictable beam positions. Metzler [25] proposed a method based on an empirical design approach, where design curves relating the phase
a) Series-fed array with polarization in the plane perpendicular to the array axis

Figure 7.2 Series-fed linear arrays of microstrip patches
velocity and the radiation conductance to the element width were formulated using measured data. The design procedure requires many experimental iterations until satisfactory results are obtained. This method is very time consuming, since the design curves need to be regenerated when any one of the design parameters (such as using different substrate material) is changed. A similar approach has been reported in [26] for the synthesis of shaped pattern array. A computer-aided design package using the transmission line model with empirically measured patch parameters has been reported by Campi [27].

In this Chapter, we propose an alternative analytical method [88] for the computer-aided design of series-fed arrays based on MNM. The purpose of developing this software is to eliminate the recourse to cut-and-try iterations in the existing methods of array design. Starting from the array specifications, the CAD procedure (discussed in this Chapter) provides the required dimensions of array elements together with the computed array characteristics. Design procedure for array elements (including the effects of losses) is discussed in Section 7.2. Computational details involved and an example of a 19-element array with a dielectric cover layer is presented in Section 7.3. The effects of the mutual coupling among array elements are considered in Section 7.4. One of the foremost advantages of the present method is its extension to sensitivity analysis of the array. This important feature is discussed in Section 7.5. The present design methodology has been verified experimentally. The results of this comparison are presented in Section 7.6.
7.2 Design methodology for series-fed arrays

For the series-fed linear array of Fig 7.2.a, the input power is fed to the array at one end. A fraction of this input power is radiated by the first element, the rest (minus losses) is transmitted to the second element with the same phenomenon occurring there, and so on, for the subsequent elements. An additional design flexibility is obtained if the array is terminated at the far end by a matched load. When the mutual coupling among array elements is not taken into account, the linear array is considered as a cascade of two-port unit cells.

Each unit cell (shown in Fig 7.3) of the array is characterized by two impedance matrices $Z_C$ and $Z_V$. As discussed in Chapter 6, the $Z_C$ matrix relates currents and voltages at the input and output ports of the unit cell. The matrix $Z_V$ relates the voltage at the radiating edges of the rectangular patch to the current at the input and output ports. As shown in Fig 7.3, a typical unit cell consists of the radiating patch with two sections of transmission line connected to it. The width of each element ($b$) and the input/output port locations ($x_1$ and $x_2$) control the amount of power radiation, and hence the amplitude distribution of the array. The lengths of the input/output transmission lines ($L_1$ and $L_2$) are varied to adjust the progressive phase along the array. The determination of two-port characteristics of each unit cell is obtained from the array specifications.

Figure 7.4 shows a flow chart for the design procedure of a series-fed array. Various design steps of the array (as depicted in the flow chart) are described in the next subsections.
7.2.1 Array specifications

As shown in the flow chart, the starting point in the design is the array specifications. The required array specifications are the beam direction (pointing of the main beam), the directive gain of the array (or power gain) or the beamwidth of the main beam and the maximum value of side lobe level.

a) Length of the array from gain/beamwidth specification

The array length is determined either from the antenna directive gain $D$ or from the half power beamwidth $BW$ specifications. For a linear array, an approximate relation between the directive gain and beamwidth is [89,p.157]

$$D = 101.5^\circ/BW$$

(7.1)

The array length $L$ is related to the directive gain by
Figure 7.4 Flow chart for the design of linear series-fed arrays

(continued on next page)
X

amplitude distribution

calculate $|S_{21}|$ (ignoring losses)

dimensions of array elements

argument of $S_{21}$, voltages

Y

progressive phase

calculate lines lengths

computation of elements dimensions (losses included)

voltage distribution and input impedance

computation of radiation characteristics

fabrication and measurement

comparison of theory and experiment

STOP

(Continued from previous page)
\[ \frac{L}{\lambda_0} = \frac{D}{2} \]  

(7.2)

where \( \lambda_0 \) is the free space wavelength. For a given directivity, expression (7.2) gives the minimum array length required to obtain the specified beamwidth for any distribution with uniform progressive phase.

b) **Interelement spacing and number of elements:**

The only constraint we impose on the uniform spacing \( d \) (as shown in *Fig 7.2.a*) is to avoid the appearance of a second main beam (grating lobe). For this purpose, we require then that [89,p.125]

\[ \frac{d}{\lambda_0} \leq \frac{1}{|1 + | \sin \theta_0 ||} \]  

(7.3)

where \( \theta_0 \) is the direction of the main beam measured from the broadside direction. The number of elements \( N \) is given by

\[ N = \text{Int}(\frac{L}{d}) - 1 \]  

(7.4)

where \( \text{Int}(x) \) is the integer part of \( x \).

c) **Phase distribution**

The linear array considered in this chapter is assumed to have a uniform progressive phase shift between the adjacent elements. This phase shift \( \alpha \) is related to the array beam pointing direction \( \theta_0 \) by

\[ \alpha = -k_0 d \sin \theta_0 \]  

(7.5)

where \( k_0 \) is the free space wavenumber and \( d \) is the interelement spacing.
d) Amplitude distribution

To achieve a low side lobe level, SLL, the amplitude distribution (power radiated by each element) needs to be adjusted properly. For the example discussed in this chapter, a Taylor distribution for the array amplitude has been used. In this type of amplitude distribution for arrays, the close-in side lobes are below a specified level. The side lobes which are further out are at increasingly lower levels. For usual practical parameter values, the beam broadening associated with the choice of SLL is negligible [89]. The determination of the amplitude distribution (element excitation) is carried out using the null matching technique [89]. The software developed also includes the option of selecting Chebyshev and uniform distributions. Other array distributions can be entered as input data to the program.

7.2.2 S-parameters for array unit cells (when losses are ignored)

The design of a linear series-fed array reduces to the determination of the two-port transmission characteristics required from each unit cell (shown in Fig 7.3). The parameters characterizing the radiating elements are; width b, length a and locations $x_1$ and $x_2$ of the input/output ports. The two sections of microstrip lines are characterized by their length $L_1$ and $L_2$ and the characteristic impedances $Z_{01}$ and $Z_{02}$ respectively. The required S-parameters of each radiating element (when losses are ignored) are discussed in the following sections.

a) Input reflection coefficient

To obtain an impedance match at the input feed of the array, we require that every cell element in the array be matched at its input port. This
input match at the individual elements ensures that the phase and the amplitude of the signal radiated from a given array element is controlled by the input signal only, and not by the signal reflected back from the next array element.

It may be noted that the values of the output reflection coefficient (of the various elements) is not relevant in the present design.

b) Transmission coefficient

To achieve a specified amplitude distribution along the array aperture, the power radiated from each element needs to be controlled. When the losses are negligible, the input power to each element is divided into the radiated power and the power transmitted to the next element. The fraction of power incident on the next element is given by $|S_{21}|^2$ of the power incident on the previous element. The total input power $P^t$ fed to the array may be expressed as (for the lossless case):

$$P^t = \sum_{i=1}^{N} C_i^2 + P_L$$

where $N$ is the total number of elements and $C_i$ is the relative amplitude of the radiation of the $i$th array element, $C_i^2$ being the power radiated. $P_L$ is the amount of power dissipated in the resistive matched load connected to the end of the array. If $P_L$ is set equal to a fraction of the input power $P^t$ ($P_L = \gamma P^t$), then the fractional power radiated $P_r(i)$ from the $i$th element may be written as

$$P_r(i) = \frac{C_i^2}{P_{in}(i)}$$

$$P_{in}(1) = P^t$$

$$P_{in}(i + 1) = P_{in}(i) - C_i^2, \quad 2 \leq i \leq n.$$  

The magnitude of the transmission coefficient $|S_{21}|$ for the $i$th radiating element
is obtained as

\[ |S_{21}(i)| = \sqrt{1 - P_r(i)} \]  \hspace{1cm} (7.9)

For the case of a series fed array without any load connected at the far end, the last element is a one-port patch and the only design requirement for this element is that \( S_{11} = 0 \).

The progressive phase shift along the array is adjusted by varying the length \( L_i \) of the transmission line connecting the \( i \)th to the \((i+1)\)th adjacent element in the array as shown in Fig 7.5. The length \( L_i \) is determined from the progressive phase \( \alpha \) as

\[ \beta L_i = -\alpha + \text{Ang}[S_{21}(i)] + \frac{1}{2} \left\{ \text{Ang}[V_e(i+1)] - \text{Ang}[V_e(i)] \right\} \]

\[ + \text{Ang}[V_e(i+2)] - \text{Ang}[V_e(i+1)] \]  \hspace{1cm} (7.10)

where \( \beta \) is the propagation constant in the transmission line, \( \text{Ang}[V_e] \) is the average phase of the voltages at the radiating edge referred to the signal at the input port and \( \text{Ang}[S_{21}(i)] \) is the phase delay of the transmitted signal through the \( i \)th element referred to its input and output ports.

### 7.2.3 Computation of array losses

The losses in microstrip antenna arrays are of three types; dielectric, conductor and surface wave losses. These losses in a unit cell of the array (shown in Fig 7.3) are obtained from the computation of the radiated power and the scattering parameters of the unit cell. Since each unit cell of the array is required to be matched, the voltage \( V(2) \) and current \( I(2) \) at the output port are related by \( V(2) = -Z_{02}I(2) \) and therefore the voltages \( V_E \) at the radiating edges are related to the current at the input port by
Figure 7.5 Phase relationship between edge voltages in two adjacent unit cells of the array

\[ V_E = |Z_V| \begin{pmatrix} 1 \\ \frac{-Z^c_{2,1}}{Z_{02} + Z^c_{2,2}} \end{pmatrix} I(1) \]  \hspace{1cm} (7.11)

For a unit power incident on the input port of the unit cell (and delivered by a source with internal resistance \( Z_s = Z_{01} \)), the current \( I(1) \) at the input port is given by

\[ |I(1)| = \frac{8Z_{01}}{|Z_{01} + Z_{in}|} \]  \hspace{1cm} (7.12)

where \( Z_{in} \) is the input impedance of the unit cell. The total radiated power by the unit cell (free space + surface wave) is related to the voltage \( V_E \) at the
radiating edges by

\[ P_T = \frac{1}{2} \text{Real} \left\{ \sum_{i=1}^{NC} V_E(i) \left[ \sum_{j=1}^{NC} Y_m^*(i,j) V_E^*(j) \right] \right\} \]  \hspace{1cm} (7.13)

where \( Y_m \) is the mutual admittance matrix characterizing the MCN network inserted between the radiating edges (including the EAN network). NC is the total number of segments at the radiating edges. The free space radiated power \( P_r \) and the surface wave power \( P_s \) are computed from (7.13) by using the approximate relation

\[ P_s/P_r = G_s/G_r \] \hspace{1cm} (7.14)

where \( G_r \) and \( G_s \) are the radiation and surface wave conductances respectively.

The net time average power transmitted to the second port (for 1 Watt incident power) is given by \( |S_{21}|^2 \) and the net time average power reflected at the input port is \( |S_{11}|^2 \). The power dissipated \( P_{\text{loss}} \) (conductor + dielectric) in the unit cell is then given by

\[ P_{\text{loss}} = 1 - |S_{21}|^2 - |S_{11}|^2 - P_T \] \hspace{1cm} (7.15)

The above results are needed for determining the transmission coefficient of unit cells when losses are included (as discussed in the next section).

### 7.2.4 \( S_{21} \) of array elements (when losses are included)

The array losses alter the array amplitude distribution and hence they must be taken into account in determining the magnitude of the transmission coefficient. The total input power fed to the array \( P^t \) is now given by

\[ P^t = \sum_{i=1}^{N} \left[ C_i^2 + P_d(i) \right] + P_L \] \hspace{1cm} (7.16)
where \( P_d(i) \) is the total power loss in the \( i \)th cell (power carried away by the surface wave launched by the patch plus the dielectric and the conductor losses in the unit cell). If we denote the ratio of power loss \( P_{loss} \) (conductor plus dielectric given by 7.15) in the \( i \)th unit cell to the radiated power \( (P_r = C_i^2) \) by that element by \( \epsilon_i \), and denote \( P_s/P_r \) given in (7.14) by \( \alpha'_i \), then the input power to the array is found as

\[
P^t = \frac{\sum_{i=1}^{N}(1 + \epsilon_i + \alpha'_i)C_i^2}{(1 - \gamma)} \tag{7.17}
\]

The input powers to the array elements are given by

\[
P_{in}(i) = \begin{cases} P^t, & \text{if } i = 1, \\ P_{in}(i - 1) - (1 + \epsilon_{i-1} + \alpha'_{i-1})C_{i-1}^2, & \text{otherwise.} \end{cases} \tag{7.18}
\]

The corresponding transmission coefficient of the \( i \)th cell in the array, which will yield the required array amplitude distribution, is given by

\[
|S_{21}(i)| = \sqrt{1 - (1 + \epsilon_i + \alpha'_i)C_i^2/P_{in}(i)} \tag{7.19}
\]

The computation of the value of \( |S_{21}| \) for each unit cell (as given in 7.19) requires the knowledge of the values of losses in the various unit cells. But these losses cannot be computed until the design dimensions of unit cells are known. Thus, the design of the array in the lossy case needs an iterative procedure. A flow chart for the design of array elements in the lossy case is shown in Fig 7.6. As shown in this flow chart, the first step in this procedure is to design the array assuming no losses. Then the losses associated with each unit cell are computed, and the array elements are redesigned using those computed losses. The iteration process is terminated when the relative difference between
Figure 7.6 Flow chart for the design of unit cells when losses are included in the design.
the computed values of $|S_{21}|$ in two successive steps is less than a prescribed value $err$ for all array elements. Since the losses are not very sensitive to small changes in elements dimensions, the number of iterations required is usually small. It may be pointed out that if the values of $\epsilon$ and $\alpha'$ are the same for all array cells, then no iterations will be required.

### 7.2.5 Computation of array characteristics

The radiation characteristics of the series-fed linear array are obtained by superposing the radiation fields from the individual unit cells. The computation of the far field of a two-port rectangular patch with a cover layer has been discussed in Chapter 6. Since the array elements do not usually have equal lengths, only one set of radiating edges can be made colinear. In the design reported in this chapter, the radiating edges which are nearer to the input and output transmission lines (edges marked 1 in Fig 7.5) are made colinear. The radiation is mainly from the two radiating edges of width $b$. The contribution of the two other (non-radiating) edges can also be included if more accurate results or the evaluation of the cross-polarization level is needed. For a fixed uniform spacing between two adjacent array elements, the interconnecting lines have to be curved to fit in the spacing between elements (as will be discussed in section 7.3.1). The radiation as well as the reflection which may be caused by these bends are minimized by choosing a high impedance value for the interconnecting lines, and by using thin substrates.

Each unit cell in the array (shown in Fig 7.7) is characterized by an impedance matrix $Z_C$ with respect to the two ports of the cell. For a one-port cell (no load) or for the load connected to the end of the array, this impedance matrix reduces to the input impedance of the one-port and to the value of the
Figure 7.7 Series-fed linear array of N microstrip patches.
resistive load respectively. Each unit cell is also characterized by the matrix $Z_1$ which relates the voltages at the radiating edges to the currents flowing into the two ports of the unit cell. Once these port currents are computed, the voltage distribution at the radiating edges of the unit cells as well as the input impedance of the array can be evaluated.

The currents are computed by grouping the voltages and currents at the connected ports and the external ports separately [37] as follows

$$
\begin{pmatrix}
V_p \\
V_c
\end{pmatrix}
= 
\begin{pmatrix}
Z_{pp} & Z_{pc} \\
Z_{cp} & Z_{cc}
\end{pmatrix}
\begin{pmatrix}
I_p \\
I_c
\end{pmatrix}
$$

(7.20)

where $V_p$ and $I_p$ are the voltage and current at the input feed. $V_c$ and $I_c$ are voltages and currents at the connected ports. The number of connected ports is equal to $(2N-2)$ for an array without a load and is equal to $2N$ for an array terminated by a matched load. Using Kirchhoff’s voltage and current laws for the connected ports, constraint matrices for the voltages and currents are written as

$$
\begin{cases}
\Gamma_1 V_c = 0, & \text{for voltages}, \\
\Gamma_2 I_c = 0, & \text{for currents}.
\end{cases}
$$

(7.21)

where $\Gamma_1$ and $\Gamma_2$ are matrices with $c/2$ rows and $c$ columns. Each row of these matrices describes a connection constraint, and the elements of every row are zero except for the two elements corresponding to ports connected together. The value of non-zero elements are 1 or -1 for $\Gamma_1$, and 1 for $\Gamma_2$. The currents at the connected ports are related to the current at the input port by the vector

$$
[C] = \begin{pmatrix}
\Gamma_1 Z_{cc} \\
j \Gamma_2
\end{pmatrix}^{-1} 
\begin{pmatrix}
-\Gamma_1 Z_{cp} \\
0
\end{pmatrix}
$$

(7.22.a)
that is,

\[ I_c = [C]I_p \] (7.22.b)

The present analysis also yields the input impedance of the array which is given by

\[ Z_p = Z_{pp} - [Z_{pc}][C] \] (7.23)

Using the value of currents given by (7.22.b), the voltages at the radiating edges of each unit cell are computed. Then the radiation characteristics (SLL, beam width, beam direction, gain, etc.) are evaluated therefrom. The currents at the connected ports can also be used to compute the value of forward and backward traveling wave variables at the connected ports and thus the reflection coefficients at the input ports of each unit cell in the array. This information is useful in sensitivity analysis and for finding the origin of any reflection in the array.

7.3 Design example and computational details

As an example of the use of the array design procedure described above and the associated computational details involved, we consider a 19-element series-fed array with a Taylor distribution. The side lobe levels are to be below 30 dB and the direction of the main beam is to be 30° off broadside. The array antenna will operate around a center frequency of 7.5 GHz. The array is to be fabricated on 5880 Duroid substrate with \( \varepsilon_{r2} = 2.2 \), \( h = 1/64 \) inch and \( \delta_d = 9 \times 10^{-4} \). The antenna is covered by a dielectric layer with \( \varepsilon_{r1} = 2.2 \) and \( d = 1/10 \) inch and \( \delta_d = 9 \times 10^{-4} \). We consider the array to be terminated in a matched load with 5% of the input power fed to the array being dissipated in this load (\( \gamma = 0.05 \)). The characteristic impedance of the interconnecting
transmission lines is 75 $\Omega$ and the spacing between elements (d shown in Fig 7.2.a) is chosen as a half wavelength in free space at f=7.5 GHz.

### 7.3.1 Aperture distribution of 19-element Taylor array

The uniform progressive phase and the relative amplitude distribution are selected for achieving a side lobe level of -30 dB and a beam direction 30° off broadside. Using equation (7.5), there are two possible values of $\alpha$ which results in 30° off broadside beam pointing:

$$
\alpha = \begin{cases} 
-90^\circ & \text{for } \theta = +30^\circ, \\
-270^\circ & \text{for } \theta = -30^\circ.
\end{cases}
$$

(7.24)

The use of $\alpha = -90^\circ$ results in interconnecting transmission lines shorter than the spacing between elements (which is $\lambda_0/2$). So, a differential phase shift of $\alpha = -270^\circ$ is used. This requires the interconnecting line length to be larger than the spacing between patches. Curved interconnecting line paths are used between adjacent patches along the array.

The amplitudes of excitation in the array (Taylor distribution) are computed by taking NA=6, where NA is the number of close-in side lobes whose levels are at -30 dB. The relative amplitude distribution, normalized to the excitation of the center element, has a symmetry around the center element. This distribution is listed in Table 7.1.

### 7.3.2 Design details and array performance

The required $|S_{21}|$ values (in the lossless case) for each array element are given by (7.9). The corresponding $|S_{21}|$ values for the amplitude distribution shown in Table 7.1 is given in Table 7.2.

For designing the array elements, to meet the $|S_{21}|$ specifications in
Table 7.1 Relative amplitude distribution for Taylor array

<table>
<thead>
<tr>
<th>Array elements</th>
<th>Amplitude distribution normalized to the center element</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 19</td>
<td>0.2664</td>
</tr>
<tr>
<td>2 &amp; 18</td>
<td>0.3048</td>
</tr>
<tr>
<td>3 &amp; 17</td>
<td>0.3900</td>
</tr>
<tr>
<td>4 &amp; 16</td>
<td>0.5112</td>
</tr>
<tr>
<td>5 &amp; 15</td>
<td>0.6384</td>
</tr>
<tr>
<td>6 &amp; 14</td>
<td>0.7531</td>
</tr>
<tr>
<td>7 &amp; 13</td>
<td>0.8536</td>
</tr>
<tr>
<td>8 &amp; 12</td>
<td>0.9343</td>
</tr>
<tr>
<td>9 &amp; 11</td>
<td>0.9840</td>
</tr>
<tr>
<td>10</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 7.2, we make use of tables containing two-port characteristics for various patch dimensions as shown in Tables 7.3 and 7.4. The introduction of this kind of tables helps in speeding up the design computation specially in the iterative design when losses are included.

Table 7.3 shows the minimum and maximum values of $|S_{21}|$, of two-port rectangular patches with a cover layer, as a function of patch width $b$. $|S_{21}|$ can be increased by decreasing the patch width and hence reducing the amount
Table 7.2 $|S_{21}|$ distribution for lossless case

| Element # | $|S_{21}|$ | Element # | $|S_{21}|$ |
|-----------|-----------|-----------|-----------|
| 1         | 0.99634   | 11        | 0.88874   |
| 2         | 0.99518   | 12        | 0.87190   |
| 3         | 0.99202   | 13        | 0.85830   |
| 4         | 0.98602   | 14        | 0.84956   |
| 5         | 0.97748   | 15        | 0.85031   |
| 6         | 0.96703   | 16        | 0.86854   |
| 7         | 0.95440   | 17        | 0.90024   |
| 8         | 0.93969   | 18        | 0.92585   |
| 9         | 0.92347   | 19        | 0.93416   |
| 10        | 0.90647   |           |           |

radiated power. However, when the width of the patches decreases, the losses increase and limit the maximum obtainable value of $|S_{21}|$. The information in Table 7.3 is used to select the initial values of width for various patches.

Table 7.4 shows the variation in $|S_{21}|$ of a two-port rectangular patch (with cover layer and $b = 5$ mm) as a function of the output port location $x_2$. The other design parameters $a$ and $x_1$ are chosen such that the input reflection coefficient $S_{11} = 0$ as discussed earlier. In the array design example considered
Table 7.3 $|S_{21}|$ values of two-port rectangular patches with 1/10 inch thick cover layer as a function of patch width $b$ (used for selecting initial values of $b$)

| $b$ (mm) | Minimum $|S_{21}|$ | Maximum $|S_{21}|$ |
|----------|-----------------|-----------------|
| 0.50     | 0.9841          | 0.9852          |
| 0.75     | 0.9826          | 0.9839          |
| 1.00     | 0.9671          | 0.9838          |
| 2.00     | 0.8842          | 0.9722          |
| 3.00     | 0.8221          | 0.9606          |
| 4.00     | 0.7079          | 0.9442          |
| 5.00     | 0.5726          | 0.9236          |
| 6.00     | 0.4318          | 0.8989          |

Here, a set of more detailed tables similar to Table 7.4 for two-port patches whose widths range from 0.5 mm to 6 mm have been generated. Those tables are not reproduced here.

For a given patch width and for a given value of $|S_{21}|$, the values of $a$, $x_1$ and $x_2$ are computed using a cubic spline interpolation scheme [85]. In all of these computations, the lengths of the interconnecting lines are computed by using $\beta L_i = -\alpha$. Also using the analysis of Chapter 6, the computed ratio of surface wave power loss to the radiated power (for the substrate and cover layer parameters mentioned earlier) is found to be $\alpha' = 0.629$ for all patches.
Table 7.4  Variation of transmission coefficient with output port locations of a two-port patch.

| $x_2$ (mm) | $x_1$ (mm) | a (mm) | $P_{rad/surf}$ | $P_{loss}$ | $|S_{21}|$ | $\angle S_{21}$ |
|------------|------------|--------|----------------|------------|-----------|--------------|
| 5.4        | 4.660      | 1.2637 | 0.4753         | 0.1968     | 0.5726    | -16.1°       |
| 5.0        | 4.395      | 1.2656 | 0.4753         | 0.1969     | 0.5726    | -15.3°       |
| 4.6        | 4.085      | 1.2681 | 0.2654         | 0.1179     | 0.7852    | -15.1°       |
| 4.2        | 3.740      | 1.2712 | 0.2033         | 0.0947     | 0.8378    | -15.2°       |
| 3.8        | 3.375      | 1.2747 | 0.1604         | 0.0787     | 0.8723    | -15.4°       |
| 3.4        | 2.990      | 1.2789 | 0.1302         | 0.0674     | 0.8957    | -15.9°       |
| 3.0        | 2.585      | 1.2834 | 0.1088         | 0.0594     | 0.9116    | -16.5°       |
| 2.6        | 2.165      | 1.2886 | 0.0932         | 0.0537     | 0.9236    | -17.5°       |
To achieve the $|S_{21}|$ distribution shown in Table 7.2, the width of the array elements are reduced gradually as we move along the array. In order to reduce the phase error related to unequal widths of adjacent patches (phase delay between the input port and the radiating edge), it is desirable to have the width of adjacent patches as close as possible. In other words, the difference between two successive element widths should be small (typically $\leq 1 \text{ mm}$). With the above constraints imposed on the choice of elements width, we select the patches (from the design tables) whose $|S_{21}|$ values closely approximate the values of the desired $|S_{21}|$ distribution in Table 7.2.

As seen from Table 7.3, the maximum $|S_{21}|$ for all widths considered is 0.9852. However the four first required values of $|S_{21}|$ shown in Table 7.2 (computed without including losses) are higher than the value of 0.9852. To satisfy the $|S_{21}|$ distribution of Table 7.2, we choose the first four elements with maximum available values of $|S_{21}|$. For obtaining the required phase difference between adjacent patches, we adjust the lengths of the input and output transmission lines (each half of the interconnecting lines between patches) using expression (7.10). The design dimensions of array elements are listed in Table 7.5.

A plot of the H-plane far field pattern of the 19-element array (with dimensions listed in Table 7.5), using the analysis of section 7.2.5, is shown in Fig 7.8. It may be noted that the array losses were ignored in the design process, but are included in the analysis leading to the radiation pattern performance. The SLL achieved is $-25 \text{ dB}$ instead of the designed value of $-30 \text{ dB}$. On the other hand, the pointing of the main beam is $-30^\circ$ as required. The increase in SLL is due to the losses in the elements which are not identical and also because the first four elements do not satisfy the $|S_{21}|$ distribution of Table 7.2.
Table 7.5 Design dimensions of 19-element series-fed array
(when losses are ignored in the design)

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>a (cm)</th>
<th>$x_1$ (cm)</th>
<th>$x_2$ (cm)</th>
<th>$L_1$ (cm)</th>
<th>$L_2$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>1.3790</td>
<td>0.2170</td>
<td>0.2200</td>
<td>1.0000</td>
<td>1.0048</td>
</tr>
<tr>
<td>0.075</td>
<td>1.3670</td>
<td>0.1750</td>
<td>0.1800</td>
<td>1.0048</td>
<td>1.0011</td>
</tr>
<tr>
<td>0.100</td>
<td>1.3507</td>
<td>0.2130</td>
<td>0.2200</td>
<td>1.0011</td>
<td>0.9977</td>
</tr>
<tr>
<td>0.100</td>
<td>1.3507</td>
<td>0.2130</td>
<td>0.2200</td>
<td>0.9977</td>
<td>0.9972</td>
</tr>
<tr>
<td>0.100</td>
<td>1.3371</td>
<td>0.3159</td>
<td>0.3229</td>
<td>0.9972</td>
<td>0.9923</td>
</tr>
<tr>
<td>0.200</td>
<td>1.3125</td>
<td>0.2776</td>
<td>0.2906</td>
<td>0.9923</td>
<td>0.9857</td>
</tr>
<tr>
<td>0.200</td>
<td>1.3021</td>
<td>0.3605</td>
<td>0.3738</td>
<td>0.9857</td>
<td>0.9808</td>
</tr>
<tr>
<td>0.300</td>
<td>1.2912</td>
<td>0.3276</td>
<td>0.3474</td>
<td>0.9808</td>
<td>0.9762</td>
</tr>
<tr>
<td>0.300</td>
<td>1.2864</td>
<td>0.3710</td>
<td>0.3927</td>
<td>0.9765</td>
<td>0.9711</td>
</tr>
<tr>
<td>0.400</td>
<td>1.2808</td>
<td>0.3400</td>
<td>0.3704</td>
<td>0.9711</td>
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</tr>
<tr>
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<td>1.2777</td>
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<td>0.4006</td>
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<td>0.9615</td>
</tr>
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<td>0.9561</td>
</tr>
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<td>0.3982</td>
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<td>0.9560</td>
</tr>
<tr>
<td>0.500</td>
<td>1.2721</td>
<td>0.3629</td>
<td>0.4081</td>
<td>0.9560</td>
<td>0.9561</td>
</tr>
<tr>
<td>0.500</td>
<td>1.2722</td>
<td>0.3622</td>
<td>0.4073</td>
<td>0.9561</td>
<td>0.9556</td>
</tr>
<tr>
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<td>1.2742</td>
<td>0.3423</td>
<td>0.3852</td>
<td>0.9556</td>
<td>0.9605</td>
</tr>
<tr>
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<td>1.2796</td>
<td>0.3510</td>
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<td>0.9605</td>
<td>0.9657</td>
</tr>
<tr>
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<td>0.3216</td>
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</tr>
<tr>
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<td>0.3449</td>
<td>0.3651</td>
<td>0.9695</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

To obtain the required value of SLL we need to redesign the array taking losses into account. New values of $|S_{21}|$ (which yield the required SLL) are computed using equation (7.19). Computation steps are repeated by using the new values of losses. The iteration is terminated when the relative difference
Figure 7.8 H-plane far field pattern of 19-element array (when losses are ignored in the design)
\( err \) between values of \( |S_{21}| \) for two successive steps is less than 0.01% for all unit cells. The final design dimensions for the 19-element array are shown in Table 7.6.

**Table 7.6 Design dimensions of 19 elements series-fed array**

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>a (cm)</th>
<th>( x_1 ) (cm)</th>
<th>( x_2 ) (cm)</th>
<th>( L_1 ) (cm)</th>
<th>( L_2 ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>1.3728</td>
<td>0.2666</td>
<td>0.2696</td>
<td>1.0000</td>
<td>1.0045</td>
</tr>
<tr>
<td>0.075</td>
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<td>0.2177</td>
<td>1.0045</td>
<td>1.0009</td>
</tr>
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<td>0.2690</td>
<td>1.0009</td>
<td>0.9983</td>
</tr>
<tr>
<td>0.100</td>
<td>1.3286</td>
<td>0.3815</td>
<td>0.3885</td>
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<td>0.9922</td>
</tr>
<tr>
<td>0.200</td>
<td>1.3122</td>
<td>0.2801</td>
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<td>0.9862</td>
</tr>
<tr>
<td>0.200</td>
<td>1.3024</td>
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<td>0.3714</td>
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<td>0.9807</td>
</tr>
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</tr>
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</tr>
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</tr>
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<td>0.3591</td>
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</tr>
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</table>
The H-plane far field pattern of the array is plotted in Fig 7.9. The SLL achieved in this case is -28.7 dB. This value is still not equal to the desired value of -30 dB. One of the contributing factors to this difference is the effect of element patterns, not included in the computation of amplitude distribution, which have a cosine distribution in the H-plane. The value of far field (computed using the array factor only) remains the same in the broadside direction and decreases near the endfire directions when element pattern is included. The element pattern contributes to a 0.8 dB increase in side lobe level near the broadside direction. The use of array elements which are not identical may also cause a slight increase in SLL. The efficiency of the array, defined as ratio of the radiated power to the input power, was computed and is 36%.

To determine the bandwidth of the 19-element series-fed linear array we need to compute the variations in array performance with frequency. The array bandwidth can be defined in terms of radiation characteristics or input impedance or both. The radiation bandwidth is limited by change in beam direction and the increase in side lobe level. The impedance bandwidth is defined as the frequency range over which VSWR is less than a specified value (usually VSWR=2).

Table 7.7 shows the variation of radiation characteristics and input $|S_{11}|$ for frequency varying from 7.4 GHz to 7.6 GHz. Over this frequency range the return loss shown in column 5 is very small. However, the direction of the main beam shifts from -36° to -24°. Since the electrical length of the array increases with frequency, as expected, the beam width decreases with frequency and changes from 8.5° to 7.3°. Column 4 shows the value of the highest side lobe as a function of frequency.
Figure 7.9 H-plane far field pattern of 19-element array (when losses are included in the design)
Table 7.7 Summary of variation of array performance with frequency

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Beam Direc.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th>Input S11 (dB)</th>
<th>SLL* (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.400</td>
<td>-36.0°</td>
<td>8.5°</td>
<td>-16.6</td>
<td>-27.9</td>
<td>-25.3</td>
</tr>
<tr>
<td>7.450</td>
<td>-33.0°</td>
<td>8.3°</td>
<td>-22.5</td>
<td>-40.9</td>
<td>-28.4</td>
</tr>
<tr>
<td>7.475</td>
<td>-31.5°</td>
<td>8.1°</td>
<td>-25.6</td>
<td>-53.7</td>
<td>-25.6</td>
</tr>
<tr>
<td>7.500</td>
<td>-30.0°</td>
<td>7.9°</td>
<td>-28.7</td>
<td>-54.8</td>
<td>-28.7</td>
</tr>
<tr>
<td>7.525</td>
<td>-28.5°</td>
<td>7.7°</td>
<td>-27.4</td>
<td>-49.7</td>
<td>-27.4</td>
</tr>
<tr>
<td>7.550</td>
<td>-27.0°</td>
<td>7.5°</td>
<td>-24.2</td>
<td>-40.6</td>
<td>-24.2</td>
</tr>
<tr>
<td>7.600</td>
<td>-24.0°</td>
<td>7.3°</td>
<td>-19.6</td>
<td>-29.8</td>
<td>-24.8</td>
</tr>
</tbody>
</table>

* SLL for lobes separated by more than 1 dB.

As shown in Fig 7.10, the close-in side lobe merges with the main beam. When the difference between the values of the dip and the peak is less than 1 dB, this side lobe is considered as a part of the main beam. If we define a side lobe as a lobe with the peak and dip values separated by more than 1 dB, the new values of the maximum SLL are shown in Column 6 of the Table. For series-fed linear arrays, the bandwidth is mostly limited by the shift in the beam direction and the increase in SLL.
Figure 7.10 H-plane far field pattern of 19-element series-fed array (f=7.45 GHz)
7.4 Role of mutual coupling (MC) in array design

Knowledge of the effects of MC among the neighboring elements in an array environment is important for the accurate design of microstrip arrays. The effects of MC on microstrip antennas are attributed to the effect of external fields and to the effect of surface wave fields due to the presence of grounded substrate(s) and cover layer(s). The characteristics of a microstrip patch in an array (e.g., input impedance and voltage distribution at the patch periphery) are different from the values for an isolated patch. This is due to the presence of other array elements at close proximity (typically $\lambda_0/2$ spacing).

The role of MC in microstrip antennas is well recognized. A collection of papers dealing with this phenomenon are included in [90]. Design procedure for a microstrip dipole linear array including the effects of MC is discussed in [91]. Experimental results for such an array are given in [92]. In general, the effects of MC are dependent on the array configuration and the beam direction [39].

Modeling of effects of MC on microstrip patches using MNM and its implementation for rectangular microstrip patches without a cover layer was discussed in Chapter 3. For microstrip patches with a cover layer, the effects of MC was investigated in Chapter 6. In this section, the MNM modeling for effects of MC on a series-fed linear array of rectangular patches is discussed. A method for the design of series-fed arrays including the effects of MC is also given.

7.4.1 Computation of array characteristics in presence of MC

When MC effects were neglected, the series-fed array has been modeled as a cascade of unit cells (as shown in Fig 7.7). Then, for evaluating the array
characteristics, these unit cells are combined using the segmentation approach discussed earlier. When MC is included, the radiation from each element is not only controlled by the signal incident at the input port (of that element) but also by the signal coupled through the radiating edges and radiated by the neighboring elements. This external coupling can be included in the array analysis model by adding one (or more) MCN network(s) to the MNM of the array (as shown in Fig 3.4 for a two-element array). The external fields of the array (radiated, surface wave, fringing fields and mutual coupling) may be represented by a single network. The Y-matrix characterizing this network consists of block sub-matrices. The sub-matrices along the diagonal represent the radiation from the patches and the mutual coupling among the same edges of the patch. The off-diagonal sub-matrices represent the mutual interaction among different patches of the array. Since mutual coupling effects between two elements decrease rapidly as their interspacing is increased, only few off-diagonal submatrices need to be evaluated. The number of neighboring elements needed to be considered depends on the array configuration and is discussed in Section 7.4.3.

The fields underneath the patch and the interconnecting transmission lines may also be characterized by a single impedance matrix. This impedance matrix is obtained by grouping the Z-matrices representing the unit cells as follows

\[
[Z] = \begin{pmatrix} Z_{pp} & Z_{pc} \\ Z_{cp} & Z_{cc} \end{pmatrix}
\]  

(7.24)

where p denotes the external ports of unit cells and c denotes the ports along the radiating edges (where the MCN network will be connected to the internal
The segmentation method described in Appendix B is used to combine the MCN network with the internal field network. This yields the Z-matrix of the array with respect to the p external ports of the unit cells. Then the adjacent ports of the unit cells are combined together using Kirchhoff’s voltage and current laws. The radiation characteristics and the input impedance are obtained by using the same approach discussed earlier in Section 7.2.5.

7.4.2 MC between two two-port rectangular patches

The analysis procedure described in the previous section, has been used for investigating the variation of effects of mutual coupling between two two-port patches as a function of their spacing. For this purpose we consider three different set of patches with different cover layer thicknesses and consider only the coupling between the radiating edges. Computations of mutual coupling including the non-radiating edges shows that their contribution is very small. The dimensions of the patches are listed in Table 7.8.

Figure 7.11 shows the variation of mutual coupling coefficient $|S_{31}|$ as a function of normalized spacing $s (=S/\lambda_0)$ between the patches (where $S$ is shown in the inset of Fig 7.11). The coupling coefficient $|S_{31}|^2$ represents the fraction of input power (to the first patch) which is coupled to the second patch through the MC contributed by the radiating edges. This power appears as a reflected signal at the input port of the second patch. Also $|S_{31}|^2$ represents the fraction of input power (to the second patch) which appears at the input port of the first patch. As seen in the figure, the MC with 1/10 inch thick cover layer is the strongest for the three cases considered. For $\lambda_0/2$ spacing (typical spacing between two adjacent elements in a series-fed array), the coupling in
Table 7.8 Design dimensions of two-port rectangular patches for different cover layer thickness d

<table>
<thead>
<tr>
<th>Patch Dimensions</th>
<th>d=0 inch (no cover)</th>
<th>d=1/10 inch</th>
<th>d=1/8 inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (mm)</td>
<td>6.06</td>
<td>6.06</td>
<td>6.00</td>
</tr>
<tr>
<td>a (mm)</td>
<td>13.371</td>
<td>1.2785</td>
<td>1.2728</td>
</tr>
<tr>
<td>$x_1$ (mm)</td>
<td>4.982</td>
<td>4.60</td>
<td>4.398</td>
</tr>
<tr>
<td>$x_2$ (mm)</td>
<td>5.50</td>
<td>5.14</td>
<td>5.00</td>
</tr>
<tr>
<td>$W^*$ (mm)</td>
<td>1.155</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

* width of 50 (Ohms) transmission lines.

The presence of 1/10 inch thick cover layer is about 1 dB higher than without cover layer.

Figure 7.12 shows a comparison between $|S_{31}|$ and $|S_{41}| (= |S_{23}|)$ for the case of patches with 1/10 inch thick cover layer. $|S_{41}|^2$ represents the power coupled from the first patch which appears at the output of the second patch. $|S_{41}|$ is almost 3 dB lower than $|S_{31}|$ for all element spacings (this comparison is true for other values of cover layer thickness as well). This is because the reflection coefficient at the output port of the patches is not zero whereas the input reflection coefficient is made to be equal zero.

Results from these computations shows that the scattering parameters of each patch remains almost unchanged. For example for patches with 1/10 inch thick cover layer and for $\lambda_0/2$ spacing, when the MC is included, $|S_{21}|$
Figure 7.11 Variation of H-plane MC coefficient $|S_{31}|$ between two two-port patches as function of the normalized spacing $S/\lambda_0$ (for different values of cover layer thickness)
Figure 7.12 Variation of H-plane MC coefficients $|S_{31}|$ and $|S_{41}|$ between two two-port patches with 1/10 inch thick cover layer as a function of the normalized spacing $S/\lambda_0$. 
changes from 0.7201 to 0.7196 and \(|S_{11}|\) changes from 0.002 to 0.001. Comparison of the coupling power \((=|S_{31}|^2 + |S_{41}|^2)\) between two two-port patches and the coupling power \((=|S_{21}|^2)\) between two one-port patches (shown in Fig 6.14), shows that the effects of MC is less for two-port patches compared to one-port patches. This is because the radiated power (normalized to the input power \(P_{in}\)) from a two-port patch (=0.2782 of \(P_{in}\)) is lower than that from a one-port patch (=0.5969 of \(P_{in}\)).

7.4.3 Effects of MC on array elements

In the previous section, we have discussed the mutual coupling between a pair of two-port patches. In this section, we study the effects of MC because of the presence of other array elements on a two-element section of an array. Also, we need to determine how many neighboring elements are directly influencing the characteristics of a given element because of mutual coupling. This is illustrated by considering a series-fed array of five identical elements as shown in Fig 7.13.

The effects of MC because of the first three elements on the subarray consisting of the last two elements are given in Tables 7.9 and 7.10. Since the array is fed in series, the power levels at the various elements are in a decreasing order, the first element radiating more power than the second and so on. From Table 7.9, we note that the effect of MC between element 3 and element 1 \((y_{m}^{13})\) is to cause changes in the transmission coefficient \(S_{41}\) of the two element sub-array and in the ratio of voltages at the edges of elements 2 and 1. Also, the input reflection \(S_{11}\) (for the two-element sub-array) increases considerably. These changes need to be compensated for.
Figure 7.13 Series-fed array of 5 identical rectangular microstrip patches showing ports and edges numbering
Table 7.9 Effects of MC because of the addition of a third element on a two-element series-fed array

<table>
<thead>
<tr>
<th>Array charact.</th>
<th>1+2 no MC</th>
<th>1+2 with MC</th>
<th>1+2+3 with MC</th>
<th>1+2+3 MC-Y_{m}^{13}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>0.002</td>
<td>0.062</td>
<td>0.0826</td>
<td>0.0581</td>
</tr>
<tr>
<td></td>
<td>$\angle 45.5^\circ$</td>
<td>$\angle 86.6^\circ$</td>
<td>$\angle 135.4^\circ$</td>
<td>$\angle 127.8^\circ$</td>
</tr>
<tr>
<td>$S_{41}$</td>
<td>0.5138</td>
<td>0.5433</td>
<td>0.5807</td>
<td>0.5713</td>
</tr>
<tr>
<td></td>
<td>$\angle -42.2^\circ$</td>
<td>$\angle -43.8^\circ$</td>
<td>$\angle -44.9^\circ$</td>
<td>$\angle -45.3^\circ$</td>
</tr>
<tr>
<td>Change* in $S_{41}$</td>
<td>—</td>
<td>+5.73 %</td>
<td>+3.75 %</td>
<td>-1.6 %</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>$\angle -1.6^\circ$</td>
<td>$\angle -1.1^\circ$</td>
<td>$\angle -0.4^\circ$</td>
</tr>
<tr>
<td>$V_{e4}/V_{e2}$</td>
<td>0.7175</td>
<td>0.7401</td>
<td>0.7486</td>
<td>0.7445</td>
</tr>
<tr>
<td></td>
<td>$\angle -201.0^\circ$</td>
<td>$\angle -199.9^\circ$</td>
<td>$\angle -198.7^\circ$</td>
<td>$\angle -199.3^\circ$</td>
</tr>
<tr>
<td>Change* in $V_{e4}/V_{e2}$</td>
<td>—</td>
<td>+3.15 %</td>
<td>+1.15 %</td>
<td>-0.55 %</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>$\angle +1.1^\circ$</td>
<td>$\angle -1.23^\circ$</td>
<td>$\angle -0.6^\circ$</td>
</tr>
</tbody>
</table>

* Compared to the previous column.

From Table 7.10, the effect of $y_{m}^{14}$ needs to be compensated for. On the other hand, the effect of $y_{m}^{15}$ is to slightly decrease the magnitude of $V_{e4}/V_{e2}$ by 0.05 % and the phase difference between $V_{e4}$ and $V_{e2}$ by 0.30°. These variations in amplitude distribution will cause a slight increase in the SLL of the 19 element series-fed array as discussed later. So the direct effect of the 1st element on the 5th element $y_{m}^{15}$ is very small and can be neglected. This means that for
Table 7.10 Effects of MC because of the addition of fourth and fifth elements on a three-element series-fed array

<table>
<thead>
<tr>
<th>Array character</th>
<th>$1+2+3+4$ with MC</th>
<th>$1+2+3+4$ MC-$Y_{m}^{14}$</th>
<th>$1+2+3+4+5$ with MC</th>
<th>$1+2+3+4+5$ MC-$Y_{m}^{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>0.0958</td>
<td>0.0823</td>
<td>0.1047</td>
<td>0.0960</td>
</tr>
<tr>
<td></td>
<td>$\angle 140.2^\circ$</td>
<td>$\angle 134.5^\circ$</td>
<td>$\angle 144.7^\circ$</td>
<td>$\angle 139.4^\circ$</td>
</tr>
<tr>
<td>$S_{41}$</td>
<td>0.5988</td>
<td>0.5940</td>
<td>0.6094</td>
<td>0.6068</td>
</tr>
<tr>
<td></td>
<td>$\angle -44.7^\circ$</td>
<td>$\angle -45.1^\circ$</td>
<td>$\angle -44.3^\circ$</td>
<td>$\angle -44.6^\circ$</td>
</tr>
<tr>
<td>Change in $S_{41}$</td>
<td>$+3.1%^{**}$</td>
<td>$+0.8%^{*}$</td>
<td>$+1.8%^{***}$</td>
<td>$-0.4%^{*}$</td>
</tr>
<tr>
<td></td>
<td>$\angle +0.2^\circ$</td>
<td>$\angle -0.4^\circ$</td>
<td>$\angle +0.4^\circ$</td>
<td>$\angle -0.35^\circ$</td>
</tr>
<tr>
<td>$V_{e4}/V_{e2}$</td>
<td>0.7509</td>
<td>0.7495</td>
<td>0.7501</td>
<td>0.7505</td>
</tr>
<tr>
<td></td>
<td>$\angle -198.0^\circ$</td>
<td>$\angle -198.4^\circ$</td>
<td>$\angle -197.6^\circ$</td>
<td>$\angle -197.9^\circ$</td>
</tr>
<tr>
<td>Change in $V_{e4}/V_{e2}$</td>
<td>$+0.31%^{**}$</td>
<td>$-0.18%^{*}$</td>
<td>$+0.1%^{***}$</td>
<td>$+0.05%^{*}$</td>
</tr>
<tr>
<td></td>
<td>$\angle +0.7^\circ$</td>
<td>$\angle +1.1^\circ$</td>
<td>$\angle +0.8^\circ$</td>
<td>$\angle -0.3^\circ$</td>
</tr>
</tbody>
</table>

* Change compared to the previous column.

** Change compared to the values in column 3 of Table 7.9

*** Change compared to the values in column 1.

the design of a given array element we need to include the MC effects of four adjacent elements on each side.

For the example considered here, the transmission coefficient $|S_{21}|$ of each element is equal to 0.717. For a 19-element Taylor array, the array elements
have high transmission coefficient as shown in Table 7.2. Therefore, we expect the effect of MC to be smaller than what we have in this example.

7.4.4 Compensations for effects of MC

So far, we have studied the effects of MC on elements of a series-fed array. In this section we develop a procedure for the compensations of the effects of MC on the array elements. Since the MC effects depend considerably on the amplitude distribution of the array, the design procedure (in presence of MC) of series-fed arrays will be developed for an actual Taylor array with 1/10 inch cover layer (shown in Table 7.6). The starting point in the design is the design of the 19 element array without including the effects of the mutual coupling. Then the mutual coupling network is included and the element dimensions, feed locations and interconnecting line lengths are modified to compensate for the effects of MC.

As discussed earlier one of the main effects of MC is to cause reflections at the input port of the unit cells. These reflected signals are undesirable, since they affect the amplitude and phase distribution and cause an increase in SLL and the appearance of a large side lobe at an angle of \(-\theta_0\) (where \(\theta_0\) is the direction of the main beam). Therefore, we require that the unit cells be redesigned for input match. For this we start from the last element in the array and work backward until all array elements are matched. This is achieved by creating a reflection at the input port of the unit cells such that this reflected signal cancels the signal received by the unit cell (which appears at the input port) due to MC to the adjacent array elements. The widths of the elements are kept unchanged. However, the input/output port locations and the length of the patches are slightly modified.
Another important effect of the MC is to decrease the amount of radiated power from the patches. This is because a fraction of the radiated power is coupled to other array elements. When MC is included, the signal transmitted to the output of the array unit cells is the sum of the signal transmitted from the input to the output port and the signals received by the radiating edges due to MC. To achieve the required array distribution we require that the total transmission coefficient of each unit cell be equal to $|S_{21}|$ computed when MC was not included. This is implemented by reducing the transmission coefficient of each unit cell starting from the first element.

Modification of the design dimensions of the unit cells may slightly alter the values of the elements in the MC admittance matrix. However, since the width of the patches are not modified, these changes will be very small and no iteration will usually be needed. The design procedure outlined above has been implemented for the case of a two-element series-fed array consisting of elements 11 and 12 of the 19-element series fed array (with dimensions given in Table 7.6).

Table 7.11 shows the dimensions of the two-element array before and after compensating for the effects of MC. As shown in Table 7.12, the effects of MC is to cause a reflection at the input ports and increase the transmission coefficient of the two-element array. However after compensation for the effects of MC, the results are identical to those without mutual coupling.

7.4.5 Effect of MC on the 19-element Taylor array

In this section we investigate the effects of MC on the 19-element series-fed array of Table 7.6. The study includes the effects on the amplitude and phase distributions and the consequential effects on the radiation charac-
Table 7.11 Design dimensions of two-element series-fed array without MC and after compensating for effects of MC

<table>
<thead>
<tr>
<th>Patches dimensions (mm)</th>
<th>Patch # 1 without MC</th>
<th>Patch # 1 with MC*</th>
<th>Patch # 2 without MC</th>
<th>Patch # 2 with MC*</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>12.775</td>
<td>12.769</td>
<td>12.756</td>
<td>12.762</td>
</tr>
<tr>
<td>b</td>
<td>4.00</td>
<td>4.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>x₁</td>
<td>3.71</td>
<td>3.71</td>
<td>3.29</td>
<td>3.34</td>
</tr>
<tr>
<td>x₂</td>
<td>4.03</td>
<td>4.07</td>
<td>3.71</td>
<td>3.715</td>
</tr>
</tbody>
</table>

* After compensating for effects of MC.

The effects of MC on the amplitude distribution of the 19 element array is shown in Fig 7.14. The percentage amplitude error shown is in comparison to the case when the MC effects are not included. We note that the maximum error (4.5%) introduced is in the 18th element of the array. Various plots in this figure refer to the cases when MC only to the next one (MC1), next two (MC2), next three (MC3) and next four (MC4) elements are considered in the analysis.
Table 7.12 Performances of two-element series-fed array in presence of MC

<table>
<thead>
<tr>
<th>Array Charact.</th>
<th>no MC</th>
<th>MC* included</th>
<th>MC** included</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>S_{11}</td>
<td>$</td>
<td>0.006</td>
</tr>
<tr>
<td>$</td>
<td>S_{21}</td>
<td>$</td>
<td>0.7791</td>
</tr>
<tr>
<td>Phase of $S_{21}$</td>
<td>-180.1°</td>
<td>-179.9°</td>
<td>-180.1°</td>
</tr>
<tr>
<td>$</td>
<td>V_{e1}/V_{e3}</td>
<td>$</td>
<td>1.578</td>
</tr>
<tr>
<td>Phase of $V_{e1}/V_{e2}$</td>
<td>-89.97°</td>
<td>-90.32°</td>
<td>-89.88°</td>
</tr>
</tbody>
</table>

* Two-element array designed without MC.

** Two-element array designed including MC.

The effect of MC on the phase distribution is shown in Fig 7.15. We note that the maximum phase error (1.2°) is introduced for the phase difference between the 16th and 17th elements of the array.

Effects of the above mentioned amplitude and phase errors on the radiation pattern and on the input $|S_{11}|$ of the 19 element array are summarized in Table 7.13. We note that the effects are negligible. The effects of MC on array performance are small because the few last elements (which are affected most) do not contribute much to the radiation pattern of the array.

It may be noted that the results presented above are computed at the
Figure 7.14 Errors in array amplitude distribution because of effects of MC among array elements
Figure 7.15 Errors in phase difference between adjacent array elements because of the effects of MC among array elements.
Table 7.13 Summary of effects of MC on radiation characteristics (f=7.5 GHz)

| Number of elem. | Beam Direct. | Beam Width | SLL (dB) | input $|S_{11}|$ (dB) |
|-----------------|--------------|------------|----------|-----------------|
| 0 (no MC)       | -30.0°       | 8.0°       | -28.7    | -54.6           |
| 1               | -30.0°       | 8.0°       | -28.4    | -52.4           |
| 2               | -30.0°       | 8.0°       | -28.5    | -52.4           |
| 3               | -30.0°       | 8.0°       | -28.6    | -52.5           |
| 4               | -30.0°       | 8.0°       | -28.6    | -51.8           |

design frequency. Effects at frequencies away from the center frequency are summarized in Table 7.14. We conclude from the above results that the effects of MC on the 19-element series-fed array are very small and no compensations for these effects is needed. The effect of mutual coupling can be larger for series-fed arrays with thicker substrates or used at higher frequencies.

7.5 Sensitivity analysis

The multiport network model has been extended for calculations of the sensitivities of the array performance with respect to various design parameters such as dielectric constant, element dimensions, etc. The method consists of the evaluation of the modified $Z_C$ and $Z_V$ matrices for each unit cell of the array. The segmentation method (described in section 7.2.4) is then used to calculate the currents at the ports of each cell. These currents are used together
Table 7.14 Summary of effects of MC on radiation characteristics as a function of frequency

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Beam Direct.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th></th>
<th>S11</th>
<th>(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.400</td>
<td>-36.0°</td>
<td>8.5°</td>
<td>-16.5</td>
<td></td>
<td>-27.7</td>
<td></td>
</tr>
<tr>
<td>7.450</td>
<td>-33.0°</td>
<td>8.2°</td>
<td>-22.0</td>
<td></td>
<td>-40.7</td>
<td></td>
</tr>
<tr>
<td>7.475</td>
<td>-31.5°</td>
<td>8.1°</td>
<td>-25.3</td>
<td></td>
<td>-50.7</td>
<td></td>
</tr>
<tr>
<td>7.500</td>
<td>-30.0°</td>
<td>7.9°</td>
<td>-28.6</td>
<td></td>
<td>-51.8</td>
<td></td>
</tr>
<tr>
<td>7.525</td>
<td>-28.5°</td>
<td>7.7°</td>
<td>-27.6</td>
<td></td>
<td>-51.2</td>
<td></td>
</tr>
<tr>
<td>7.550</td>
<td>-27.0°</td>
<td>7.5°</td>
<td>-24.7</td>
<td></td>
<td>-41.0</td>
<td></td>
</tr>
<tr>
<td>7.600</td>
<td>-24.5°</td>
<td>7.3°</td>
<td>-20.0</td>
<td></td>
<td>-30.5</td>
<td></td>
</tr>
</tbody>
</table>

with $Z_C$ and $Z_V$ matrices to compute the voltage distribution at the radiating edges of array elements. This yields the modified radiation performance (beam direction, beam width and SLL) and the input impedance of the array.

The above procedure has been implemented for a 19-element series-fed array without a cover layer [93]. In this section we consider the 19-element array with a cover layer, whose dimensions are given in Table 7.6. The aim of this study is to investigate the practical feasibility of the low SLL array with cover layer using the currently available technology and the approximate formulation for edge admittance. In the following sections, sensitivities of the series-fed array with respect to critical design parameters are discussed.
7.5.1 Tolerance in dielectric constant

The manufacturers specifications for the dielectric constant value are given at few values of frequency and usually within 1-2% accuracy. The actual value of the dielectric constant varies from one dielectric sheet to another and is also a continuous function of frequency. An accurate method for measuring the real part of the dielectric constant at the operating frequency uses a ring resonator [94]. The accuracy of this method is limited by measurement errors and by the accuracy of design formulas used to extract the value of the dielectric constant from the measured resonance frequency of the ring resonator.

Table 7.15 shows the modified values of beam direction, SLL and input impedance when $\varepsilon_r$ (for both the substrate and the cover layer) changes from 2.2 to 2.211. The effects of this change on edge admittance and MC is small. The main effects will be on the fields underneath the patch. The corresponding effects on the array is to cause the design frequency $f_0$ to shift from 7.5 GHz to 7.48 GHz. This is close to that given by $f_0 \times \sqrt{\varepsilon_r/(\varepsilon_r + \Delta \varepsilon)}$. The SLL changes from -28.6 to -28.26 dB.

7.5.2 Uncertainty in dielectric and conductor losses

The presence of dielectric and conductor losses in microstrip arrays reduces the efficiency and increases the bandwidth of those arrays. The effects of those losses are also to disturb the amplitude distribution along the array. Table 7.16 shows the effects of changes in the values of dielectric and conductor losses on the array performance. Even for 50 % changes in values of losses, the effects on radiation characteristics and input impedance are very small. This may be due to the fact that the presence of losses affects equally the radiated power and the transmitted power and the same amount of percentage losses
Table 7.15 Modified values of array characteristics for dielectric constant of 2.211

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Beam Direct.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th>$Z_{in}$ (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>-29.0°</td>
<td>7.80°</td>
<td>-28.68</td>
<td>74.4 - j0.14</td>
</tr>
<tr>
<td>7.49</td>
<td>-29.5°</td>
<td>7.80°</td>
<td>-28.66</td>
<td>74.2 + j0.15</td>
</tr>
<tr>
<td>7.48</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.26</td>
<td>74.0 + j0.08</td>
</tr>
<tr>
<td>7.47</td>
<td>-30.5°</td>
<td>7.80°</td>
<td>-26.72</td>
<td>74.1 - j0.14</td>
</tr>
</tbody>
</table>

has been used for all elements.

Table 7.16 Modified values of array characteristics with variation in values of dielectric and conductor losses

<table>
<thead>
<tr>
<th>% Change in losses</th>
<th>Beam Direct.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th>$Z_{in}$ (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.59</td>
<td>74.3 - j0.16</td>
</tr>
<tr>
<td>10.0</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.60</td>
<td>74.2 + j0.17</td>
</tr>
<tr>
<td>50.0</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.70</td>
<td>74.4 + j0.15</td>
</tr>
</tbody>
</table>

7.5.3 Uncertainty in edge capacitance

The edge capacitance is one of the critical parameters in the design of
microstrip antennas and arrays because of the inherent narrow antenna bandwidth. The edge capacitance for rectangular patches with cover layer has been determined from the measured resonance frequencies of unloaded patches (as discussed in Chapter 6). Inaccuracy in the values of edge capacitance may be due to errors in measured resonance frequency or to the simplified transmission line model used in the computation. Table 7.17 shows the modified array characteristics for a 5% increase in the actual value of edge capacitance. The effect of this error is a shift in the design frequency from 7.5 GHz to 7.48 GHz and a slight improvement in the value of SLL.

Table 7.17 Modified values of array characteristics with 5% increase in the value of edge capacitance

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Beam Direct.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th>$Z_{in}$ (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>-28.5°</td>
<td>7.80°</td>
<td>-26.88</td>
<td>74.5 - j0.28</td>
</tr>
<tr>
<td>7.49</td>
<td>-29.5°</td>
<td>7.80°</td>
<td>-28.41</td>
<td>74.5 + j0.08</td>
</tr>
<tr>
<td>7.48</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.69</td>
<td>74.5 + j0.20</td>
</tr>
<tr>
<td>7.47</td>
<td>-30.5°</td>
<td>7.80°</td>
<td>-28.64</td>
<td>74.4 + j0.01</td>
</tr>
</tbody>
</table>

7.5.4 Uncertainty in edge conductances and mutual coupling

As discussed in Chapter 6, the edge conductances (both radiation $G_r$ and surface wave $G_s$) and the MC admittance have been computed by replacing the fields at the edges by equivalent magnetic current line sources using the
equivalence principle. This approximation is very accurate for thin substrates and becomes less accurate for thicker substrates. Thus, for any practical dielectric parameters, this approximation introduces errors in the computed edge conductances and the MC admittance. Modified values of radiation characteristics and input impedance have been computed for a 10% uncertainty in both $G_r$ and $G_s$ respectively. The effect of these uncertainties is negligible. The effect of a change of 10% in the mutual coupling admittance on the array characteristics has been computed and was seen to be negligible also.

7.5.5 Dimensional tolerances

The measured dimensions of an array after fabrication are usually different from the desired design dimensions. This is mainly due to etching problems. For large arrays, the effect is not uniform along the array length. In the present day technology, fabrication tolerances are usually within 1 mil. We consider here the case where the effect is uniform and all dimensions after fabrication are 1 mil larger than the designed dimensions. The patch length $a$, the patch width $b$ and the width of the interconnecting lines are increased by 1 mil. The input/and output transmission lines lengths are increased by 0.5 mil each.

Table 7.18 shows the effect of change in the design dimensions on the array performance. As seen from this Table, this effect consists of a shift in the design frequency $f_0$ from 7.5 GHz to 7.49 GHz and the SLL increases to -28.45 dB. This shift in the design frequency is very close to that given by $f_0 \times (a/(a + 1 \text{ mil}))$. 
Table 7.18 Modified values of array characteristics with 1 mil increase in the design dimensions

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Beam Direct.</th>
<th>Beam Width</th>
<th>SLL (dB)</th>
<th>$Z_{in}$ (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>-29.5°</td>
<td>7.80°</td>
<td>-28.55</td>
<td>72.5 + j0.10</td>
</tr>
<tr>
<td>7.49</td>
<td>-30.0°</td>
<td>7.80°</td>
<td>-28.45</td>
<td>72.3 + j0.15</td>
</tr>
<tr>
<td>7.48</td>
<td>-30.5°</td>
<td>7.80°</td>
<td>-27.59</td>
<td>72.5 + j0.07</td>
</tr>
<tr>
<td>7.47</td>
<td>-31.0°</td>
<td>7.80°</td>
<td>-26.17</td>
<td>72.4 + j0.38</td>
</tr>
</tbody>
</table>

7.6 Comparison with experiment

The design procedure of microstrip series-fed array has been verified by comparing the results obtained with experiment. The fabrications and measurements for the results reported in this chapter have been carried out at the Naval Weapon Center, China Lake. For these experiments, the same dielectric material for both the substrate and the cover layer with $\varepsilon_r=2.2$ and loss tangent $\delta_d = 0.0009$ has been used. The substrate thickness is 1/64 inch and the cover layer thickness is 1/10 inch. The value of $\varepsilon_r=2.2$ has been verified by measurement of resonance frequency of ring resonator.

7.6.1 Two-element series-fed array

Experimental results for a sub-array of two elements (number 11 and 12 of Table 7.6) with 1/10 inch cover layer have been compared with theoretical calculations. Since the dimensions of the fabricated two-patch were slightly
different (because of etching tolerances, etc.) from the original design dimensions, it was necessary to rerun the theoretical results for the actual dimensions measured from the fabricated array. The two-element array layout and the measured dimensions after fabrication are shown in Fig 7.16.

Theoretical and measured values of input reflection coefficient $|S_{11}|$ are compared in Fig 7.17. The measured minimum value of $|S_{11}|$ is -24 dB, whereas the theoretical minimum value is -45 dB. This discrepancy may be due to changes in the values of impedances of the input and output quarter-wave matching networks (because of etching) or to the effects of connectors. The measured resonance frequency (for minimum $|S_{11}|$) is 7.57 GHz, which is in excellent agreement with the theoretical value of 7.58 GHz (0.13 % error). This is expected, since the edge capacitance (critical parameter in the determination of resonance frequency) has been evaluated experimentally as discussed in Chapter 6. From the variation of $|S_{11}|$ with frequency shown in Fig 7.17, we calculate the measured bandwidth (for $VSWR \leq 2$) to be 310 MHz. The computed bandwidth is found to be 340 MHz.

Figure 7.18 shows a comparison of the calculated and measured magnitude of the transmission coefficient $|S_{21}|$ (transmission from the input port of the first patch to the output port of the second patch). The two variations with frequency follow the same trend, but the experimental measured values are 1 dB higher than the theoretical values. Part of this discrepancy may be attributed to the adhesive thin film used for joining the cover layer with the substrate.

Figures 7.19 and 7.20 show a comparison between the theoretical and the measured H-plane far field patterns of the two-element array at $f=7.55$
Dimensions are in (mm)

\[ h = \frac{1}{64}'' \quad d = \frac{1}{10}'' \]
\[ \varepsilon_{r1} = \varepsilon_{r2} = 2.2 \]

Figure 7.16 Dimensions of the fabricated two-element series-fed array
Figure 7.17 Comparison of experimental and theoretical values of $|S_{11}|$ for the two-element series-fed linear array
Figure 7.18 Comparison of experimental and theoretical values of $|S_{21}|$ for the two-element series-fed linear array
Figure 7.19 Comparison of measured and theoretical H-plane far-field pattern of two-element series-fed array (f=7.55 GHz)
Figure 7.20 Comparison of measured and theoretical H-plane far field pattern of two-element series-fed array (f=7.58 GHz)
GHz and $f=7.58$ GHz respectively. The agreement is better at the resonance frequency ($f=7.58$ GHz) than away from resonance at $f=7.55$ GHz.

### 7.6.2 19-Element series-fed array

The computer aided-design procedure developed in this report has been verified also by comparing the measured and the calculated performance of the 19-element series-fed array with Taylor distribution (designed in Section 7.3). Table 7.19 shows the measured dimensions ($a$, $b$, $x_1$ and $x_2$ shown in Fig 7.3) of the array elements after fabrication. After fabrication, the widths of the 75 Ω interconnecting microstrip lines are not uniform along the length of the lines. Average values $W_1$ and $W_2$ for the input and the output sections of the input/output lines taken along the lengths of the lines are also given in Table 7.19 and have been used for recomputing the theoretical results. The designed value of width of the 75 Ω interconnecting lines is 0.0508 cm. The measured dimensions ($a$, $b$, $x_1$ and $x_2$) shown in Table 7.19 are very close to the designed dimensions shown in Table 7.6, except for the first element where values of $a$, $x_1$ and $x_2$ are lower than the corresponding designed values (due to mask problems). To transform from the 75 Ω impedance of the lines to a 50 Ω input impedance and a 50 Ω load, quarter wave transformers are used at both the input and the output ports of the array. The theoretical computations have been redone using the measured dimensions of the array elements.

Figure 7.21 shows a comparison between the measured and the calculated values of the magnitude of the reflection coefficient ($|S_{11}|$) at the input port of the array. Very good agreement for $|S_{11}|$ is obtained for values of frequencies between 7.25 GHz and 7.7 GHz. The mismatch at the input of the array is caused mainly by the incorrectly fabricated first array element.
Table 7.19 Measured dimensions of 19-element series-fed array after fabrication

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>a (cm)</th>
<th>$x_1$ (cm)</th>
<th>$x_2$ (cm)</th>
<th>$w_1$ (cm)</th>
<th>$w_2$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0523</td>
<td>1.1321</td>
<td>0.0286</td>
<td>0.0262</td>
<td>0.0569</td>
<td>0.0541</td>
</tr>
<tr>
<td>0.0711</td>
<td>1.3604</td>
<td>0.2112</td>
<td>0.2173</td>
<td>0.0559</td>
<td>0.0554</td>
</tr>
<tr>
<td>0.0963</td>
<td>1.3421</td>
<td>0.2633</td>
<td>0.2710</td>
<td>0.0549</td>
<td>0.0550</td>
</tr>
<tr>
<td>0.0949</td>
<td>1.3274</td>
<td>0.3812</td>
<td>0.3901</td>
<td>0.0546</td>
<td>0.0554</td>
</tr>
<tr>
<td>0.1971</td>
<td>1.3053</td>
<td>0.2756</td>
<td>0.2930</td>
<td>0.0561</td>
<td>0.0570</td>
</tr>
<tr>
<td>0.1959</td>
<td>1.2987</td>
<td>0.3555</td>
<td>0.3706</td>
<td>0.0563</td>
<td>0.0563</td>
</tr>
<tr>
<td>0.2983</td>
<td>1.2929</td>
<td>0.2984</td>
<td>0.3155</td>
<td>0.0565</td>
<td>0.0580</td>
</tr>
<tr>
<td>0.2989</td>
<td>1.2880</td>
<td>0.3492</td>
<td>0.3703</td>
<td>0.0557</td>
<td>0.0570</td>
</tr>
<tr>
<td>0.2989</td>
<td>1.2809</td>
<td>0.3889</td>
<td>0.4092</td>
<td>0.0547</td>
<td>0.0565</td>
</tr>
<tr>
<td>0.3990</td>
<td>1.2751</td>
<td>0.3433</td>
<td>0.3732</td>
<td>0.0540</td>
<td>0.0585</td>
</tr>
<tr>
<td>0.3973</td>
<td>1.2753</td>
<td>0.3721</td>
<td>0.4021</td>
<td>0.0545</td>
<td>0.0560</td>
</tr>
<tr>
<td>0.4021</td>
<td>1.2753</td>
<td>0.3260</td>
<td>0.3715</td>
<td>0.0519</td>
<td>0.0545</td>
</tr>
<tr>
<td>0.4985</td>
<td>1.2720</td>
<td>0.3424</td>
<td>0.3869</td>
<td>0.0547</td>
<td>0.0559</td>
</tr>
<tr>
<td>0.4966</td>
<td>1.2738</td>
<td>0.3467</td>
<td>0.3947</td>
<td>0.0538</td>
<td>0.0538</td>
</tr>
<tr>
<td>0.4981</td>
<td>1.2730</td>
<td>0.3395</td>
<td>0.3851</td>
<td>0.0545</td>
<td>0.0602</td>
</tr>
<tr>
<td>0.5009</td>
<td>1.2766</td>
<td>0.3120</td>
<td>0.3536</td>
<td>0.0538</td>
<td>0.0563</td>
</tr>
<tr>
<td>0.3974</td>
<td>1.2794</td>
<td>0.3237</td>
<td>0.3607</td>
<td>0.0504</td>
<td>0.0523</td>
</tr>
<tr>
<td>0.3970</td>
<td>1.2855</td>
<td>0.2603</td>
<td>0.2942</td>
<td>0.0484</td>
<td>0.0513</td>
</tr>
<tr>
<td>0.2949</td>
<td>1.2847</td>
<td>0.3358</td>
<td>0.3581</td>
<td>0.0513</td>
<td>0.0518</td>
</tr>
</tbody>
</table>

Figure 7.22 shows a comparison between the measured and the calculated values of the magnitude of the transmission coefficient from the input port to the output port of the array. The two curves follow the same trend. However, the measured values are almost 4 dB lower than the corresponding
Figure 7.21 Comparison of experimental and theoretical values of $|S_{11}|$ for the 19-element series-fed array
Figure 7.22 Comparison of experimental and theoretical values of $|S_{21}|$ for the 19-element series-fed array
theoretical values. This discrepancy in $|S_{21}|$ may be due to the extra losses in the connectors or to errors in the estimation of the radiation and the surface wave conductances of the array elements. It may be noted that the difference between the theoretical $|S_{21}|$ values of -14 dB (3.98 %) and the corresponding measured values -18 dB (1.58 %) is only 2.4 % of the input power to the array. The radiation from the non-radiating edges, which was not included in the theoretical computations, may be partly responsible for this difference.

Figures 7.23 and 7.24 show the measured H-plane far-field patterns of the array at $f=7.5$ GHz and $f=7.45$ GHz respectively. Measurements of these radiation patterns have been carried out using a 13-foot indoor range at the Naval Weapon Center (China Lake, CA).

Figures 7.25 and 7.26 show the corresponding calculated far-field patterns at $f=7.5$ GHz and $f=7.45$ GHz respectively. Table 7.20 shows a comparison between the measured and the calculated radiation characteristics of the array. The appearance of the large side lobes at $-33^\circ$ in Fig 7.23 and $-40^\circ$ is due to a back traveling wave caused by the reflections at the input of the various array elements and moving towards the input port. These side lobes are properly predicted by the theoretical computations also. The measured side lobes at angles larger than $60^\circ$, which are not predicted by the theory, are due to the scattering of surface waves from the edges of the finite substrate. We conclude that, a good agreement between theory and experiment is obtained for the 19-element array and this verifies the accuracy of the CAD procedure developed in this report.
Figure 7.23 Measured H-plane far-field pattern of 19-element series-fed array with Taylor distribution (f=7.50 GHz)
Figure 7.24 Measured H-plane far-field pattern of 19-element series-fed array with Taylor distribution (f=7.45 GHz)
Figure 7.25 Calculated H-plane far-field pattern of 19-element series-fed array with Taylor distribution (f=7.50 GHz)
Figure 7.26: Calculated H-plane far-field pattern of 19-element series-fed array with Taylor distribution (f=7.45 GHz)
Table 7.20 Comparison of measured and calculated radiation performance of 19-element array

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam direct.</td>
<td>Beam width</td>
</tr>
<tr>
<td>7.5</td>
<td>32.5°</td>
<td>8.0°</td>
</tr>
<tr>
<td>7.45</td>
<td>35.5°</td>
<td>8.0°</td>
</tr>
</tbody>
</table>

7.7 Discussions

The multiport network model has been used for the computer-aided analysis and design of microstrip series-fed arrays. The purpose of developing this CAD software is to eliminate the recourse to try-and-cut experimental iterations needed in the existing methods of arrays design. When the effects of mutual coupling among the array elements are negligible, the array is modeled as a cascade of two-port cells. Therefore, the design of the array for given specifications reduces to the determination of two-port characteristics of the unit cells.

The losses in an array (dielectric, conductor and surface wave) alter the array amplitude and phase distributions and hence cause an increase in SLL and reduce the array efficiency. A method for including the array losses in the design of arrays has been presented.
Effects of mutual coupling on the performance of a 19-element series-fed linear array with Taylor amplitude distribution have been investigated in detail. Computations show that the effects of MC are to cause errors in the amplitude and the phase distributions of the array. These effects are mostly on few elements near the load end of the array. The corresponding effects of MC on array performance are negligible since the contribution of these elements to the radiation is small. The effect of mutual coupling can be higher for thicker substrates and for higher frequencies.

The multiport network model has been extended to the sensitivity analysis of the array. Results obtained show that the critical parameters in the design are the dielectric constant and the edge capacitance. Therefore, these parameters need to be accurately determined (may be found experimentally as has been done in this report) for an accurate design of the array. Sensitivity, with respect to the design dimensions, shows the need for an accurate etching of the microstrip series-fed array.

The proposed computer-aided design software has been verified by careful experiments. Good agreement between theory and experiment for the scattering parameters and the radiation pattern of a two-element series-fed array and a 19-element series-fed array with Taylor distribution has been obtained.
CHAPTER VIII

CONCLUDING REMARKS

In this Chapter, a summary of the work described in this report is presented. First, a discussion of the multiport network model developed for the analysis and conclusions pertinent to specific applications are given. Then, suggestions for possible improvements of the method and other future work are presented.

8.1 Multiport network modeling approach

A Multiport Network Model (MNM) has been proposed for the modeling and the analysis of microstrip patch antennas and arrays. In this approach, the microstrip patches are analyzed by modeling the internal and external fields separately in terms of multiport networks. Therefore, the analysis and the design of the patches reduce to that of an equivalent multiport network. Analysis techniques available for multiport network such as sensitivity analysis and optimization procedures can be readily applied to microstrip patches and arrays when the MNM approach is used. Therefore, the multiport network modeling approach is ideally suited for the computer-aided design of microstrip antennas and arrays.

In the MNM method, the effect of fringing electric field at an edge of a microstrip patch is accounted for by using an edge capacitance. The power losses by radiation and coupling into surface waves are taken into account by
using edge conductances. Also, an equivalent edge inductance has been introduced in the MNM to take into account the effect of fringing magnetic field at the edges of microstrip patch antennas. This fringing inductance is needed for edges with non-uniform voltage distribution (for example, for radiating edges of a rectangular patch, and for a circular patch). For a rectangular patch, the fringing inductance per unit length of the edge can be evaluated from the knowledge of the fringing capacitance. The same approach has been used for circular microstrip patches.

For electrically thin substrates, the solution of the internal field by modeling the patch as a planar component with magnetic walls is accurate. On the other hand, accurate formulas are not available for the edge admittance. However, the approach presented in this report allows more accurate expressions for the edge admittance to be incorporated as and when they become available. Also, this model can include the value of the edge capacitance computed from experimental measurement (as discussed in Appendix C).

8.2 Applications of the method

The multiport network modeling approach for radiating microstrip patches has been used in Chapter IV through Chapter VI for the analysis and the design of multiple-port patches and in Chapter VII for the CAD of series-fed arrays. A summary of the results obtained during these studies is given in the following sub-sections.

8.2.1 Effect of mutual coupling

In Chapter III, the MNM has been extended to model the mutual coupling in microstrip antennas. The mutual coupling is characterized by an
admittance matrix, whose elements represent the coupling among various edges of a patch or patches of an array. The elements of this admittance network can be computed by replacing the aperture field at the edges by equivalent magnetic current line sources. This approach has been implemented for the case of rectangular patches. Good agreement between calculated and measured values of coupling coefficient between two identical probe-fed rectangular patches (without any dielectric cover layer) is obtained. The proposed model for the mutual coupling is compatible with the CAD analysis of microstrip arrays discussed in Chapter VII.

8.2.2 Two-port rectangular patches

The multiport network model has been used for the analysis of two-port rectangular patches. Such two-port patches form elements of series-fed microstrip arrays (discussed in Chapter VII). Results presented in Section 4.3 show that it is possible to match a two-port patch at the input port and, at the same time, achieve the required transmission coefficient to the second port in order to taper the amplitude distribution of the array appropriately. Usually the design of two-port rectangular patches (used as elements of a series-fed array) for specified transmission characteristics requires the optimization of the design parameters. A very good starting point in the optimization process is the use of the results based on the dominant mode analysis (discussed in Section 4.2). The power radiated from the non-radiating edges is 17 dB less than that from the radiating edges and has been neglected in the analysis.

8.2.3 Radiation from feedlines

The effects of the radiation from microstrip feed lines of a two-port
rectangular patch fed along the non-radiating edges has been investigated. The radiation is computed by taking ports along the length of the lines and evaluating the voltages at those ports. The radiated fields from the transmission lines has the same polarization as that of the radiated field by the patch. Example of a two-port patch, with $a=1.3403$ cm, $b=0.5$ cm, $x_1=0.4038$ cm, $x_2=0.4409$ cm, $\epsilon_r=2.2$, $h=1/64$ inch and $f=7.5$ GHz, has been considered. The characteristic impedances of the input and the output microstrip lines are 50 $\Omega$. The total field radiated by the feed lines is 35 dB down or less in the H-plane compared to the field radiated by the patch. However, in the E-plane it is 35 dB down in the broadside direction and increases to -20 dB at 90° from broadside.

### 8.2.4 Circular microstrip patches

In Chapter 5, the MNM approach has been applied for the analysis of one-port and two-port circular microstrip patches. Two different formulations for the computations of the elements of the $Z$-matrix of a circular segment have been investigated and compared. As done for rectangular segments, the Green's function for circular segments, usually expressed as double summation, has been reduced to a single summation using the mode matching technique. Computation of the elements of the $Z$-matrix of the circular segment using this single summation is very fast compared to that when using the double summation. Results obtained for two-port circular patches show that is possible to control the transmission coefficient by varying the angular separation between the input and the output ports. However, to achieve a match at the input port, the characteristic impedances of the input and the output transmission lines have to be selected appropriately for every angular separation.

For two-port circular patches with a shorted radial wall, wide range of
$|S_{21}|$ values can be obtained for the same values of input and output transmission lines. Also, the analysis of circular patches has been carried out using the dominant mode analysis. This approximate analysis is useful when optimization is used for computing the design dimensions of a circular patch.

### 8.2.5 Patches with thick dielectric cover layer

Analysis of microstrip patches with a thick dielectric cover layer has been carried out in Chapter VI. The radiation and the surface wave conductances as well as the mutual coupling are computed by replacing the aperture fields at the edges by equivalent magnetic current line sources. This approximation is very good for microstrip patches on thin substrates. The key step in the analysis is the determination of the electromagnetic fields due to a magnetic dipole over ground plane with a dielectric cover layer. The mutual coupling in presence of a cover layer has also been investigated. Comparison of MC between one-port patches, for different values of cover layer thickness, show that for small values of spacing (less than $\lambda_0/4$) between the patches the mutual coupling in the presence of a $1/10$ inch thick cover layer is around 1 dB higher than when no cover layer is present. However, for large spacing (larger than $\lambda_0$) the mutual coupling in the presence of a cover layer is almost 2.5 dB higher because of the presence of surface waves whose field decays as $1/\sqrt{\rho}$. The surface wave also reduces the efficiency of microstrip patches.

### 8.2.6 Series-fed linear arrays

The MNM approach has been successfully used for the computer-aided analysis and design of series-fed linear arrays. The purpose of developing this software is to eliminate the recourse to cut-and-try experimental iterations in
the existing methods of array design. When mutual coupling effects are negligible, the array is modeled as a cascade of two-port patches. The design of the array for given specifications thus reduces to the accurate design of two-port patches (discussed in details in Chapters 4, 5 and 6).

The array losses alter the array amplitude distribution and hence cause an increase in SLL and a decrease in array efficiency. A method for including the array losses in the design of arrays has been presented.

Effects of mutual coupling on the performance of a series-fed linear array has been discussed in details. Computations show that the effect of MC is to cause errors in the amplitude and phase distributions of the array. These effects are mainly on the elements near the load end of the array. The corresponding effects on the array performance are very small since the contribution of these elements (near the load end) to the radiation is small.

The MNM approach has been extended to the sensitivity analysis of the series-fed linear array. Results obtained show that the critical parameters in the design are: the dielectric constant and the edge capacitance. Therefore, these parameters need to be accurately determined for an accurate design of the array.

8.2.7 Experimental verifications

The proposed design methodology has been verified by careful experiments. Very good agreements between experiment and theory have been obtained for the scattering parameters of two-port rectangular patches with and without a dielectric cover layer, and also for one-port and two-port circular patches. Good agreement has been obtained for the mutual coupling between one-port rectangular microstrip patches with and without a cover layer. The
accuracy of the computer-aided design software for series-fed arrays has been verified by the very good agreement between the calculated and the measured scattering parameters and the radiation pattern of a two-element and a 19-element (with Taylor distribution) series-fed arrays with cover layer.

8.3 Advantages of the proposed modeling and analysis approach

Several applications of the proposed modeling and analysis approach have been summarized in Section 8.2. Major advantages of this approach may be summed up as follows. (i) This approach provides a simple but efficient and accurate method for the analysis of microstrip patches and arrays. (ii) The approach is applicable to the analysis of microstrip patches (on thin substrates) with arbitrary shapes. (iii) The approach is well suited for the analysis of multiple-port microstrip patches (used as array elements). (iv) The approach takes accurately into account the effect microstrip feed junction reactances. (v) The effects of mutual coupling may be incorporated when designing microstrip patches and arrays. (vi) This model is suitable for the analysis and design of microstrip patches of arbitrary shape covered with thick dielectric layer(s). (vii) This approach is well suited for the development of CAD procedures for microstrip patches and arrays.

8.4 Suggestions for further work

In the MNM approach, the radiating microstrip patches are modeled in terms of networks. Characterizations of these networks have been given in this report. However improvements in the characterization of the external field networks (EAN and MCN) are desired as discussed in Section 8.4.1. Some examples of further applications of the MNM approach are given in Section
8.4.2.

8.4.1 Improved characterization of EAN and MCN networks

For the accurate modeling of fringing fields using MNM, it is required to add an edge inductance to the edge admittance network (EAN) to account for the fringing magnetic fields. It has been shown that for rectangular microstrip patches, the fringing inductance can be computed from the fringing capacitance. A method for the computation of fringing inductance for an arbitrary shaped patch would be needed. An approach described by King [95] for the computation of the self inductance of an arbitrary shaped wire may possibly be useful for this computation.

Computation of mutual coupling in microstrip patches in terms of MCN network has been carried out by approximating the fringing fields by equivalent magnetic current sources. This approximation is not valid for the computation of the mutual susceptance between two adjoining sections of an edge. Usually the voltage along an edge of a patch has a slow variation and adjoining sections are at almost the same voltage level. Also, it may be recalled that the electrostatic and inductive couplings among these sections get incorporated in the computations of the edge capacitance and edge inductance respectively. The effect of the additional reactive part of the MC is expected to be small. However for more accurate computations, this effect may be included. A better approximation is to replace the line source of magnetic current by a infinite aperture in the patch plane in the direction normal to the edge. The field distribution in this aperture may be approximated by a quasi-static field distribution, and used for the computation of mutual coupling. A similar approach has been used in [74].
8.4.2 Further applications of MNM approach

Detailed analysis has been presented for two-port rectangular patches with and without cover layer. The approach presented in this report is also applicable when one wants to design patches with three or more ports. Examples of these patches are used in the multiple resonator broad-band microstrip antennas discussed in [51] and the dual polarized doubly series-fed array discussed in [7].

The CAD approach used for the design of series-fed arrays can be extended to the analysis of corporate-fed linear and 2-dimensional arrays. Junction reactances and radiations from the power dividers and from the bends can be accurately taken into account in the MNM. In a corporate-fed 2-dimensional array, the feed lines are very close to the patches and since some of these lines are usually carrying signals much larger than that existing in the patch strong coupling may exist among the feed lines and the patches. This type of coupling can also be conveniently included in the MNM approach.

The effect of mutual coupling between one-port rectangular patches has been investigated and good agreement between theory and experiment was obtained. A similar approach can be used to study the effects of mutual coupling between two arbitrary shaped patches. The approach can be in particular used for circular patches (discussed in Chapter V).

Multiport network model for microstrip patches fed by coaxial probes or microstrip lines presented in this report. The method can be extended to the analysis of microstrip patches fed by proximity coupling to a microstrip line in the same plane [39]. This can be done by modeling the gap between the patch and the microstrip line by a capacitive network (similar to that done for the coupling between rectangular patches in [34]).
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APPENDIX A

GREEN'S FUNCTION APPROACH FOR COMPUTATION OF Z-MATRIX OF PLANAR SEGMENTS

In this appendix, a review of the computation of the Z-matrix of a planar segment using the Green's function approach is presented. The method is applicable for shapes for which the wave equation can be solved for using separation of variables technique. Expressions for the Green's functions for rectangular and circular segments using this technique are given.

A.1 Z-matrix of a planar network with magnetic boundary

When a planar component is excited by a current density $J_z$ (oriented in the $z$-direction) at any arbitrary point $(x_0, y_0)$ inside the periphery as shown in Fig. A.1, the wave equation for the voltage (in rectangular coordinate system) is written as [37]

$$ (\nabla_T^2 + k^2) V(x, y) = -i\omega \mu h J_z(x_0, y_0) \quad (A.1) $$

where $\nabla_T^2 \equiv \partial_x^2 + \partial_y^2$ and $k = \omega \sqrt{\mu \epsilon}$. The magnetic boundary condition at the wall is satisfied by taking $\partial V / \partial n = 0$ all along the patch periphery. The solution of equation (A.1) in terms of the Green's function for an arbitrary source distribution inside the patch is written as

$$ V(x, y) = \int \int_D G(x, y|x_0, y_0) J_z(x_0, y_0) dx_0 dy_0 \quad (A.2) $$
where D denotes the area of the patch enclosed by the magnetic wall. The integration in (A.2) is everywhere zero except at the source locations. The Green's function G is the solution of the wave equation for the delta function excitation source

\[
(\nabla_T^2 + k^2) G(x, y | x_0, y_0) = -i\omega \mu \delta(x - x_0) \delta(y - y_0) \quad (A.3)
\]

The function G is chosen to satisfy the same boundary condition as V. When the planar component is fed on the periphery only, the boundary condition for the voltage at a coupling port is
\[
\frac{\partial V}{\partial n} = i\omega \mu h J_z
\]  

(A.4)

When the planar component is excited at its periphery only, the voltage at any location at the periphery is related to the Green's function by

\[
V(s) = \int_{c} G(s|s_0) J_z(s_0) ds_0
\]  

(A.5)

where \(s\) and \(s_0\) are distances measured along the periphery \(c\). The integration in (A.5) reduces to an integration along the coupling ports only since \(J_z = 0\) at the rest of the periphery. So if \(n\) coupling ports of width \(W_j\) are located at the periphery, \(V(s)\) is rewritten as:

\[
V(s) = \sum_{j=1}^{j=n} \int_{W_j} G(s|s_0) J_z(s_0) ds_0
\]  

(A.6)

The current flowing in at the coupling port \(j\) is related to the current density \(J_z(s_0)\) by

\[
I_j = \int_{W_j} J_z(s_0) ds_0
\]  

(A.7)

When the width of the coupling port is small, the current density can be taken to be uniform along the width. Otherwise, the width of the port is subdivided into smaller sections, such that the current density can be assumed to be uniform along the width of each of the sections. Then the current \(I_j\) given by (A.7) reduces to

\[
I_j = W_j J_z(s_0)
\]  

(A.8)
A voltage $V_i$ at the $i^{th}$ coupling port is defined as the average voltage over the width of the port. Using equations (A.6) and (A.8) the voltage $V_i$ is given by

$$V_i = \sum_j \frac{I_j}{W_i W_j} \int_{W_i} \int_{W_j} G(s | s_0) ds_0 ds$$  (A.9)

The element $Z_{ij}$ of the impedance matrix $Z$ relating the voltage at port $i$ to the current at port $j$ is given by

$$Z_{ij} = \frac{1}{W_i W_j} \int_{W_i} \int_{W_j} G(s | s_0) ds_0 ds$$  (A.10)

### A.2 Computation of Green’s function

The Green’s function for a planar component is computed by expanding it into eigenfunctions $\psi_m$ which satisfy

$$\nabla^2_T \psi_m + k^2_m \psi_m = 0$$  (A.11)

$\psi_m$ satisfies the boundary condition $\partial \psi_m / \partial n = 0$ at the periphery. $k_m$ is the eigenvalue corresponding to the eigenfunction $\psi_m$. The set of eigenfunctions $\psi_m$ are assumed to form a complete orthonormal set

$$\int_D \int \psi_m^{*} \psi_n dx dy = \begin{cases} 1; & \text{if } n=m, \\ 0; & \text{otherwise.} \end{cases}$$  (A.12)

So, $G(x, y | x_0, y_0)$ can be expanded in series of $\psi_m$ as

$$G(x, y | x_0, y_0) = \sum_{m} A_m(x_0, y_0) \psi_m(x, y)$$  (A.13)

Inserting (A.13) into (A.3) and using relation (A.12), the coefficients $A_m$ are
given by

\[ A_m = \frac{i \omega \mu h \psi_m^*(x_0, y_0)}{k_m^2 - k^2} \]  \hspace{1cm} (A.14)

So, the required Green's function expansion is

\[ G(x, y; x_0, y_0) = i\omega \mu h \sum_{m=0}^{\infty} \frac{\psi_m(x, y) \psi_m^*(x_0, y_0)}{k_m^2 - k^2} \]  \hspace{1cm} (A.15)

where \( \psi_m^* \) is the complex conjugate of \( \psi_m \).

### A.3 Green's function for a rectangular planar segment

The Green's function for the rectangle shown in *Fig. A.2* with magnetic boundary wall is

\[ G(x, y; x_0, y_0) = \frac{i\omega \mu h}{ab} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\sigma_m \sigma_n \cos(k_x x_0) \cos(k_y y_0) \cos(k_x x) \cos(k_y y)}{k_x^2 + k_y^2 - k^2} \]  \hspace{1cm} (A.16)

![Figure A.2 Rectangular planar segment](image-url)
where $k_x = m\pi/a$, $k_y = n\pi/b$ and

$$\sigma_m = \begin{cases} 1; & \text{if } m=0, \\ 2; & \text{otherwise}. \end{cases}$$

Parameters $a$ and $b$ are the dimensions of the rectangle along $x$ and $y$-direction respectively. $h$ is the height of the substrate material separating the patch from the ground plane and $\varepsilon_r$ is the complex dielectric constant of the substrate.

A.4 Green's function for circular planar segment

The Green's function for a circular segment shown in Fig A.3 is given by

$$G(\rho, \phi; \rho_0, \phi_0) = \frac{h}{i\omega\varepsilon\pi a^2} + i\omega\mu h \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n J_n(k_{nm}\rho_0)J_n(k_{nm}\rho)\cos[n(\phi - \phi_0)]}{\pi(a^2 - n^2/k_{nm}^2)(k_{nm}^2 - k^2)J_n^2(k_{nm}a)}$$

(4.17)
where \( J_n \) represents Bessel's function of the \( n^{th} \) order, and \( K_{nm} \) satisfy

\[
\frac{\partial}{\partial \rho} J_n(k_{nm} \rho) \bigg|_{\rho=a} = 0 \quad (A.18)
\]

where \( a \) is the radius of the circular segment. The subscript \( m \) in \( k_{nm} \) denotes the \( m^{th} \) root of (A.18). For the zeroth order Bessel's function (\( n=0 \)), the first root of (A.18) is taken to be the non-zero root.
APPENDIX B

SEGMENTATION METHOD

The mathematical formulation of the segmentation technique is illustrated here by combining two multiport subnetworks (shown as A and B in Fig B.1) to yield the overall network shown in Fig B.2. The subnetworks A and B are characterized by the impedance matrices \(|Z_A|\) and \(|Z_B|\) respectively.

In the application of the segmentation method [49], a reduction in the computational effort is achieved if the connected ports (denoted by \(q\) for the subnetwork A and by \(r\) for the subnetwork B) are suitably regrouped. This is done in such a way that \(q_1\) and \(r_1\) ports are connected together, \(q_2\) and \(r_2\) ports are connected together and so on. The external (unconnected) ports for each subnetwork (denoted by \(p_1\) for A and \(p_2\) for B) are numbered first. The voltage variables are related to the current variables by the following impedance matrix

\[
|Z| = \begin{pmatrix}
Z_{pp} & Z_{pq} & Z_{pr} \\
Z_{qp} & Z_{qq} & Z_{qr} \\
Z_{rp} & Z_{rq} & Z_{rr}
\end{pmatrix}
\]  \hspace{1cm} (B.1)

where \(p = p_1 + p_2\) and \(q=r\). The interconnection constraint, using Kirchhoff's circuit law, is that the voltages at two connected ports are equal and the sum of currents at two connected ports is zero

\[
\begin{cases}
V_q = V_r \\
I_q + I_r = 0.
\end{cases}
\]  \hspace{1cm} (B.2)
Figure B.1 Two planar components with ports numbering

Figure B.2 Combination of segments A and B of Figure B.1
Using relations (B.1) and (B.2) we obtain the Z-matrix \( Z_p \) (with respect to the \( p \) external ports) for the overall network shown in Fig B.2 as

\[
Z_p = Z_{pp} + (Z_{pq} - Z_{pr})(Z_{qq} - Z_{qr} - Z_{rq} + Z_{rr})^{-1}(Z_{rp} - Z_{qp}) \tag{B.3}
\]

The voltages \( V \) at the connected ports \( (q=r) \) are related to the current \( I_p \) at the external ports \( (p=p_1 + p_2) \) by

\[
V = [Z_{qp} + (Z_{qq} - Z_{qr}) (Z_{rq} + Z_{rr})^{-1} (Z_{rp} - Z_{qp})]I_p \tag{B.4}
\]

In most cases of interest, we have \( Z_{qr} = Z_{rq} = 0 \) since \( q \) and \( r \) ports belong to two physically separate segments. For these cases, equations (B.3) and (B.4) reduce to

\[
Z_p = Z_{pp} + (Z_{pq} - Z_{pr})(Z_{qq} + Z_{rr})^{-1}(Z_{rp} - Z_{qp}) \tag{B.5}
\]

\[
V = [Z_{qp} + (Z_{qq} - Z_{qr}) (Z_{qq} + Z_{rr})^{-1} (Z_{rp} - Z_{qp})]I_p \tag{B.6}
\]

Equations (B.5) and (B.6) are the working relations in the segmentation technique.
APPENDIX C

EXPERIMENTAL DETERMINATION OF EDGE CAPACITANCE OF RECTANGULAR MICROSTRIP PATCHES

The multiport network model of rectangular microstrip patches needs the knowledge of edge capacitance $C$ for the modeling of fringing field at the radiating edges. In this appendix, we present a method for the determination of edge capacitance from the measurement of resonance frequency of unloaded rectangular microstrip patches. Figure C.1 shows a rectangular microstrip patch loosely coupled to feed lines via a small gap of width $s$. At the resonance frequency of the dominant mode $TM_{10}$, the field is uniform along the patch width and hence the antenna can be modelled as a transmission line as shown in Fig C.1.

The MCN network represents the external coupling among the radiating edges of the patch. The $Y$-matrix characterizing the MCN network with respect to port 1 and 2 is computed as

$$ [Y]_{MCN} = \begin{pmatrix} Y_E & Y_m \\ Y_m & Y_E \end{pmatrix} \quad (C.1) $$

where $Y_E = G + i\omega C$, $G$ is the total conductance of an edge ($G = G_r + G_s$) and $C$ is the edge capacitance to be computed. $Y_m$ is the mutual coupling admittance between the two radiating edges of the patch. The $Y$-matrix for the equivalent transmission line with respect to port 1 and 2 is given by
Figure C.1 Equivalent transmission line model of unloaded rectangular microstrip patch
\[
[Y^*]_{TL} = \begin{pmatrix}
-iY_0 \cot(\beta a) & iY_0 / \sin(\beta a) \\
iY_0 / \sin(\beta a) & -iY_0 \cot(\beta a)
\end{pmatrix}
\]  \hspace{1cm} (C.2)

where $\beta$ and $Y_0$ are the propagation constant and the characteristic admittance of the equivalent transmission line of width $b$ respectively.

The input impedance $Y_{in}$ at port 1 with the port 2 open circuited is given by

\[
Y_{in} = \frac{[Y_E - iY_0 \cot(\beta a)]^2}{[Y_E - iY_0 \cot(\beta a)]} - \frac{[Y_m + iY_0 / \sin(\beta a)]^2}{[Y_E - iY_0 \cot(\beta a)]} \hspace{1cm} (C.3)
\]

The resonance condition for the circuit shown in Fig C.1 is that the imaginary part of $Y_{in}$ is equal to zero at resonance. The resonance frequency is determined experimentally by measuring both the reflection and transmission coefficients. At resonance, the reflection coefficient is minimum and the transmission coefficient is maximum. The value of the gap width $s$ in Fig C.1 is selected by increasing $s$ until no further change in the resonance frequency is observed. The variation of the edge capacitance with frequency is assumed to be linear around the measured frequency.
APPENDIX D

ELECTROMAGNETIC FIELDS OF A MAGNETIC DIPOLE OVER GROUND PLANE WITH A DIELECTRIC COVER LAYER

The electromagnetic (EM) fields of a magnetic dipole over a ground plane with a dielectric cover layer can be computed by using the z-component of Hertz potentials of electric type $\Pi_e$ and of magnetic type $\Pi_m$. The two scalar potential $\Pi_{e2}$ and $\Pi_{m2}$ are known as Whittaker potentials and totally describe the EM fields in the source free region above the ground plane [87]. The source, which is the magnetic dipole, is included as a part of electromagnetic field boundary condition as shown later. Fig D.1 shows a magnetic dipole over ground plane with a cover layer and located at the origin of the system of coordinates. The magnetic current density is expressed as

$$ \mathbf{J}_m = \bar{a}_z P_m \delta(x)\delta(y)\delta(z) \quad (D.1) $$

where $P_m$ is the dipole moment. The time dependence of the field is assumed to be of the form $e^{i\omega t}$ and will be omitted throughout the derivation. Since the configuration shown in Fig D.1 has no conducting boundaries in the $x$ and $y$ directions, solution of the EM fields (which satisfy Maxwell equations) is greatly simplified by defining a Fourier transform pair as
Figure D.1 Magnetic current element over ground plane with a dielectric cover layer

\[
\begin{align*}
\bar{f}(x, y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{f}(\alpha, \beta) e^{-ik_0(\alpha x + \beta y)} d\alpha d\beta \\
\tilde{f}(\alpha, \beta) &= \left(\frac{k_0}{2\pi}\right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) e^{ik_0(\alpha x + \beta y)} dx dy
\end{align*}
\]  
(D.2)

The components of the EM fields are related to \(\Pi_{ez}\) and \(\Pi_{mz}\) by

\[
\begin{align*}
\bar{E} &= \nabla \times \nabla \times (\Pi_{ez} \bar{a}_z) - ik_0 \eta_0 \nabla \times (\Pi_{mz} \bar{a}_z) \\
\bar{H} &= \nabla \times \nabla \times (\Pi_{mz} \bar{a}_z) + \frac{ik_0}{\eta_0} \varepsilon_r \nabla \times (\Pi_{ez} \bar{a}_z)
\end{align*}
\]  
(D.3)

In cylindrical coordinates, the z-components and the tangential components are
given by
\[
\begin{cases}
E_z = -\nabla_t^2 \Pi_{ez} \\
H_z = -\nabla_t^2 \Pi_{mz}
\end{cases}
\quad (D.4)
\]
\[
\begin{cases}
\overline{E}_t = \nabla_t (\frac{\partial}{\partial z} \Pi_{ez}) - ik_0 \eta_0 \nabla_t \times (\Pi_{mz} \bar{a}_z) \\
\overline{H}_t = \nabla_t (\frac{\partial}{\partial z} \Pi_{mz}) + \frac{ik_0}{\eta_0} \epsilon_r \nabla_t \times (\Pi_{ez} \bar{a}_z)
\end{cases}
\quad (D.5)
\]
where \(\epsilon_r\) is given by
\[
\epsilon_r = \begin{cases}
1; & z > d, \\
\epsilon_{r1}; & z \leq d.
\end{cases}
\quad (D.6)
\]
From (D.4), we notice that \(\Pi_{ez}\) describes TM-to-z mode \((H_z = 0)\) and \(\Pi_{mz}\) describes the TE-to-z mode (or \(E_z = 0\)). The Whittaker potentials \(\Pi_{ez}\) and \(\Pi_{mz}\) satisfy Helmholtz equation
\[
\left(\nabla_t^2 + \frac{\partial^2}{\partial z^2} + k^2\right)\Pi_{ez, mz} = 0
\quad (D.7)
\]
The presence of the excitation source is included in the boundary conditions for the EM fields. The boundary conditions for the EM fields at the source and on the ground plane is
\[
\bar{a}_z \times (\overline{E}_t^1 - \overline{E}_t^2) = -\overline{J}_m
\quad (D.8)
\]
The boundary conditions for the EM fields at the air-dielectric interface are
\[
\begin{cases}
\overline{E}_t^1 = \overline{E}_t^2 \\
\overline{H}_t^1 = \overline{H}_t^2
\end{cases}
\quad (D.9)
\]
The superscripts 1 and 2 denotes the upper and lower region of an interface respectively. Using Maxwell equations, the following relations among the tan-
gential components and the z-components of the EM fields are obtained

\[
\begin{align*}
\nabla_t \cdot \bar{E}_t &= -\frac{\partial}{\partial z} E_z \\
\nabla_t \cdot \bar{H}_t &= -\frac{\partial}{\partial z} H_z \\
\n\bar{a}_z \cdot \nabla_t \times \bar{E}_t &= -i k_0 \eta_0 H_z \\
\n\bar{a}_z \cdot \nabla_t \times \bar{H}_t &= +i \frac{k_0}{\eta_0} \epsilon_r E_z \\
\end{align*}
\]

\[ (D.10) \]

The boundary conditions in terms of \( E_z \) and \( H_z \) (For TM and TE modes) can be derived by multiplying both sides of (D.8) and (D.9) by \( (\nabla_t \cdot ) \) and also by \( (\bar{a}_z \cdot \nabla_t \times ) \). We note that both the divergence and the curl of the fields are needed in order to totally describe the tangential fields and hence the boundary conditions. Making use of relations (D.10), the following boundary conditions for \( E_z \) and \( H_z \) are obtained.

(i) TE mode case (boundary conditions for \( H_z \)):

\[
\begin{align*}
\frac{ik_0 \eta_0 H_z}{\partial z} &= -\nabla_t \cdot \bar{J}_m; \quad \text{at } z=0, \\
H_z \text{ and } \frac{\partial}{\partial z} H_z \text{ are continuous; at } z=d. \\
\end{align*}
\]

\[ (D.11) \]

(ii) TM mode case (boundary conditions for \( E_z \)):

\[
\begin{align*}
\frac{\partial}{\partial z} E_z &= \bar{a}_z \cdot \nabla_t \times \bar{J}_m; \quad \text{at } z=0, \\
\epsilon_r E_z \text{ and } \frac{\partial}{\partial z} E_z \text{ are continuous; at } z=d. \\
\end{align*}
\]

\[ (D.12) \]

Field solution in the Fourier transform plane:

Replacing \( \Pi_{ez} \) and \( \Pi_{mz} \) in (D.7) by their Fourier transform as defined in (D.2) and interchanging the order of spatial derivative and the spectral integration, Helmholtz equation in the spectral domain becomes

\[
\left[ \frac{\partial^2}{\partial z^2} + k_0^2 (\epsilon_r - \alpha^2 - \beta^2) \right] \bar{\Pi}_{ez,mz} = 0
\]

\[ (D.13) \]

Using (D.4), \( \bar{E}_z \) and \( \bar{H}_z \) are related to \( \bar{\Pi}_{ez} \) and \( \bar{\Pi}_{mz} \) in the spectral domain by
\[
\begin{align*}
\begin{cases}
\tilde{E}_z = k_0^2(\alpha^2 + \beta^2)\tilde{\Pi}_{ez} \\
\tilde{H}_z = k_0^2(\alpha^2 + \beta^2)\tilde{\Pi}_{mz}
\end{cases}
\tag{D.14}
\end{align*}
\]

Using (D.11), (D.12) and (D.14) the following boundary conditions for \(\tilde{\Pi}_{ez}\) and \(\tilde{\Pi}_{mz}\) are obtained

\[
\begin{align*}
\frac{\partial}{\partial z} \tilde{\Pi}_{ez} = \frac{ik_0\beta P_m}{4\pi^2(\alpha^2 + \beta^2)}; & \quad \text{at } z=0, \\
\epsilon_r \tilde{\Pi}_{ez} \text{ and } \frac{\partial}{\partial z} \tilde{\Pi}_{ez} \text{ are continuous;} & \quad \text{at } z=d.
\end{align*}
\tag{D.15}
\]

and

\[
\begin{align*}
\tilde{\Pi}_{mz} = \frac{\alpha I_m}{4\pi^2\eta_m(\alpha^2 - \beta^2)}; & \quad \text{at } z=0, \\
\tilde{\Pi}_{mz} \text{ and } \frac{\partial}{\partial z} \tilde{\Pi}_{mz} \text{ are continuous;} & \quad \text{at } z=d.
\end{align*}
\tag{D.16}
\]

General solutions for Helmholtz equation (D.13) for \(\tilde{\Pi}_{ez}\) and \(\tilde{\Pi}_{mz}\) are respectively

\[
\tilde{\Pi}_{ez} = \begin{cases}
\tilde{\Pi}_{e0} e^{-k_0 u_0 (z-d)}; & \quad z \geq d, \\
\tilde{\Pi}_{e1} [\sinh(k_0 u_1 z) + \Gamma_e \cosh(k_0 u_1 z)]; & \quad 0 \leq z \leq d.
\end{cases}
\tag{D.17}
\]

\[
\tilde{\Pi}_{mz} = \begin{cases}
\tilde{\Pi}_{m0} e^{-k_0 u_0 (z-d)}; & \quad z \geq d, \\
\tilde{\Pi}_{m1} [\sinh(k_0 u_1 z) + \Gamma_m \cosh(k_0 u_1 z)]; & \quad 0 \leq z \leq d.
\end{cases}
\tag{D.18}
\]

For the fields to satisfy the radiation condition at infinity \((z \to \infty)\), we need the following choice for \(u_0\)

\[
u_0 = \begin{cases}
+\sqrt{\alpha^2 + \beta^2 - 1}; & \quad \text{if } \alpha^2 + \beta^2 > 1, \\
+i\sqrt{1 - \alpha^2 - \beta^2}; & \quad \text{if } \alpha^2 + \beta^2 < 1.
\end{cases}
\tag{D.19}
\]

The sign of \(u_1 (= \sqrt{\alpha^2 + \beta^2 - \epsilon_r})\) is irrelevent in the computation of the EM fields. The constants \(\tilde{\Pi}_{e0}, \tilde{\Pi}_{e1}\) and \(\Gamma_e\) are obtained from the boundary conditions (D.15) as follows
\[
\begin{align*}
\Gamma_c &= -\frac{u_1 \cosh(k_0 u_1 d) + \epsilon_{r1} u_0 \sinh(k_0 u_1 d)}{D_{TM}} \\
\tilde{\Pi}_{e1} &= \frac{i \beta P_m}{4\pi^2(\alpha^2 + \beta^2)u_1} \\
\tilde{\Pi}_{e0} &= -\frac{\epsilon_{r1} u_1 \tilde{\Pi}_{e1}}{D_{TM}}
\end{align*}
\] (D.20)

where \(D_{TM}\) is given by

\[
D_{TM} = \epsilon_{r1} u_0 \cosh(k_0 u_1 d) + u_1 \sinh(k_0 u_1 d)
\] (D.21)

The constants \(\tilde{\Pi}_{m0}, \tilde{\Pi}_{m1}\) and \(\Gamma_m\) are obtained from satisfying the boundary conditions (D.16) as

\[
\begin{align*}
\Gamma_m &= -\frac{D_{TE}}{u_0 \cosh(k_0 u_1 d) + u_1 \sinh(k_0 u_1 d)} \\
\tilde{\Pi}_{m1} &= \frac{\alpha P_m}{4\pi^2\eta_0 \Gamma_m(\alpha^2 + \beta^2)} \\
\tilde{\Pi}_{m0} &= \frac{u_0 \tilde{\Pi}_{m1}}{u_0 \cosh(k_0 u_1 d) + u_1 \sinh(k_0 u_1 d)}
\end{align*}
\] (D.22)

where \(D_{TE}\) is given by

\[
D_{TE} = u_0 \sinh(k_0 u_1 d) + u_1 \cosh(k_0 u_1 d)
\] (D.23)

**Computations of \(\Pi_{ez}\) and \(\Pi_{mz}\) in the spatial domain**

Using the following transformations of coordinates

\[
\begin{align*}
\alpha &= \lambda \cos(t) \\
\beta &= \lambda \sin(t) \\
x &= \rho \cos(\phi) \\
y &= \rho \sin(\phi)
\end{align*}
\] (D.24)

and using the inverse Fourier transform defined in (D.2), \(\Pi_{ez}\) and \(\Pi_{mz}\) are given
by
\[
\begin{cases}
\Pi_{ez} = \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \tilde{F}_e(\lambda^2, z) e^{-ik_0\lambda\rho \cos(t-\phi)} \sin(t)(d\lambda)(dt) \\
\Pi_{mz} = \int_{-\infty}^{+\infty} \int_{0}^{2\pi} \tilde{F}_m(\lambda^2, z) e^{-ik_0\lambda\rho \cos(t-\phi)} \cos(t)(d\lambda)(dt)
\end{cases}
\]  \hfill (D.25)

where
\[
\tilde{F}_e(\lambda^2, z) = \begin{cases}
\frac{-i\epsilon_{e,1} P_m}{4\pi^2 \eta_0 D_{TM}} e^{-k_0u_0(z-d)}, & z \geq d, \\
\frac{-i P_m}{4\pi^2 \eta_0 D_{TM}} \{\epsilon_{r,1} u_0 \sinh[..] + u_1 \cosh[..]\}, & 0 \leq z \leq d.
\end{cases}
\]  \hfill (D.26)

and
\[
\tilde{F}_m(\lambda^2, z) = \begin{cases}
\frac{u_0 P_m}{4\pi^2 \eta_0 D_{TE}} e^{-k_0u_0(z-d)}, & z \geq d, \\
\frac{P_m}{4\pi^2 \eta_0 D_{TE}} \{u_0 \sinh[..] + u_1 \cosh[..]\}, & 0 \leq z \leq d.
\end{cases}
\]  \hfill (D.27)

where $[..] = k_0 u_1(d-z)$. The integration with respect to the variable $t$ can be carried out by making the change of variable $(t-\phi) = \theta$ and using the following identity [71,p.360]
\[
\int_{0}^{2\pi} e^{iz \cos(\theta)} \cos(n\theta) d\theta = i^n \pi J_n(z)
\]  \hfill (D.28)

where $J_n(z)$ is Bessel function of the first kind of order $n$. The following formulas are obtained for $\Pi_{ez}$ and $\Pi_{mz}$
\[
\begin{cases}
\Pi_{ez}(\phi, \rho, z) = -2i\pi \sin \phi \int_{0}^{\infty} \tilde{F}_e(\lambda^2, z) J_1(k_0\lambda\rho)(d\lambda) \\
\Pi_{mz}(\phi, \rho, z) = -2i\pi \cos \phi \int_{0}^{\infty} \tilde{F}_m(\lambda^2, z) J_1(k_0\lambda\rho)(d\lambda)
\end{cases}
\]  \hfill (D.29)

From expression (D.29) for $\Pi_{ez}$ and $\Pi_{mz}$, the electromagnetic field
components are computed using equations (D.4) and (D.5).

The functions $\tilde{F}_e(\lambda^2, z)$ and $\tilde{F}_m(\lambda^2, z)$ contain the multivalued function $u_0$ (defined in D.19). The integrals (D.29) have poles $\lambda_p$ when either $D_{TM}$ (given by D.21) or $D_{TE}$ (given by D.23) are zero. These poles are located along the real axis between 1 and $\sqrt{\varepsilon_{r1}}$. The complex $\lambda$-plane with the branch cuts (which uniquely define $u_0$) and the integration path is shown in Fig D.2.