MIMICAD TECHNICAL REPORT NO. 15

Effects of Packages on Spurious Couplings Among Microstrip Discontinuities

by

Sung H. Lee
K.C. Gupta

MIMICAD Center
Department of Electrical and Computer Engineering
University of Colorado
Boulder, Colorado 80309-0425

May 1992
This technical report forms a part of the final report for MIMICAD Center Project 90-1 entitled, "Numerical Modeling of Complex, Dielectric Loaded Microwave Packages" (Co-principal Investigators: David Chang, Dick Booton, and K.C. Gupta) carried out during the June 1990 - May 1992 time period. Other activities of this project based on edge-element analysis of microwave packages are reported separately.

Software based on the analysis presented in this report is documented in "Coupling Analysis with Package Program (CAP User's Guide" by Sung Lee, May 1992, which is included as an addendum to this rept.
# Contents

1. Introduction

1.1 Background .................................................. 1
1.2 Previous Work Related to the Effect of Packages on Spurious Spurious Couplings ........................................ 1
1.3 Approach Used in the Report ................................. 2
1.4 Organization of the Report .................................... 4

2. Planar Model and Segmentation Method for Impedance Matrix of a Given Microstrip Circuit .................................. 6

2.1 Planar Model and Segmentation Method ...................... 6
2.2 Equivalence Theorem to Evaluate Equivalent Magnetic Currents for Calculation of Fields in the Enclosure ............. 8

3. Coupling Between Discontinuities in a Rectangular Enclosure .... 11

3.1 Modal Fields for a Rectangular Enclosure and Normalization 12
3.1.1 Modal field expressions .................................. 13
3.1.2 Computation of normalization factors for modal fields 13
3.1.3 Numerical verification for normalization procedure ... 16
3.2 Total Fields Produced by a Magnetic Current Element .... 19
3.3 Computation of Modal Amplitude Coefficients for a Magnetic Current Excitation .................................... 20
3.3.1 Magnetic current vector .................................. 22
3.3.2 Amplitude coefficients for various modes ............... 22
3.4 Equivalent Dielectric Constants for the Cavity Model ...... 23
3.4.1 Motivation of calculating equivalent dielectric constants 24
3.4.2 Equivalent dielectric constant for a mode ............... 25
3.5 Convergence of Results of Total Fields ....................... 26
3.6 Induced Currents in Neighboring Discontinuities and Mutual Coupling Matrix .......................................... 28
3.7 Conductor Loss in Walls of the Package ...................... 29
3.8 Examples .................................................. 30
   3.8.1 Comparison between the Em and MNM results for the
case of two open ends inside a rectangular package 31
   3.8.2 Comparison of coupling coefficient $S_{13}$ between Em
and MNM programs for the case of two right-angled
bends .................................................. 33

4. Coupling Effects among Discontinuities in a Triangular Package 35
   4.1 Source-Free Fields .............................................. 36
   4.2 Normalization .................................................. 39
   4.3 Fields Produced by a Magnetic Current Element ............. 40

5. Concluding Remarks .............................................. 41
   5.1 Summary of Main Results ...................................... 41
      5.1.1 Coupling between discontinuities in a rectangular
package .................................................. 41
      5.1.2 Coupling effects between discontinuities in a
right-angled isosceles triangular package ....................... 42
      5.1.3 Software Development ...................................... 43
   5.2 Suggestions for the Follow-Up Work ......................... 44
      5.2.1 Computation of effective dielectric constant for
individual modes .............................................. 44
      5.2.2 Development of fields excited by a magnetic current
in packages with different geometries .......................... 45
      5.2.3 Investigation of coupling matrix [Y] with an enclosure
5.2.4 Investigation of coupling region in an enclosure .......... 46
      5.2.5 Software extensions ...................................... 46

References .................................................. 47

APPENDICES:
   A: Normalization For Source-Free Fields in a Rectangular
      Cavity .................................................. 49
   B: Normalization For Source-Free Fields in a
      Right-Angled Isosceles Triangular Cavity ................. 51
1. Introduction

1.1 Background

Geometrical discontinuities between distributed circuit elements, between lumped circuit elements, and between distributed and lumped circuit elements always exist in microwave circuits. Some of these discontinuities are: open circuits, short circuits, bends, step changes or junctions. In fact any deviation from a straight uniform microstrip line can be considered as a discontinuity. Most discontinuities produce undesirable fields as the results of scattering and reflection. The major effects of discontinuities on circuit performance are additional mismatches and additional signal loss in the circuit due to radiation losses and undesired interaction between different parts of the circuit due to external electromagnetic coupling. In this report we are interested in the spurious coupling caused by discontinuities. The coupling associated with these discontinuities is called parasitic (spurious), as it is not introduced intentionally. Coupling is the interaction between the two circuit elements. It generally occurs in an uncontrolled manner between the neighboring circuit components and is mainly caused by the discontinuities in the circuit. The coupling effect from discontinuities becomes more critical at higher frequencies because higher order modes can be excited much easier or when circuit density is increased so that discontinuities are located closer to each other. Incorporation of mutual coupling among the discontinuities can be modeled by computing the current induced at one discontinuity by a voltage (magnetic current) distribution at the other discontinuities.

In many practical applications, MICs or MMICs entail a metallic enclosure which serves the purposes of hermetic sealing, mechanical strength, electromagnetic shielding, connector mounting, and ease of handling of the module. Then packaging is absolutely necessary in many cases. However, using packages brings some undesired side effects; the presence of conducting top and side walls lowers both the characteristic impedance and the effective microstrip permittivity, which is due to the increased portion of electric field in the air region. Also, at microwave frequencies, the package walls can cause signal
reflection and signal loss. The electromagnetic energy also can be coupled to packaging modes in the package. Specially when a metal package is large enough to support resonant modes within the frequencies of operation of the enclosed circuit, the coupling effects between the circuit and these resonant cavity modes may disturb circuit operation considerably. The enclosure can be viewed as a resonant cavity with standing waves inside a cavity.

There are two aspects to be considered in designing the physical dimensions of the package enclosure. If the dimension is bigger and any one of resonant mode is excited, then the coupling effect is stronger than that in the absence of package resonance. On the other hand, if the dimensions are too small, then the characteristic parameters of the microstrip line would need to be modified. Thus it is desirable that an enclosure shouldn't be too small to affect the given circuit characteristic parameters over the tolerance limit and it shouldn't be too large to locate the self resonances of the cavity in the useful bandwidth of the circuit.

In this report we investigate the coupling effects from discontinuities inside an enclosure. We have studied two different geometries of the package; one of them is a rectangular cavity and the other is a right-angled isosceles triangular package. We have some analytical derivation and numerical examples for a rectangular cavity case in this report and mathematical derivation for the triangular case.

1.2 Previous work related to the effect of packages on spurious couplings

As discussed above, the effect of a metallic enclosure can be categorized in two parts:

1. Effects on the transmission line parameters of a microstrip line
2. Effects on the coupling between discontinuities of a circuit

There are several previous reports which have studied the characteristics of transmission line parameters of microstrip line in a closed metallic enclosure.
A part of them formulate the analytical solutions by assuming only the top and the bottom conductors and the others use numerical method. Usually, however, if the distance of the microstrip line to the package walls is greater than about five times of the substrate height then the effect on the characteristic parameters of the microstrip line is negligible. In such a case, the main effect of packaging is coupling generated by resonant modes of the enclosure.

There are relatively fewer research references for the study of packaging effects on the parasitic coupling of microstrip line circuits than those on the characteristics of transmission line parameters. These are summarized below.

A numerical algorithm to find resonant modes and the quality factors for a given metal rectangular enclosure is given in [1]. The paper suggests that using a dielectric substrate coated with a resistive film is an inexpensive alternative to the microwave absorber for reducing the circuit coupling to a resonant mode.

A moment method formulation which models microstrip circuits in a lossy enclosure is presented in [2](using rooftop current basis functions). This paper showed that the coupling of power to a resonant mode can be reduced by repositioning certain circuit features within the enclosure.

There are a number of papers [3,4] which present generalized techniques for the analysis of microwave circuits in an enclosure. Reference [3] presents a method based on an integral equation approach. The integral equation is derived by an application of the reciprocity theorem and then solved by the method of moments. It is shown that an erratic current condition and an optimum sampling range exist. Also, the convergence and the stability of the solution are explored.

1.3 Approach used in the report

The analysis of the coupling effects of packages developed in this project is approached by combining the method of planar analysis for microstrip line circuits and the method of modal expansion of fields for coupling computation inside a given enclosure to calculate scattering matrix \([S]\) for the network in the presence of an enclosure. In the method of planar analysis, it is assumed that the dimensions of the package are large enough so that the analysis for the circuit behavior in the open space is good approximation for the actual case of including a metallic enclosure. For modal expansion of fields in the enclosure, the equivalence theorem has been used to model the fringing fields at microstrip edges as equivalent magnetic currents which act as sources for fields inside a package. The magnetic current sources excite modal fields in the enclosure. Induced current in the neighboring ports are computed and the admittance matrix \([Y]\) is calculated to model the coupling. The overall approach for evaluating spurious coupling between two components A and B involves the following steps:

1. Divide Component A in regular segments and calculate \([Z_a]\) matrix.
2. Divide Component B in regular segments and calculate \([Z_b]\) matrix.
3. Represent the edge fields by small sections of magnetic current line sources of length \(dl\).
4. Calculate magnetic field excited inside the enclosure by a magnetic current element at the \(j\)-th subsection of component A.
5. Calculate the induced current \(J_s\) in the \(j\)th element of the component B.
6. Calculate the \([Y_{ij}]\) element of spurious coupling admittance matrix.
7. Use the same procedure to compute all other elements of the coupling matrix \([Y]\).
8. Invert the matrix \([Y]\) to obtain matrix \([Z_c]\).
9. Use the segmentation method to combine matrices \([Z_a]\) and \([Z_c]\) to yield the matrix \([Z_{ac}]\).
10. Use the segmentation method to combine matrix \([Z_{ac}]\) with \([Z_b]\) to produce \([Z]\)-matrix representation of the coupled elements.
11. Transform [Z]-matrix to [S]-matrix to yield performance including coupling between components A and B.

For the triangular case, only a right-angled isosceles triangle has been studied. Ray approach has been used to calculate field pattern inside the triangular wave guide and then boundary conditions are applied by placing two conducting walls in the z-direction. Once source-free modal fields are known, finding total fields $E$ and $H$ follows the same procedure as that used for the rectangular case.

1.4 Organization of the report

The rest of the report is organized in the following way:

Chapter 2 includes discussions about finding [Z] matrices for a given microstrip line circuit using planar model and segmentation method.

Chapter 3 presents the mathematical derivation and numerical implementation for the coupling effect between discontinuities in a rectangular enclosure.

Chapter 4 has a similar discussion as in Chapter 3 but for a right-angled isosceles triangular enclosure. However, a numerical implementation for triangular packages is not included in this report.

Chapter 5 contains a brief summary and conclusions and some remarks related to the further work.
2. Planar Model and Segmentation Method for Impedance Matrix of a given Microstrip Circuit

2.1 Planar Model and Segmentation Method

Microstrip line circuits may also be called planar circuits. Planar circuit is defined as the circuit which has dimensions comparable to the wavelength in two dimensions but a much smaller thickness in one direction. The concept of planar circuits has found many applications in microwave integrated circuits, reduced height waveguide circuits and microstrip antennas. The basic equation describing the electromagnetic field in the planar circuit is the two dimensional wave equation (Helmholtz equation). The circuit characteristics are obtained as the solution of the Helmholtz equation under the given boundary conditions.

Planar waveguide model is used for the analysis of planar circuits and plays a role of basis for the multiport network model [MNM]. The model consists of two parallel conductors bounded by magnetic walls in the transverse directions. The electric and magnetic fields inside the planar model are uniform in the transverse directions. The fringing fields are accounted for by an effective width of the line. The effective dimensions in the discontinuity region are obtained by extrapolating the effective edges for the connecting lines as shown in Figure 2-1. The dispersion effect can also be implemented easily in this model since \( Z_0 \) and \( \varepsilon_{re} \) are functions of frequency and \( W_{eff} \) is a function of \( Z_0 \) and \( \varepsilon_{re} \).

A network technique, called segmentation method, has been used to solve the complex geometries of given circuits. For example, when the given circuit has a composite geometry it can be broken into several simple structures for which Green's functions are available and so [Z] matrices can be found. Once the [Z] matrix characterization for each one of segments is known, the individual [Z] matrices can be combined together to produce the [Z] matrix characterization of the overall composite configuration. Figure 2-2 shows an example of segmentation method. A chamfered right-angled bend is segmented into three segments; two rectangular segments and a triangular segment.
Figure 2-1  : Effective dimensions for discontinuity area

Figure 2-2  : An example of segmentation
Line thickness and conductor loss from microstrip line are considered in the calculation of \([Z]\) matrices but not accounted for the calculation of coupling in an enclosure. Microstrip dispersion effect has not been implemented in the numerical computations presented in this report.

2.2 Equivalence theorem to evaluate equivalent magnetic currents for calculation of fields in the enclosure.

Equivalence theorem has been applied to model fringing fields as equivalent magnetic currents at the edges of the microstrip line circuits. From the Huygen's principle, these equivalent magnetic currents can be considered as sources for fields in the enclosure. This equivalent sources produce the same field distributions as the actual source does and will be used to evaluate modal fields in an enclosure later in Chapter 3.

For details, let's consider an example of a microstrip line circuit in a cavity as shown in Figure 2-3. Assume that the height of the substrate is small enough. Let's place an equivalent surface S-S as shown in Figure 2-3. Neglecting the thickness of the microstrip conductor, the level of the surface is the same as the conductor. Both surface electric current and surface magnetic current density vectors \(J_S\) and \(M_S\) can exist over the regions 1-2 and 3-4, while only electric current density \(J_S\) can exist in the region 2-3 since \(E_{tan}=0\). As \(H_{tan}\) is negligible over the surface 1-2 and 3-4, the \(J_S\) distribution over the surface can be ignored and only \(M_S\) can be considered in the region 1-2 and 3-4. Since we are not interested in the field distribution in the region below the surface and equivalence theorem allows us to assume any values for the \(E\) and \(H\) field in that region, we can use Love's equivalence principle. That case is equivalent to having a perfect conductor in the region below the surface S-S. Over the surface of perfect conductor, \(J_S\) induces images in the opposite direction so that it is cancelled out. Therefore, we have only \(M_S\) over the surface of a perfect conductor. The magnitude of the \(M_S\) is proportional to the \(E_{tan}\) over S. This is an example of equivalent magnetic current source. For coupling computation it is not necessary to compute the magnitude of the magnetic current explicitly because voltage is cancelled out in the computation of admittance. The general \(E\) field distribution
near the edge of the microstrip line in open space or in large enough enclosure is given in Figure 2-4.

Note that computations presented in this report are based on approximating the magnetic currents as line currents rather than a 2-dimensional distribution because the fringing field decays rapidly as it moves away from the edge as shown in Figure 2-4. Thus the fringing field is modeled as a pair of line currents at the edges of the microstrip line as shown in Figure 2-5.

The magnetic current is formulated to be a vector $\mathbf{M}$ such that

$$
\mathbf{M} = \mathbf{E}_{\text{tan}} \times \mathbf{n}
$$

(2.2.0.1)

where $\mathbf{n}$ is in z direction as in Figure 2-5 and $\mathbf{E}_{\text{tan}}$ is over the surface S-S. (Note that the vector notations $\mathbf{M}$ (Bold Face) and $\mathbf{M}$ are interchangeably used for the same vector. This notation is also used for other vectors throughout in this report.)

Figure 2-6 shows the final form of the model for calculating the package fields of the microstrip line circuit in an enclosure. Note that the enclosure dimension in the height direction is reduced by the amount of the substrate height. Later we need to consider equivalent dielectric constant for this model. This equivalent dielectric constant yields the same resonance frequencies as in the case of actual enclosure.
Figure 2-3: Equivalent surface on a microstrip circuit

Figure 2-4: E field distribution on an edge of a microstrip circuit

Figure 2-5: A pair of equivalent magnetic current on an edge of a microstrip circuit

(a) modeling  (b) original

Figure 2-6: Modeling of a microstrip line in an enclosure
3. Coupling between discontinuities in a rectangular enclosure

In this chapter, coupling between discontinuities in a rectangular enclosure is modelled as an admittance matrix $[Y]$ using modal fields expansion. The geometry of the enclosure and the coordinate system are shown in Figure 3-1.

![Figure 3-1: Rectangular cavity geometry](image)

The source-free modal fields are derived first by solving a wave equation with appropriate boundary conditions. These individual modes are normalized to form a complete orthonormal set.

The equivalent magnetic current sources representing microstrip circuit edge fields are located in the x-y plane and expressed as a delta function. This magnetic current is used to compute amplitude of each mode. Then the total $\mathbf{E}$ and $\mathbf{H}$ fields in an enclosure are obtained from an infinite summation of source-free modal fields multiplied by amplitude of respective modes.
The total $H$ field induces surface electric current in the metallization in the neighborhood of a discontinuity. Then the coupling matrix $[Y]$ is obtained as the ratios of voltages and currents for a given pair of ports.

Practical cavities, being made of metal, dissipate energy in their walls. This finite loss limits the quality factor $Q$ of the enclosure to a finite value. The derivation of equivalent loss tangent as the inverse of a $Q$ factor is described in this chapter. However, in the numerical implementation in this report, the wall losses are incorporated by taking an arbitrary loss tangent value.

Two examples for coupling among discontinuities are given in this Chapter. One of them has two open-ends microstrip line structure in a rectangular enclosure and the other has two right-angled bends in another rectangular enclosure. Their coupling coefficient $S_{12}$ or $S_{13}$ is computed for each example from approach developed here and compared to the results obtained from Em program.

3.1 Modal fields for a rectangular enclosure and normalization

The wave equation is solved for z-components of $E$ and $H$ fields first. Then the other components of $E$ and $H$ are obtained by solving Maxwell’s equations. The field configurations inside a rectangular enclosure of Figure 3-1 may be either TE or TM mode with respect to $z$ direction. These fields are derived in a manner similar to that used waveguide fields. The difference is that we must allow for standing waves, instead of traveling waves along one of the directions ($z$ direction in this report). We must also impose additional boundary conditions along the front and back walls. To meet this boundary conditions the planes must be separated by an integral number of half-wavelengths. This will only be the case at certain frequencies. Thus waves can be excited in the enclosure, dominantly only at a number of discrete frequencies, where the enclosure resonates.

The normalization factors $A$ and $B$ are obtained by evaluating the energy in the enclosure, and normalizing to a unit level. In this way the values of the field
components can be obtained for a given amount of electromagnetic energy in the enclosure.

Coefficients of \( \mathbf{E} \) field for randomly chosen mode numbers for the purpose of verifying the program are given in Table 3-1. If these values are substituted back into the integration process the electrical energy of each mode will be 1.

3.1.1 Modal field expressions

The wave equation for this system may be written as:

\[
(\nabla^2 + \omega^2 \mu \varepsilon) \begin{pmatrix} E_z \\ H_z \end{pmatrix} = 0
\]

\((3.1.1.1)\)

where \( z \) axis is chosen as shown. This wave equation can be solved by the separation of variables with proper boundary conditions to get the solutions

\[
E_z = A \sin(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z)
\]

\((3.1.1.2)\)

\[
H_z = B \cos(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z)
\]

\((3.1.1.3)\)

where

\[
\alpha_1 = \frac{m \pi}{a}, \quad \alpha_2 = \frac{n \pi}{b}, \quad \alpha_3 = \frac{p \pi}{c}
\]

\((3.1.1.4)\)

and

\( A \) and \( B \) are arbitrary constants.

Also, \( a, b, \) and \( c \) are the dimensions of the enclosure in \( x, y \) and \( z \) direction respectively and \( m, n, \) and \( p \) are integers.
In the waveguide case, the characteristic equation is solved for the phase constants at arbitrary frequencies. In the enclosure case, the characteristic equations are solved for eigenvalues of the frequencies (resonance frequencies).

The transverse fields components can be obtained in terms of \( E_z \) and \( H_z \) by applying Maxwell's equations to get the following:

\[
E_x = \frac{1}{k^2} \left( -\alpha_1 \alpha_3 A + j \omega \mu \alpha_2 B \right) \cos(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \tag{3.1.1.5}
\]

\[
E_y = \frac{1}{k^2} \left( \alpha_2 \alpha_3 A - j \omega \mu \alpha_1 B \right) \sin(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \tag{3.1.1.6}
\]

\[
H_x = \frac{1}{k^2} \left( j \omega \alpha \alpha_2 A - \alpha_1 \alpha_3 B \right) \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \tag{3.1.1.7}
\]

\[
H_y = \frac{1}{k^2} \left( -j \omega \alpha \alpha_2 A - \alpha_2 \alpha_3 B \right) \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \tag{3.1.1.8}
\]

where

\[
k^2 = \alpha_1^2 + \alpha_2^2 \tag{3.1.1.9}
\]

A time variation \( e^{j \omega t} \) is understood for all field components. \( A(\text{V/m}) \) and \( B(\text{A/m}) \) are normalizing factors. Whatever the relative values of \( A \) and \( B \), the field components satisfy Maxwell's equations. When \( A = 0 \), it is TE mode fields; and TM mode fields when \( B = 0 \).

By taking normalization factors either \( A=0 \) or \( B=0 \), the fields may be classified as TE and TM modes as given in the following.

(a) TM mode fields
\[ E_x = \frac{1}{k^2} (-\alpha_1 \alpha_3) A \cos(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \]  
\[ (3.1.1.10) \]

\[ E_y = \frac{1}{k^2} (-\alpha_2 \alpha_3) A \sin(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \]  
\[ (3.1.1.11) \]

\[ E_z = A \sin(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]  
\[ (3.1.1.12) \]

\[ H_x = \frac{1}{k^2} (j \omega \varepsilon \alpha_2) A \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \]  
\[ (3.1.1.13) \]

\[ H_y = \frac{1}{k^2} (-j \omega \varepsilon \alpha_1) A \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]  
\[ (3.1.1.14) \]

\[ H_z = 0 \]  
\[ (3.1.1.15) \]

(b) TE mode fields

\[ E_x = \frac{1}{k^2} (j\omega \mu_0 \alpha_2) B \cos(\alpha_1 x) \sin(\alpha_2 y) \sin(\alpha_3 z) \]  
\[ (3.1.1.16) \]

\[ E_y = \frac{1}{k^2} (-j\omega \mu_0 \alpha_1) B \sin(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \]  
\[ (3.1.1.17) \]

\[ E_z = 0 \]  
\[ (3.1.1.18) \]

\[ H_x = \frac{1}{k^2} (-\alpha_1 \alpha_3) B \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \]  
\[ (3.1.1.19) \]

\[ H_y = \frac{1}{k^2} (-\alpha_2 \alpha_3) B \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]  
\[ (3.1.1.20) \]
\[ H_z = B \cos(\alpha_1 x) \cos(\alpha_2 y) \sin(\alpha_3 z) \] (3.1.1.21)

The constants A and B will be normalized to the unit value later in Sec 3.1.2. For the solutions of the wave equation with appropriate boundary conditions the indices m, n, and p can be any integers. However, not all modes can exist. Examination of the field components shows that for TM mode all fields components vanish if m or n is equal to zero, while for TE modes all the field components vanish if p=0. Also, no new information is obtained by taking negative values of m,n, and p. Therefore we limit them to positive or zero values.

It is proven in the numerical implementation that if any two of the indices m, n, and p are equal to zero, then all fields vanish; and if p = 0 then only TM mode fields exist, and if m or n is equal to zero then only TE mode fields exist.

3.1.2 Computation of Normalization factors for modal fields

Let's consider the enclosure structure of Figure 3-1 enclosing a homogeneous and isotropic medium with dielectric constant \( \varepsilon \). We wish to normalize the mode vectors such that the following orthonormality relationships are satisfied.

\[
\int_{v} \varepsilon \mathbf{E}_i^* \mathbf{E}_j \, dv = \begin{cases} 0 & \text{if } i \neq j \\ 1 & i = j \end{cases}
\] (3.1.2.1)

where \( v \) is the volume of the enclosure and \( i \) and \( j \) are mode indices. This integral can be derived from Maxwell equations

\[
\nabla \times \mathbf{E}_i = -j \omega \mu \mathbf{H}_i \quad \text{and} \quad \nabla \times \mathbf{H}_i = j \omega \varepsilon \mathbf{E}_i.
\] (3.1.2.2)

Normalizing \( \mathbf{E} \) also normalizes the \( \mathbf{H} \) because
\[
\int_v \varepsilon |\vec{E}_i|^2 \, dv = \int_v \mu |\vec{H}_i|^2 \, dv
\] (3.1.2.3)

That is, the time-average electric and magnetic energies are equal. Hence the orthonormality of \( \vec{H} \) corresponding to the orthonormal \( \vec{E} \) are

\[
\int_v \mu \vec{H}_i^* \vec{H}_j \, dv = \begin{cases} 
0 & \text{if } i \neq j \\
1 & \text{if } i = j
\end{cases}
\] (3.1.2.4)

Note that in general each set of indices \( m, n \) and \( p \) represents two modes \( \text{TE}_{mnp} \) or \( \text{TM}_{mnp} \). In the following, the results from the derivation of normalization are given.

(1) TM mode case

By integrating and completing the algebra necessary for the integral equation,

\[
\varepsilon \int_v |\vec{E}|^2 \, dv = 1
\] (3.1.2.5)

we obtain the following results in various possible cases:

(a) when \( m, n \neq 0 \) and \( p = 0 \)

\[
\varepsilon \left( \frac{2}{4} \frac{a \cdot b \cdot c}{4} \right) = 1
\] (3.1.2.6)

Thus, by solving the equation for \( A \) we get

\[
A = \frac{2}{\sqrt{a \cdot b \cdot c \cdot \varepsilon}}
\] (3.1.2.7)
(b) when m, n and p ≠ 0

\[
\varepsilon \left( \frac{a^2 b c}{8} + \frac{a^2 b c \alpha_1 \alpha_3^2}{k^4} + \frac{a^2 b c \alpha_2 \alpha_3^2}{k^4} \right) = 1
\]  

(3.1.2.8)

Thus

\[
A = \frac{2\sqrt{2}k}{\sqrt{a b c \varepsilon \omega \sqrt{\mu \varepsilon}}}
\]  

(3.1.2.9)

c) otherwise

\[A = 0\]  

(3.1.2.10)

(2) TE mode case

By taking E field components for TM mode and following the same procedure as in TM case, the results are given in the following.

(a) when m = 0, n and p ≠ 0

\[
\varepsilon \left( \frac{\omega_0^2 \mu \alpha_2}{k^4} \frac{a b c}{4} \right) = 1
\]  

(3.1.2.11)

Thus by solving for B

\[
B = \frac{2k^2}{\sqrt{a b c \mu (\alpha_2^2 + \alpha_3^2) \alpha_2}}
\]  

(3.1.2.12)

(b) when n=0, m and p ≠ 0
\[ B = \frac{2k^2}{\sqrt{abc \mu (\alpha_1 + \alpha_3)}} \alpha_1 \]  \hspace{1cm} (3.1.2.13)

(c) when \( m, n, \) and \( p \neq 0 \)

\[ \varepsilon \left( \frac{2 \omega_1^2 \mu \alpha_2^2 abc}{4K} \right) = 1 \]  \hspace{1cm} (3.1.2.14)

Thus

\[ B = \frac{2\sqrt{2}k}{\sqrt{abc \mu \omega_1 \mu \varepsilon}} \]  \hspace{1cm} (3.1.2.15)

d) otherwise

\[ B = 0 \]  \hspace{1cm} (3.1.2.16)

3.1.3 Numerical Verification for the normalization procedure

The numerical implementation has been verified by reversing the procedures of the normalization. That is, numerical values for each part of the computation have been plugged back into the normalization integral expression to get back unit values which represent electric and magnetic energies. The Table 3-1 shows the numerical values of coefficients of each component of \( E \) field computed from the program for each mode.
<table>
<thead>
<tr>
<th>mode</th>
<th>Square of absolute value of Coeff. of $E_x$ comp.</th>
<th>Square of absolute value of Coeff. of $E_y$ comp.</th>
<th>Square of absolute value of Coeff. of $E_z$ comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM $11_0$</td>
<td>0</td>
<td>0</td>
<td>3.076186E16</td>
</tr>
<tr>
<td>TE $01_1$</td>
<td>3.07618E16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TM $11_1$</td>
<td>1.77643E16</td>
<td>3.01973E16</td>
<td>1.356199E16</td>
</tr>
<tr>
<td>TE $11_1$</td>
<td>3.87362E16</td>
<td>2.27875E16</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3-1

The kinds of mode indexes in Table 3-1 have been selected to cover many general cases. Magnetic energy is also proved to be 1 by using $H$ field instead of $E$ field. It may be noted that

\[ \int_0^a \sin^2 \left( \frac{m \pi}{a} x \right) dx = \int_0^a \cos^2 \left( \frac{m \pi}{a} x \right) dx = \frac{a}{2} \]  

(3.1.2.17)

is used in the integration process.

3.2 Total fields produced by a magnetic current element.

In Chapter 2, we obtained a rectangular enclosure model with equivalent magnetic current sources from a given physical enclosure package containing a microstrip line circuit. For this enclosure model the total fields $E$ and $H$ are expressed as the summation of mode fields multiplied by an amplitude coefficients for each mode. A general geometry of a cavity with magnetic current source is shown in Figure 3-2.

Let total field $H$ be an infinite series of modal fields $H_i$'s with unknown corresponding constant $A_i$'s as given below:

\[ \vec{H} = \sum_i A_i \vec{H}_i \]  

(3.2.0.1)
Substituting this $\mathbf{H}$ into the wave equation,

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{H} \right) - \omega^2 \mu \mathbf{H} = -j \omega \mathbf{M}$$

(3.2.0.2)

Since for all modes denoted by integers $i$, it is true when there is no source that

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{H} \right) = \omega_i^2 \mu \mathbf{H}_i$$

(3.2.0.3)

we can have the following expression by substitution

$$\sum_i A_i (\omega - \omega_i)^2 \mu \mathbf{H}_i - j \omega \mathbf{M}$$

(3.2.0.4)

Then by using the orthonormality of $\mathbf{H}_i$'s

$$A_i = \frac{j \omega}{\omega - \omega_i} \int_v \mathbf{M}^* \mathbf{H}_i^* \, dv$$

(3.2.0.5)

Therefore, the total fields $\mathbf{E}$ and $\mathbf{H}$ are
\[ \vec{E} = \sum \frac{j \omega_i \vec{E}_i}{\omega - \omega_i} \int_v \vec{M} \circ \vec{H}_i^* \, dv \]  
\[ \vec{H} = \sum \frac{j \omega \vec{H}_i}{\omega - \omega_i} \int_v \vec{M} \circ \vec{H}_i^* \, dv \]

where

\( \vec{M} \) is the magnetic current source and
\( \vec{H}_i^* \) is the complex conjugate of \( \vec{H}_i \).

3.3 Computation of modal amplitude coefficients for a magnetic current excitation

In this section we want to calculate amplitudes of individual modes excited by a magnetic current source. For this purpose, magnetic current is formulated as a vector in the space and then this vector is integrated over the volume of the enclosure after it is dot-produced with the conjugate of the modal field \( \vec{H}_i \).

3.3.1 Magnetic Current Vector

As it is mentioned earlier, the edge voltages of the microstrip line circuits are modeled as equivalent magnetic currents. The edge voltage is equivalent to the integral of tangential components of fringing \( \vec{E} \) fields. This magnetic current vector itself can be formulated as 3-dimensional. However, this source is assumed to reside on the bottom conductor at \( z=0 \) in our model. For this case, the \( z \) component of \( \vec{H} \) field is always zero. Therefore, we need to concern only on the \( x \) and \( y \) components.

\[ \vec{M} = a_x M_x + a_y M_y \]  

where
\[ M_x = -V_o \sin \theta \delta(x - a_o) \delta(y - b_o) \delta(z) \text{ if } a_o - \frac{w |\sin \theta|}{2} < x < a_o + \frac{w |\sin \theta|}{2} \tag{3.2.1.2} \]

\[ M_y = V_o \cos \theta \delta(y - y') \delta(x - a_o) \delta(z) \text{ if } b_o - \frac{w |\cos \theta|}{2} < y' < b_o + \frac{w |\cos \theta|}{2} \tag{3.2.1.3} \]

where

\( a_o \) and \( b_o \) are \( x, y \) locations of source element, \( \theta \) is defined as shown in Figure 3-3, and \( V_o \) is the magnitude of the edge voltage at the port location.

![Figure 3-3](image)

Figure 3-3  How angle \( \theta \) is defined

3.3.2 Amplitude Coefficients for various modes

The amplitudes of modes excited by the magnetic current source depend mainly on the two factors. One of them is operating frequency; how close it is to a resonance frequency. The other is the location of the source point. The excitation depends on the value of the modal field \( H_i \) at the location of source point.

Amplitude coefficient for each mode is calculated by carrying the following integration:
\[ \frac{j \omega}{2} \int_{\omega - \omega_i}^{\omega + \omega_i} M^* H^*_i \, dv = \frac{j \omega}{2} \left( \int_v M_x H_x^* \, dv + \int_v M_y H_y^* \, dv \right) \]  

(3.3.2.1)

where

\[ H_x = H_1 \sin(\alpha_1 x) \cos(\alpha_2 y) \cos(\alpha_3 z) \]  

(3.3.2.2)

\[ H_y = H_2 \cos(\alpha_1 x) \sin(\alpha_2 y) \cos(\alpha_3 z) \]  

(3.3.2.3)

By replacing \( x = a_o \), \( y = b_o \), and \( z = 0 \) in the expressions for \( H_x \) and \( H_y \) and then carrying out integrations for \( x \) and \( y \) components separately, we get the results of the following:

\[ \int_v M_x H_x^* \, dv = -\frac{H_1}{\alpha_1} 2V_o \sin \theta \cos(\alpha_2 b_o) \sin(\alpha_1 a_o) \sin \left( \frac{\alpha_1 d l_x}{2} \right) \]  

(3.3.2.4)

\[ \int_v M_y H_y^* \, dv = \frac{H_2}{\alpha_2} 2V_o \cos \theta \cos(\alpha_1 a_o) \sin(\alpha_2 b_o) \sin \left( \frac{\alpha_2 d l_y}{2} \right) \]  

(3.3.2.5)

where

\[ d l_x = w \sin \theta, \quad d l_y = w \cos \theta \]  

(3.3.2.6)

with \( w \) as length of the magnetic current element and \( \theta \) defined as in Fig 3-3.

### 3.4 Equivalent Dielectric Constants for the Cavity Model

#### 3.4.1 Motivation of calculating equivalent dielectric constant

Let's first compare our model enclosure Figure 2-6(a) to the actual enclosure Figure 2-6(b). First we can notice that the height of the model enclosure is shorter than that of actual enclosure as much as the height of the substrate height. The reason was we put an equivalent surface at the level of conductor of microstrip line. We neglected the thickness of the conductor. Second, the model enclosure is filled with homogeneous dielectric while the actual enclosure is filled with two different dielectric materials. By remembering
that resonance frequencies are functions of enclosure dimensions, it is obvious that the resonance frequencies are different between in the model and in the actual. In order to take account of the effect of the difference on resonance frequencies in the modeling, we need to find equivalent dielectric constant with which the model enclosure will produce the same resonance frequencies as the original enclosure.

3.4.2 Equivalent dielectric constant for a mode

Let $\varepsilon_{req}$ be the equivalent dielectric constant for the enclosure model. The total fields are dominated by a resonant mode with the closest resonance frequency to a given operating frequency. For evaluation of $\varepsilon_{req}$, we first consider effective dielectric constant and then take account of the difference of height. In this report, $\varepsilon_{req}$ has the same value as $\varepsilon_{re}$ and the difference of height of the enclosure is not counted. Because of this approximation for $\varepsilon_{req}$, the resonance frequencies in the model are higher than the those from the Em program. This is shown in Table 3-2. Note that $\varepsilon_{re}$ is defined to be effective dielectric constant for a cavity filled with inhomogeneous materials and that the effective dielectric constants for the modes in the actual enclosure can be all different from each other because the mode field patterns are all different among the modes.

In the computation of $\varepsilon_{re}$, when the field distribution is relatively simple for a given mode, we can get good approximation to $\varepsilon_{re}$ for the mode. For example, TM$_{110}$ mode has $E$ field distribution given in the following:

$$E_z = \sin\left(\frac{\pi}{a}x\right) \sin\left(\frac{\pi}{b}y\right)$$  \hspace{1cm} (3.4.2.1)

As seen from Equation (3.4.2.1), the electrical field is vertical from top to bottom conductors normally and becoming zero at the side walls as required by the perfect conductor. The magnetic field lines are in horizontal (x-y) plane and surround the vertical displacement current resulting from the time rate of changes of $E_z$. There are equal and opposite charges on top and bottom conductors.
because of the normal electric field entering here. Thus the top and bottom conductors act as capacitor plates.

For this mode, effective dielectric constant is defined in terms of effective capacitance concept with uniform E field between the top and bottom plates, so that we can calculate equivalent capacitance for this parallel plate filled with inhomogeneous medium. Two examples of resonance frequency calculated based on \( \varepsilon_{re} \) calculations are give in Table 3-2. One of them is \( \text{TM}_{110} \) mode and the other is \( \text{TE}_{011} \) mode. The detailed parameters and dimensions are given in Sec. 3.8. Resonance frequencies obtained from electromagnetic simulation program Em are also included in this Table.

<table>
<thead>
<tr>
<th>MNN</th>
<th>Em</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{TM}_{110} )</td>
<td>( \text{TE}_{011} )</td>
</tr>
<tr>
<td>7.9602 GHz</td>
<td>9.7937 GHz</td>
</tr>
<tr>
<td>( \text{TM}_{110} )</td>
<td>( \text{TE}_{011} )</td>
</tr>
<tr>
<td>7.9600 GHz</td>
<td>9.7796 GHz</td>
</tr>
</tbody>
</table>

Table 3-2

As shown in the Table, resonance frequencies have differences less than 0.2 MHz for \( \text{TM}_{110} \) and 15 MHz for \( \text{TE}_{011} \).

As mentioned earlier, values of \( \varepsilon_{re} \) are all different for various modes. At the resonance frequency, the resonant mode is dominating other modes so that \( \varepsilon_{re} \) for the resonant mode can be used for computations for other modes in the series. Possibilities of further refinement of this model are discussed in Chapter 5.

### 3.5 Convergence of results for total fields

The convergence of results has been investigated for the MNN program. Since the total fields are infinite series, they are numerically obtained by a finite number of elements of the series. However, we need to ensure the convergence of the sum.
In the numerical implementation process, the convergence is dependent on the operating frequency and the number of terms of the series. Generally speaking, the series converges very fast if the operating frequency is close to a resonance frequency since there will be at least one dominant mode. In such cases, the series converges with about 10, 10, 10 terms for each index m, n, and p around the resonant mode. The error is less than 0.1 %. When the operating frequency is away from a resonance frequency, the convergence requires more terms in the summation. That means we need more terms in the series to ensure convergence. The following Tables show some examples of convergence. The Table 3-3 is for the case of two Open-Ends and Table 3-4 is for Right-Angled Bends. The parameters and the geometries are given in Sec 3.8 later. These Tables show the dependency of convergence on the operating frequency and the number of modes in the summation. The sampled frequencies are around the two resonant frequency $F=7.9602$ GHz of TM$_{110}$ mode and $F=8.936$ GHz of TE$_{011}$ mode.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>p</th>
<th>$S_{12}$ at $F=7.6$ GHz</th>
<th>$S_{12}$ at $F=7.95$ GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>-84.03 (dB)</td>
<td>-50.86 (dB)</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>-82.64</td>
<td>-50.86</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>40</td>
<td>-82.17</td>
<td>-50.85</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>80</td>
<td>-81.92</td>
<td>-50.84</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>-83.02</td>
<td>-50.87</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>10</td>
<td>-82.92</td>
<td>-50.87</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>10</td>
<td>-82.91</td>
<td>-50.87</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>10</td>
<td>-88.43</td>
<td>-50.97</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>10</td>
<td>-85.94</td>
<td>-50.93</td>
</tr>
<tr>
<td>80</td>
<td>10</td>
<td>10</td>
<td>-86.03</td>
<td>-50.94</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>80</td>
<td>-84.03</td>
<td>-50.90</td>
</tr>
</tbody>
</table>

Table 3-3

From these two Tables, it is concluded that the series can be approximated with 10 number of terms for each index when the operating frequency is close to
a resonance frequency. When the operating frequency is away from a resonance frequency so that the s-parameter values are smaller than -50 dB, it is better to increase the number of terms in the series computation.

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>p</th>
<th>$S_{12}$ at F=8.8 GHz</th>
<th>$S_{12}$ at F=8.9 GHz</th>
<th>$S_{12}$ at F=8.93 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
<td>-54.81 (dB)</td>
<td>-43.92 (dB)</td>
<td>-28.85 (dB)</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>-54.71</td>
<td>-43.89</td>
<td>-28.85</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>40</td>
<td>-54.65</td>
<td>-43.88</td>
<td>-28.85</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>80</td>
<td>-54.61</td>
<td>-43.86</td>
<td>-28.84</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>-54.74</td>
<td>-43.90</td>
<td>-28.85</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>10</td>
<td>-54.73</td>
<td>-43.90</td>
<td>-28.85</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>10</td>
<td>-54.76</td>
<td>-43.91</td>
<td>-28.85</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>10</td>
<td>-54.91</td>
<td>-43.95</td>
<td>-28.86</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>10</td>
<td>-54.99</td>
<td>-43.97</td>
<td>-28.86</td>
</tr>
<tr>
<td>80</td>
<td>10</td>
<td>10</td>
<td>-55.03</td>
<td>-43.98</td>
<td>-28.86</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>80</td>
<td>-55.08</td>
<td>-44.00</td>
<td>-28.87</td>
</tr>
</tbody>
</table>

Table 3-4

3.6 Induced currents in a neighboring discontinuities and mutual coupling matrix

As discussed in Sec. 3.2, the amplitude of excitation of magnetic current is multiplied to the source-free mode fields. The summation of all the modes generates total fields in the enclosure. The magnetic fields induces an electric surface current in the neighboring circuit. The induced current density is expressed as

$$\vec{J} = \vec{n} \times \vec{H}$$  \hspace{1cm} (3.6.0.1)
In the coupling network matrix \([Y]\), the element \(y_{ij}\) is obtained from the current induced on the \(j\)-th subsection of the circuit as a result of a voltage \(V_i\) at the \(i\)-th subsection. The current density is assumed to be uniform along the subsection. We have

\[
y_{ij} = \frac{J_i \cdot dl_j}{V_i} \tag{3.6.0.2}
\]

where \(dl_j\) is the width of the \(j\)-th subsection. Since \([Y]\) is independent of magnitude of voltages, \(V_i\) is taken to be unity for computations of \(y_{ij}\)'s.

3.7 Conductor loss in walls of the package.

The conductor loss in walls can be incorporated into the model by an equivalent loss tangent by finding quality factor \(Q\) of the enclosure. This \(Q\) factor is the inverse of loss tangent.

The quality factor \(Q\) for the cavity is proportional to volume and inversely proportional to surface. Resonant mode dominates the loss from the walls so that equivalent loss tangent values is calculated for various resonant modes. The process of computing equivalent loss tangent is given in the following.

First, power dissipated in the walls is calculated by integrating tangential components of \(H\) fields from six conducting walls

\[
P_d = \frac{R_s}{2} \int_{\text{walls}} |\vec{H}|^2 \, ds \tag{3.7.0.1}
\]

where

\[
R_s = \sqrt{\frac{\omega \mu}{2 \sigma}} \tag{3.7.0.2}
\]
which is the surface resistivity of the walls.

Also we need to know about the energy stored in the enclosure which may be computed at the instant time when \( E \)-field is maximum and \( H \)-field is zero. It is obtained by integrating the \( E \) field in the volume.

\[
W_e = \frac{\varepsilon}{4} \int \frac{-2}{v} dv
\]

(3.7.0.3)

Quality factor of the enclosure can be calculated as shown below. Its inverse is the equivalent loss tangent.

\[
Q = \frac{2 \omega_r W_e}{P_d} = \frac{1}{\tan \delta}
\]

(3.7.0.4)

This loss tangent value is used to find the imaginary part of the complex dielectric constant as:

\[
\varepsilon = \varepsilon - j \varepsilon = \varepsilon (1 - j \tan \delta)
\]

(3.7.0.5)

3.8 Examples

Two different circuits in the different dimensions of rectangular packages are considered as examples; two open-ends and two right-angled bends. The transmission coefficients \( S_{12} \) or \( S_{13} \) is computed and compared with the results from electromagnetic simulation using Em code. Note that Em program performs an electromagnetic analysis of a microstrip circuit to calculate S-parameters by solving the current distribution in the entire microstrip metallization while MNM program computes S-parameters by setting ports around the discontinuity area only. This means that coupling values from Em is the result from the entire metallization and coupling from MNM is the result just from the discontinuity. MNM approach assumes that coupling effect is dominated from the discontinuity area and it is cancelled in other region.
3.8.1 *Comparison between the Em and MNM results for the case of two open-ends inside a rectangular package*

The comparison has been made for a case of two open-ends microstrip lines in a rectangular package. The transmission coefficient $S_{21}$ has been computed from both the Em and MNM programs. The frequency range of analysis covers two resonance frequencies. One of them is TM$_{110}$ and the other is TE$_{011}$ mode. In the calculation of resonance frequency of the enclosure, an effective dielectric constant value for the package is used. This effective dielectric constant of the package is computed by considering dielectric substrate located between the top and bottom conductors. The conductor loss is accounted for in the microstrip circuit. The wall losses are not included in the present computations. The conductivity value used for the microstrip line is taken to be 5.8 E7 Simens/meter. Comparisons of the results in two different frequency ranges are plotted in Figures 3-4 and 3-5 with parameters used for the computations given in Table 3-5.

Figure 3-4 shows the comparison near the TM$_{110}$ resonance frequency. The graphs “em3105.dat” and “op3n.dat” show the $S_{21}$ values computed from the Em and MNM programs, respectively. At the peak of resonance curve the $S_{21}$ value using Em is -0.5 dB and the corresponding value using MNM approach is -0.28 dB. Off the resonance frequency, the differences between the two results are about 10 to 13 dB. The resonance frequencies differ by about 0.2 MHz, perhaps because of the effective dielectric constant approximation used.

Figure 3-5 shows the comparison in the neighborhood of TE$_{011}$ resonance frequency. “Em105x.dat” and “op3ox.dat” curves represent Em and MNM results, respectively, as in Figure 3-5. The $S_{21}$ value using Em is -0.06 dB and corresponding value using the MNM approach is -1.7 dB at the resonance frequency. Away from the resonance frequency, the difference between the two programs is about 13 to 16 dB. The resonance frequencies in this case differ by about 20 MHz, again due to the approximation of the effective dielectric constant.
Comparison for TM110 between Em and MNM programs

Comparison for TE011 between Em and MNM programs
<table>
<thead>
<tr>
<th>Package Dimension</th>
<th>Parameters</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a = 30.9 \text{ mm (along } x))</td>
<td>(\varepsilon_r = 2.2)</td>
<td>(7.6 &lt; f &lt; 8.3) , (9. &lt; f &lt; 10.2)</td>
</tr>
<tr>
<td>(b = 23.7 \text{ mm (along } y))</td>
<td>(h \text{ (sub. hght) = 0.1 mm})</td>
<td>(f = 7.960 \text{ GHz for } \text{TM}_{110})</td>
</tr>
<tr>
<td>(d = 20.1 \text{ mm (along } z))</td>
<td>(w = 0.3 \text{ mm})</td>
<td>(f = 9.793 \text{ GHz for } \text{TE}_{011})</td>
</tr>
</tbody>
</table>

Table 3-5

<table>
<thead>
<tr>
<th>For (\varepsilon_r = 2.2)</th>
<th>(\text{TM}_{110})</th>
<th>(\text{TE}_{011})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_{\text{req}} = 1.0027)</td>
<td>(\varepsilon_{\text{req}} = 1.006)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-6

3.8.2 *Comparison of coupling coefficient \(S_{13}\) between Em and MNM programs for the case of two right-angled bends*

Comparison between Em and MNM results has also been made for a case of two right-angled bends in microstrip lines in a rectangular package. The transmission coefficient \(S_{13}\) has been computed to compare coupling effects calculated from Em and MNM programs. The frequency range is chosen to be 8.6 GHz to 9.4 GHz, which covers a resonant mode \(\text{TE}_{011}\). The effective dielectric constant was calculated as discussed in two open-ends case. The wall loss is not included in the present computations. A comparison of the results is plotted in Figure 3-6 and parameters used for the computation are given in Table 3-7.

<table>
<thead>
<tr>
<th>Package Dimension</th>
<th>Parameters</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a = 30.5 \text{ (mm) along } x)</td>
<td>(\varepsilon_r = 2.2)</td>
<td>(f = 8.936 \text{ for } \text{TE}_{011})</td>
</tr>
<tr>
<td>(b = 30.5 \text{ (mm) along } y)</td>
<td>(d = 5.8 \text{ S/m})</td>
<td>(8.6 &lt; f &lt; 9.4 \text{ (GHz)})</td>
</tr>
<tr>
<td>(d = 20.1 \text{ (mm) along } z)</td>
<td>(w = 0.5) , (h = 0.1)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-7
Figure 3-6 shows the comparison near the $\text{TE}_{011}$ resonance frequency. The graphs "s134892.dat3" and "ems1348.dat3" show the $S_{13}$ values computed from MNM and Em programs, respectively. As it is seen, at the peak of resonance curve, the $S_{13}$ value using Em is -6.73 dB at 8.933 GHz and -6.13 at 8.936 GHz from MNM. Off-resonance frequency, the differences between the two programs are about 8 - 13 dB. The resonance frequency is different by about 3 MHz.

Comparison for $S_{13}$ between Em and MNM programs
4. Coupling effects among discontinuities in a triangular package

The coupling effects among discontinuities of a microstrip circuit in a triangular enclosure is studied in this chapter. The procedure is similar to that for rectangular case. In this report, the right-angled isosceles triangular cavity is investigated. The structure is shown in Figure 4-1.

![Diagram of a right-angled triangular cavity](image)

Figure 4-1: Right-Angled Triangular Cavity

The ray method has been used to find modal fields in the triangular wave guide. Then boundary condition is considered to find field in a right-angled isosceles triangular cavity by placing two conducting walls in the wave propagation direction.

After source-free fields have been derived, normalization of the modal fields is carried out and then total fields excited by an equivalent magnetic current source are evaluated. There are no numerical implementation and results included in this report. The overall algorithm is summarized in the following:

1. Find source-free modes in the cavity using Ray method with conducting walls at \( x=0, y=0, \) and \( x+y=a, \) and \( B.C \) at \( z=0 \) and \( c, \) for conducting walls.
2. Normalize each mode
3. Derive modal expansion for total fields

4.1 Source-Free Fields

Figure 4-2: Reflected rays inside a right-angled triangular

Figure 4-3: Covering Space

Consider a plane wave 1 in Figure 4-2 propagating at an angle $\theta$ between the $y$-axis and the projection of the ray in the $xy$-plane will have

$$H_z^{inc} = H_1 e^{-j k_c (x \sin \theta - y \cos \theta)}$$

(4.1.0.1)
Considering reflections of this ray with respect to the wave guide walls, Figure 4-2 shows that there are other seven ray directions generated inside the waveguide. Thus, the general solution for $H_z$ should be

$$
H_z = H_1 e^{-j k_e (x \sin \theta - y \cos \theta )} + H_2 e^{-j k_e (x \sin \theta + y \cos \theta )} + H_3 e^{-j k_e (\cdot x \sin \theta + y \cos \theta )} + H_4 e^{-j k_e (\cdot x \sin \theta - y \cos \theta )} + H_5 e^{-j k_e (x \cos \theta + y \sin \theta )} + H_6 e^{-j k_e (x \cos \theta - y \sin \theta )} + H_7 e^{-j k_e (\cdot x \cos \theta + y \sin \theta )} + H_8 e^{-j k_e (\cdot x \cos \theta - y \sin \theta )}
$$

(4.1.0.2)

where $H_i$, $i=1,2,...,8$ of each plane wave corresponds to the ray $i$ in Figure 4-2. To determine the values for $k_C$ and $\theta$ we need to apply boundary condition at each of the walls in turn. The tedious process of tracing successive reflection can be eased by using a covering space for the triangle as suggested in Figure 4-3. The covering space is obtained by repeatedly unfolding or reflecting an image of the original triangle about one or another of its sides in such a way as to cover the entire plane. The $H_z$ field must satisfy boundary conditions such that
\[ \text{at } x = 0, \nabla H_z \cdot \vec{a}_y = \frac{\partial H_z}{\partial y} \]
\[ \text{at } y = 0, \nabla H_z \cdot \vec{a}_x = \frac{\partial H_z}{\partial x} \]
\[ \text{at } x + y = a, \nabla H_z \cdot \frac{1}{\sqrt{2}} (\vec{a}_x + \vec{a}_y) = \frac{1}{\sqrt{2}} \left( \frac{\partial H_z}{\partial x} + \frac{\partial H_z}{\partial y} \right) = 0 \] (4.1.0.3)

This leads to:

\[ H_z = B \left\{ \sin(\alpha_1 x) \sin(\alpha_2 y) + (-1)^{m+n} \sin(\alpha_2 x) \sin(\alpha_1 y) \right\} \] (4.1.0.4)

where
\[ \alpha_1 = \frac{m \pi}{a}, \quad \alpha_2 = \frac{n \pi}{a} \] (4.1.0.5)

To satisfy the boundary conditions for electric walls at \( z = 0 \) and \( c \), we have

\[ H_z = B \left\{ \sin(\alpha_1 x) \sin(\alpha_2 y) + (-1)^{m+n} \sin(\alpha_2 x) \sin(\alpha_1 y) \right\} \sin(\alpha_3 z) \] (4.1.0.6)

In the similar way we get

\[ E_z = A \left\{ \sin(\alpha_1 x) \sin(\alpha_2 y) - (-1)^{m+n} \sin(\alpha_2 x) \sin(\alpha_1 y) \right\} \cos(\alpha_3 z) \] (4.1.0.7)

Now from the Maxwell's equations, the transverse components are obtained as following:
\( H_x = \frac{1}{k^2} \left[ \left\{ \alpha_1 \alpha_3 B \cos(\alpha_1 x) \sin(\alpha_2 y) + (-1)^{m+n} \alpha_2 \alpha_3 B \cos(\alpha_2 x) \sin(\alpha_1 y) \right\} \cos(\alpha_3 z) \\
+ j \omega A \left\{ \alpha_2 \sin(\alpha_1 x) \cos(\alpha_2 y) - (-1)^{m+n} \sin(\alpha_2 x) \cos(\alpha_1 y) \right\} \cos(\alpha_3 z) \right] \) 

(4.1.0.8)

\( H_y = \frac{1}{k^2} \left[ \left\{ \alpha_2 \alpha_3 B \sin(\alpha_1 x) \cos(\alpha_2 y) + (-1)^{m+n} \alpha_1 \alpha_3 B \sin(\alpha_2 x) \cos(\alpha_1 y) \right\} \cos(\alpha_3 z) \\
- j \omega A \left\{ \alpha_2 \cos(\alpha_1 x) \sin(\alpha_2 y) - (-1)^{m+n} \cos(\alpha_2 x) \sin(\alpha_1 y) \right\} \cos(\alpha_3 z) \right] \) 

(4.1.0.9)

4.2 Normalization

Since the procedure is the same as given for rectangular case, only the final results are given here. Note that, of course, the modal fields distributions are different from those for the rectangular cavity.

(1) when \( m \neq n \)

\[ A = \sqrt{\frac{\lambda_o \left( \frac{m^2 + n^2}{a^2} \right)}{\epsilon_o V_o}} \] 

(4.2.0.1)

(2) otherwise

\[ A = 0 \] 

(4.2.0.2)

(3) for all \( m \) and \( n \)
\[ B = \sqrt{\frac{\lambda_c \left( \frac{m}{a} \right)^2 \frac{n}{a}}{\mu_0 V_o}} \]  

(4.2.0.3)

4.3 Field produced by a magnetic current element

For this step also, the procedure is also the same as for the rectangular case. Thus Chapter 3 can be referred for details. We have,

\[ \bar{E} = \sum_i B_{ie} \bar{E}_i \]

\[ \bar{H} = \sum_i B_{ih} \bar{H}_i \]

(4.3.0.1)

where \( i = m, n, p \) and

\[ B_{ie} = \frac{j \omega_i}{2} \int \frac{\vec{M} \cdot \vec{H}_i}{\omega - \omega_i} \, dv \]

\[ B_{ih} = \frac{j \omega}{2} \int \frac{\vec{M} \cdot \vec{H}_i}{\omega - \omega_i} \, dv \]

(4.3.0.2)
5. **Concluding remarks**

In this Chapter, we summarize the main results of this project and include some discussions for the follow-up work related to the project.

5.1 **Summary of main results**

The summary of results is divided into three categories:

1. Coupling effects in a rectangular package,
2. Coupling effects in a right-angled isosceles triangular package, and
3. Software Development

5.1.1 **Coupling between discontinuities in a rectangular package**

The method used involves an equivalent magnetic current representation of the microstrip line fields and modal expansion of the fields produced in the package. First source-free modal fields $E_j$ and $H_j$ are normalized to satisfy orthonormality relationship so that total fields $E$ and $H$ excited by magnetic current sources can be expressed as sum of package modes. Then, the convergence test on a series expression of total fields $E$ and $H$ has been made. We can ensure the convergence of the series when the operating frequency is close to a resonance frequency. In such cases, ten terms for each $m, n$ and $p$ indexes are enough to get the convergence. When the operating frequency is away from a resonant frequency, more terms for the series are required unless we are willing to compromise some accuracy in the computation. Based on the experience of the various computations carried out, it may be reasonable to say that convergence can be assured if the coupling value in dB unit is greater than -50 dB. It may be pointed out that lower coupling values are not of much practical importance.

The two factors mentioned above are critical for verification of the code developed. A few comparisons have been made for the coupling values between
MNM approach and Em** code. As discussed, there is a reasonable agreement around the resonance frequencies and some differences are observed when the operating frequency is away from the resonance frequencies.

Conductor loss from walls of the cavity has been included in the analytical derivation. In this derivation, quality factor of the cavity with given parameters is computed for each mode and then it is transformed into an equivalent loss tangent value. These loss tangent values are used as the imaginary part in the complex dielectric constant for various modes of the package.

One of the main advantages of the software developed compared to the full-wave analysis method is shorter computing time. MNM program is much faster than Em program. An example of comparison for CPU times for the two approaches is made for the case of two right-angled bends in a rectangular cavity. The results and parameters used for this comparison are listed in the Tables 5-1 and 5-2.

<table>
<thead>
<tr>
<th></th>
<th>MNM</th>
<th>Em</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU Time Elapsed</td>
<td>9.7 seconds</td>
<td>27465.2 seconds</td>
</tr>
</tbody>
</table>

Table 5-1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>εr</td>
<td>12.5</td>
<td>a (in x axis)</td>
</tr>
<tr>
<td>h</td>
<td>0.125 mm</td>
<td>b (in y axis)</td>
</tr>
<tr>
<td>σ</td>
<td>5.8 E7 Simens/m</td>
<td>d (in z axis)</td>
</tr>
</tbody>
</table>

Table 5-2

5.1.2 Coupling effects between discontinuities in a right-angled isosceles triangular package.

The procedure for triangular packages is the same as for the rectangular case. We used ray method to find source-free modes for this case. The report

*Registered Trademark of Sonnet Software, Inc., Liverpool, NY.
includes derivation of source-free modal fields, normalization and the total fields produced by magnetic currents. This derivation is not yet implemented in the present software code.

5.1.3 Software Development

Computer programs related to these algorithms have been developed. The software is in FORTRAN language and implemented in the HP 9000 workstation network in the MIMICAD Center.

Programs can be divided into three main functions:
(1) Characteristic parameters for a given microstrip line
(2) [Z] matrix computation of a given circuit
(3) [Y] matrix computation for coupling in an enclosure

First the program computes [Z] matrices for given microstrip line circuits. The program implements Green’s function approach and segmentation method for the [Z] matrix computation. Before the program computes [Z] matrix, it requires input file such as number of segments and location of ports, and characteristic parameter values of a microstrip line. Then it computes [Z] matrix for a single segment such as a rectangular geometry. After finding all the [Z] matrices corresponding to each segment, it combines all the segments together to compute the overall [Z] matrix using segmentation method.

The coupling matrix is computed using modal expansion in a rectangular package. First, E and H fields are calculated. Then, induced currents are computed from H field. Then [Y] matrix is constructed as the ratio between induced currents and edge voltages. This [Y] matrix is transformed into [Z] matrix.

All the [Z] matrices are combined by segmentation method to produce an overall [Z] matrix, which is transformed into [S] matrix.
In short, the currently developed software can analyze coupling effects in a rectangular package containing simple circuits which can be segmented into rectangular segments.

5.2 Suggestions for the follow-up work

5.2.1 Computation of effective dielectric constant for individual modes

The effective dielectric constant in a partially filled cavity can be calculated by solving the related boundary value problem. The structure is shown in Figure 5-1.

![Diagram of a rectangular cavity](image)

**Figure 5-1**

The procedure for any TM$_{mn}$ or TE$_{mn}$ mode (with respect to z direction) may be summarized as follows. First, find fields in each region of dielectric and air. Then apply boundary conditions at the dielectric - air interface such that tangential E and H fields are continuous. This will yield a characteristic equation. Solution of the characteristic equation yields the resonance frequency. Comparison with the corresponding values for a homogeneous filled cavity yields equivalent dielectric constant.
5.2.2 Development of fields excited by a magnetic current in packages with different geometries.

The ray method used in the isosceles triangular cavity case can be applied in other shapes of triangular packages such as;

(1) 30-60-90 degree triangular cavity
(2) 60-60-60 degree triangular cavity

5.2.3 Investigation of coupling matrix [Y] with an enclosure

Mutual coupling matrix [Y] used in this work has the structure of Figure 5-2 in most cases since self admittance and mutual admittance for sections of the same discontinuity edges are accounted in the planar model for microstrip circuits. However, the matrix configuration shown in Figure 5-3 is another possible structure for [Y] matrix where coupling among various edge sections of the same discontinuity are also included. In one of the cases studied, it was noticed that the [S] values obtained with this modification are different from that obtained with the present method. Further investigations of this feature are desirable. In these figures, “x” means possibly non-zero element and “0” means zero.

\[
[Y] = \begin{bmatrix}
0 & 0 & 0 & x & x & x \\
0 & 0 & 0 & x & x & x \\
x & x & x & 0 & 0 & 0 \\
x & x & x & 0 & 0 & 0 \\
x & x & x & 0 & 0 & 0 \\
x & x & x & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
[Y] = \begin{bmatrix}
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
x & x & x & x & x & x & x \\
\end{bmatrix}
\]

Figure 5-2

Figure 5-3
5.2.4 Investigation of coupling region in an enclosure

We assumed that the coupling effect is dominated from the discontinuities in the circuits as it is generally true for an open space case. However, when the cavity size is getting smaller, the coupling effects of a package may not be dominated only from discontinuities. The effects can be produced from other parts of the circuit also.

5.2.5 Software Extensions

Here are possible areas for programming extensions. These extensions will broaden the scope of the coupling analysis simulation in an enclosure with more efficient and accurate results.

(1) Include triangular segments in \([Z]\) matrix computation,
(2) Complete implementation for incorporating loss in the package walls,
(3) Upgrade the computation of characteristic parameters for the case of including an enclosure, and
(4) Develop easy interface for input and output files.
REFERENCE


Appendices:

A Normalization for a source-free fields in a rectangular cavity

Normalization relation is given as

$$\mu \int_{V} |\vec{H}|^2 \, dv = 1$$  \hspace{1cm} (A.1)

The source-free mode fields are give in Chapter 3 for H fields. By assuming B=0 TM mode fields are given also in chapter 3. Here in the following the square of the absolute value of each component is given.

$$|H_x|^2 = \frac{2 \omega \varepsilon A}{k^4} \alpha_2^2 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \cos^2(\alpha_3 z)$$  \hspace{1cm} (A.2)

$$|H_y|^2 = \frac{2 \omega \varepsilon A}{k^4} \alpha_1^2 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) \cos^2(\alpha_3 z)$$  \hspace{1cm} (A.3)

Since it is true that

$$\mu \int_{V} |\vec{H}|^2 \, dv = \mu \int_{V} (|H_x|^2 + |H_y|^2) \, dv$$

$$= \frac{2 \omega \varepsilon A}{k^4} \alpha_2^2 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \cos^2(\alpha_3 z)$$

$$+ \frac{2 \omega \varepsilon A}{k^4} \alpha_1^2 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) \cos^2(\alpha_3 z)$$  \hspace{1cm} (A.4)
Now consider the cases of different combination of m, n, and p values which can be either zero or non-zero. The results are given in chapter 3.

Calculating normalizing factor B can be made in the same way. That is,

$$\varepsilon \int_v |\mathbf{E}|^2 \, dv = 1$$  \hspace{1cm} (A.5)

By setting A=0 in the source-free mode field $\mathbf{E}$ expression and squaring the absolute value of the components,

$$|E_x|^2 = \frac{\omega^2}{k^4} \mu \frac{B^2}{\alpha_2 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) \sin^2(\alpha_3 z)}$$ \hspace{1cm} (A.6)

$$|E_y|^2 = \frac{\omega^2}{k^4} \mu \frac{B^2}{\alpha_1 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \sin^2(\alpha_3 z)}$$ \hspace{1cm} (A.7)

From

$$\varepsilon \int_v |\mathbf{E}|^2 \, dv = \varepsilon \int_v (|E_x|^2 + |E_y|^2) \, dv$$

$$= \frac{\omega^2}{k^4} \mu \frac{B^2}{\alpha_2 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) \sin^2(\alpha_3 z)}$$

$$+ \frac{\omega^2}{k^4} \mu \frac{B^2}{\alpha_1 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \sin^2(\alpha_3 z)}$$  \hspace{1cm} (A.8)
Also the case by case corresponding m,n and p values are given in Chapter 3.

B. Normalization for a source-free fields in a right-angled isosceles triangular cavity

\[ \mu \int_{V} \left| \mathbf{H} \right|^2 dV = 1 \]  

(B.1)

From the source-free mode fields derived in chapter 4, we have

\[ H_x = \frac{j \omega \varepsilon A}{k^2} \left\{ \alpha_2 \sin(\alpha_1 x) \cos(\alpha_2 y) - (-1)^{m+n} \alpha_1 \sin(\alpha_2 x) \cos(\alpha_1 y) \right\} \cos(\alpha_3 z) \]  

(B.2)

\[ H_y = -\frac{j \omega \varepsilon A}{k^2} \left\{ \alpha_1 \cos(\alpha_1 x) \sin(\alpha_2 y) - (-1)^{m+n} \alpha_2 \cos(\alpha_2 x) \sin(\alpha_1 y) \right\} \cos(\alpha_3 z) \]  

(B.3)

\[ H_z = 0 \]  

(B.4)

Then it is easy to see that

\[ \left| H_x \right|^2 = -\frac{2}{k^4} \left\{ \frac{2}{k^4} \left( \alpha_2^2 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) + \alpha_1^2 \sin^2(\alpha_2 x) \cos^2(\alpha_1 y) \right) \cos^2(\alpha_3 z) \right\} \]  

(B.5)

\[ \left| H_y \right|^2 = -\frac{2}{k^4} \left\{ \frac{2}{k^4} \left( \alpha_1^2 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) + \alpha_2^2 \cos^2(\alpha_2 x) \sin^2(\alpha_1 y) \right) \cos^2(\alpha_3 z) \right\} \]  

(B.6)
\(|H_4|^2 = 0\)

By using the facts of integration

\[
\int \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \cos^2(\alpha_3 z) \, dv = \frac{a^2 c}{16} \quad \text{where} \quad \begin{cases} 
0 \leq x \leq a \\
0 \leq y \leq a-x \\
0 \leq z \leq c 
\end{cases}
\]

\[
\int \sin(\alpha_1 x) \sin(\alpha_2 x) \cos(\alpha_1 y) \cos(\alpha_2 y) \, dv = \begin{cases} 
a^2 \frac{a}{8} & \text{if } m = n \\
0 & \text{if } m \neq n 
\end{cases}
\]

we obtained that

\[
\mu \int_{v} |\vec{H}_1|^2 \, dv = \mu \int_{v} (|H_x|^2 + |H_y|^2) \, dv
\]

\[
= \begin{cases} 
\frac{2^2 \omega^2 \varepsilon \mu a^2 c^2}{k^4} \left( \alpha_1^2 + \alpha_2^2 - 2 \alpha_1 \alpha_2 \right) & \text{if } m = n \\
\frac{2^2 \omega^2 \varepsilon \mu a^2 c^2}{k^4} \left( \alpha_1^2 + \alpha_2^2 \right) & \text{if } m \neq n
\end{cases}
\]

\[
= \begin{cases} 
0 & \text{if } m = n \\
\frac{\omega^2 \mu \varepsilon A^2 a^2 c}{k^2} & \text{if } m \neq n
\end{cases}
\]

(B.7)
By normalizing to 1, we can calculate normalizing factor $A$ such that

$$\frac{A^2 \omega^2 \mu \varepsilon a^2 c^2}{8 k^2} = 1$$  \hspace{1cm} (B.8)

Therefore,

$$A = \sqrt{\frac{\lambda_o \left( \frac{|m|^2}{a} + \frac{|n|^2}{a} \right)}{\varepsilon V_o}}$$  \hspace{1cm} (B.9)

In the same way $B$ can be computed;

$$\varepsilon \int_V |E|^2 \, dv = 1$$  \hspace{1cm} (B.10)

The components of mode field $E$ we have as in Chapter 4

$$E_x = \frac{j \omega \mu B}{k^2} \left( \alpha_2 \sin(\alpha_1 x) \cos(\alpha_2 y) + (-1)^{m+n} \alpha_1 \sin(\alpha_2 x) \cos(\alpha_1 y) \right) \sin(\alpha_3 z)$$  \hspace{1cm} (B.11)

$$E_y = \frac{j \omega \mu A}{k^2} \left( \alpha_1 \cos(\alpha_1 x) \sin(\alpha_2 y) + (-1)^{m+n} \alpha_2 \cos(\alpha_2 x) \sin(\alpha_1 y) \right) \sin(\alpha_3 z)$$  \hspace{1cm} (B.12)

$$E_z = 0$$  \hspace{1cm} (B.13)

Therefore,
\[ |E_x|^2 = \frac{\omega^2 \mu B^2}{k^4} \left( \alpha_2 \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) + \alpha_1 \sin^2(\alpha_2 x) \cos^2(\alpha_1 y) \right) \sin^2(\alpha_3 z) \]

\[ |E_y|^2 = \frac{\omega^2 \epsilon B^2}{k^4} \left( \alpha_1 \cos^2(\alpha_1 x) \sin^2(\alpha_2 y) + \alpha_2 \cos^2(\alpha_2 x) \sin^2(\alpha_1 y) \right) \sin^2(\alpha_3 z) \]

\[ |E_z|^2 = 0 \]

Again using the integration facts

\[ \int_v \sin^2(\alpha_1 x) \cos^2(\alpha_2 y) \cos^2(\alpha_3 z) \, dv = \frac{a^2 c}{16} \quad \text{where} \quad \begin{cases} 0 \leq x \leq a \\ 0 \leq y \leq a-x \\ 0 \leq z \leq c \end{cases} \]

\[ \int_v \sin(\alpha_1 x) \sin(\alpha_2 x) \cos(\alpha_1 y) \cos(\alpha_2 y) \, dv = \left\{ \begin{array}{ll} \frac{a^2}{8} & \text{if } m = n \\ 0 & \text{if } m \neq n \end{array} \right\} \]

we have

\[ \varepsilon \int_v |\vec{E}|^2 \, dv = \varepsilon \int_v (|E_x|^2 + |E_y|^2) \, dv \]
\[ \begin{cases} \frac{2 \omega \varepsilon \mu a^2 c}{k^4} \left( \alpha_1^2 + \alpha_2^2 + 2 \alpha_1 \alpha_2 \right) & \text{if } m = n \\ \frac{2 \omega \varepsilon \mu a^2 c}{k^4} \left( \alpha_1^2 + \alpha_2^2 \right) & \text{if } m \neq n \end{cases} \]  

(B.16)

\[ \begin{cases} \frac{2 \omega \mu B^2 a^2 c}{k^2} \left( \alpha_1 + \alpha_2 \right)^2 & \text{if } m = n \\ \frac{2 \omega \mu B^2 a^2 c}{k^2} & \text{if } m \neq n \end{cases} \]  

(B.17)

By normalizing to 1, we find normalizing factor B such that
\[ \frac{2 \omega \varepsilon \mu a^2 c}{8 k^2} = 1 \]  

(B.18)

Therefore,
\[ B = \sqrt{\frac{\lambda_o \left\{ \frac{m^2}{a} + \frac{n^2}{a} \right\}}{\mu \nu_o}} \]  

(B.19)
Coupling Analysis with Package Program (CAP)
User's Guide

Sung Lee

MIMICAD Center
Department of Electrical and Computer Engineering
Univ. of Colorado at Boulder

May 1992
Contents

1. Introduction---------------------------------------------------------------1
   1.1 Organization of this user's guide
   1.2 Overview of CAP package
   1.3 Procedure for using CAP program

2. CAP program--------------------------------------------------------------6

3. MCZM subprogram--------------------------------------------------------7

4. TWODCAP subprogram------------------------------------------------------8

5. EFFDIMS program---------------------------------------------------------10

6. Preparing Data Files----------------------------------------------------11

7. Example-----------------------------------------------------------------14

8. References---------------------------------------------------------------32

List of Figures

Figure 1: Flow chart of CAP program --------------------------------------5

Figure 2: Configuration of the example -----------------------------------14

Figure 3: Geometry of two open-ends --------------------------------------15

List of Tables

Table 1: Parameters used in the example -----------------------------------15
Chapter 1

Introduction

Coupling analysis with Package (CAP) is a program to perform coupling computation from discontinuities of a microstrip circuit in an enclosure using mutiport network approach and modal expansion. The analytical derivation of CAP is described in [1]. The program works only with rectangular cavity package.

This user's guide is intended for use by audience with some background in electromagnetics and microwave theories.

1.1 Organization of this user's guide

(1) Introduction

This is the chapter you are now reading. It describes what the user's guide contains and the procedure for using CAP. The chapter also gives an overview of CAP package.

(2) CAP program

This chapter discusses the main part of CAP package i.e., computation of S-parameters for the given circuit in an enclosure.

(3) MCZM subprogram

This chapter discusses the second part of CAP package i.e., computation of impedance matrix [Z] as coupling network, which is inverse of coupling admittance matrix [Y]. The elements of [Y] is computed from total H field in an enclosure. The total field is computed by using modal expansion in a specified rectangular cavity.

(4) TWODCAP subprogram
This chapter discusses the first part of CAP package i.e., computation of \( [Z] \) matrix of the microstrip circuit configuration using the Green's function approach.

(5) EFFDIMS program

This chapter discusses the supplementary part of CAP package i.e., computation of effective dimensions and characteristic parameters of the microstrip line. This task is performed by EFFDIMS program.

(6) Preparing data file

This chapter discusses the user's data file for running CAP program. It explains how the user specify the configuration to be analyzed and other necessary parameters for the microstrip circuit.

(7) Example

This chapter presents an example of using CAWP to compute coupling S-parameters from discontinuities of the two open ends microstrip circuit in the rectangular enclosure.

1.2 Overview of CAP package

The CAP package computes S-parameters for a given microstrip circuit in the rectangular cavity package. The program uses multi-port network and modal expansion approaches. The input to the CAP program contains number of frequencies, starting frequency, incremental frequency and input files for TWODCAP and MCZM programs.

The user may need to run EFFDIMS program to compute effective dimensions and necessary parameters to prepare the input file to run TWODCAP program. The TWODCAP program produces the overall impedance matrix \([Z]\) of a circuit element. For example, if the user has two right-angled bends, the user needs to run TWODCAP program twice to get two \([Z]\) matrix corresponding each right-angled bend. After computing all the \([Z]\) matrices for corresponding circuit elements, MCZM program produces mutual coupling matrix \([Z]\) among the
discontinuities of the circuit in the rectangular cavity enclosure. Finally, CAP program combines all the \([Z]\) matrices produced to yield an overall \([Z]\) matrix. This \([Z]\) matrix is transformed into \([S]\) matrix for final output.

The current CAP program can analyze microstrip circuit configuration which can be broken up into rectangular segments only. However, the program can be modified easily to include other type of segments such as three different triangular segments, circular disc segments, circular sector segments, annular ring segments, and annular sector segments.

The program takes into account the dispersion effect but not the conductor loss from walls of the enclosure.

The current program employs no error checking and messages. Hence, it is very inconvenient for the user when trying to correct a problem.

The program can be run either interactively or automatically. To get familiar with the program quickly, it is better to run the program at least once interactively while referring to the data files given as an example in Chapter 6.

The package is written in standard FORTRAN 77. It is developed on the Hewlett-Packard workstation HP-9000 series 300. It has been run in IBM comparable PC with Intel 80486 processor and with Microsoft Fortran 5.0 compiler. It has not been used on any other machines.

1.3 Procedure for using CAP

Once the design specifications are known, one can start analyzing the circuit of discontinuity configuration by following the procedure described below. Flow chart for the entire program is given in Figure 1.

**STEPS:**

1. Divide Component A in a regular segments and calculate \([Z_a]\) matrix.
2. Divide Component B in a regular segments and calculate \([Z_b]\) matrix.
3. Represent the edge fields by small sections of magnetic current line sources of length dl.
4. Calculate magnetic field excited inside the enclosure by a magnetic current element at the j-th subsection of component A.
5. Calculate the induced current $J_S$ in the jth element of circuit B.
6. Calculate the $[Y_{ij}]$ element of mutual coupling admittance matrix.
7. Use the same procedure to compute all other elements of the coupling matrix $[Y]$.
9. Use the segmentation method to combine $[Z_A]$ and $[Z_C]$ to produce $[Z_{AC}]$.
10. Use the segmentation method to combine matrix $[Z_{AC}]$ with $[Z_B]$ to produce $[Z]$-matrix representation of the coupled elements.
Figure 1: Flow chart of the CAP program
Chapter 2

CAP program

This chapter discusses the CAP program. Understanding of this program requires understanding of TWODCAP and MCZM programs. Those materials are explained in following chapters.

Basically, this is the program managing all the necessary subprograms related to the CAP. In other words, it calls necessary subprograms whenever they are needed.

First, the program reads data from standard input device. The data are contained in the file \textit{opmcn.in} given an an example in Chapter 6.

The program continues with computation of $[Z]$ matrix for each component of the circuit. The current version of CAP can have maximum two components such as two open-ends or two right-angled bends etc.. However, it can be easily modified to analyze more complex circuits. For example, make a loop to call TWODCAP program as many as the number of components of a circuit. At this time, the program assumes automatically two circuit components so that it calls TWODCAP subprogram twice.

After $[Z]$ matrix computations, it calls MCZM subprogram to compute coupling matrix $[Y]$. For the coupling computation, ports need to be located in the region of discontinuities. The $[Z]$ matrix computed is the inverse matrix of admittance matrix $[Y]$.

The three different $[Z]$ matrices from TWODCAP and MCZM are now combined by segmentation technique. The details of the segmentation technique may be referred to [3].

Finally, the overall $[Z]$ matrix is transformed into $[S]$ matrix. This is the final output of the CAP program.
The program produces several output files during the computation. The list of output file names are given in Chapter 6 as an example. The file names shown with "out" in *opmcn.in* and *colfile.in* are given as output files.
Chapter 3

MCZM subprogram

This chapter discusses the MCZM program. Understanding of this program is required if the user runs CAP program. The program returns coupling impedance matrix [Z].

The program is used to compute the frequency dependent admittance matrix [Y]. It first reads input file, for example, opmczm.in, given in Chapter 6. Total H field is computed first using modal expansion for the rectangular cavity. The modes in the cavity are excited by magnetic current, which is equivalence source to fringing edge fields in the microstrip circuit. Total E filed is also computed at the same time. The induced current is computed from H field. The ratio of induced current and edge voltage between the ports is given as an element of [Y] matrix. The details of the background can be referred to Chapter 3 in [1].
Chapter 4

TWODCAP subprogram

This chapter discusses the TWODCAP subprogram. This is a modified version of MANYSEG program to be called into the CAP program. Thus, the details of the program are referred to Chapter 4 in [2].

To run this program, the user need to run EFFDIMS program first to obtain effective dimensions and characteristic parameters for the given microstrip line. The program returns impedance matrix for the given configuration.

As brief summary of TWODCAP, the routine analyzes the configuration by characterizing and combining the segments, making up the configuration, two at a time. These combinations are carried out by using the impedance matrix characterization of each segment. The result of these combinations is the impedance matrix characterization of the whole configuration.
Chapter 5

EFFDIMS program

This chapter is referred to Chapter 3 in [2].

As quick summary, The program EFFDIMS is used to compute the frequency dependent parameters for rectangular transmission line segments, and the effective radius for circular segments and circular sector segments. The formulas for computing the effective dielectric constant and the characteristic impedance are given in [4].
Chapter 6

Preparing data files

This chapter discusses how to prepare input files to run CAP program. The CAP program requires five different files for input data; the names of the files can be chosen arbitrarily. The file names given in this document are just examples. The contents of those files are given below as examples. The comments contained in the files shouldn't be included for practical use.

**File_name: opmcn.in**

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Number of frequencies</td>
</tr>
<tr>
<td>7.6d9</td>
<td>Starting frequency for analysis</td>
</tr>
<tr>
<td>0.1d9</td>
<td>Incremental frequency</td>
</tr>
<tr>
<td>opsmat.out</td>
<td>A name for S-parameter output</td>
</tr>
<tr>
<td>1</td>
<td>Index number of i for S_{ij}</td>
</tr>
<tr>
<td>2</td>
<td>Index number of j for S_{ij}</td>
</tr>
<tr>
<td>colfile.in</td>
<td>A name of data file containing the rest of the data file</td>
</tr>
</tbody>
</table>

**File_name: colfile.in**

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>za.out</td>
<td>A name of [Z] matrix of a circuit component</td>
</tr>
<tr>
<td>openda.out</td>
<td>A name for [Z] from 2dcap program</td>
</tr>
<tr>
<td>y</td>
<td>Yes for input file</td>
</tr>
<tr>
<td>openda.in</td>
<td>A name of input file for 2dcap</td>
</tr>
<tr>
<td>zb.out</td>
<td>A name of [Z] matrix of a circuit component</td>
</tr>
<tr>
<td>opendb.out</td>
<td>A name for [Z] from 2dcap program</td>
</tr>
<tr>
<td>y</td>
<td>Yes for input file</td>
</tr>
<tr>
<td>opendb.in</td>
<td>A name of input file for 2dcap</td>
</tr>
<tr>
<td>zmc.out</td>
<td>A name of coupling [Z] matrix output</td>
</tr>
<tr>
<td>opmczm.in</td>
<td>A name of input file for coupling computation</td>
</tr>
<tr>
<td>opmczm.out</td>
<td>A name of output file for coupling computation</td>
</tr>
<tr>
<td>zmc.out</td>
<td>A name of segmentation output</td>
</tr>
<tr>
<td>zmcab.out</td>
<td>A name of segmentation output</td>
</tr>
</tbody>
</table>
### File name: openda.in

| 1 | ; Number of segments |
| 0.D0,5.8e7,0.1e-3,3e-6 | ; $\delta$ (loss tangent), $\sigma$ (conductivity), $h$ (height of substrate), $t$ (thickness of conductor) |
| 2 | ; Number of external ports |
| 1.87,50,0 | ; $\varepsilon_{re}$, ohm, length of ref. plane |
| 1.87,50,0 | ; $\varepsilon_{re}$, ohm, length of ref. plane |
| y | ; Yes for intermediate results |
| 1 | ; Number of segments |
| 10.2E-3,0.55E-3,1.87 | ; Dimensions and ere |
| 2 | ; Number of interconnected ports |
| 1,0.E0,0.E0,0.E0,0.55E-3 | ; Num. of ports & the sec. occupied |
| 1,10.2E-3,0.55E-3,10.2E-3,0.E-3 | ; Num. of ports & the sec. occupied |
| n | ; No for renumbering |

### File name: opendb.in

| 1 | ; Number of segments |
| 0.D0,5.8e7,0.1e-3,3e-6 | ; $\delta$ (loss tangent), $\sigma$ (conductivity), $h$ (height of substrate), $t$ (thickness of conductor) |
| 2 | ; Number of external ports |
| 1.87,50,0 | ; $\varepsilon_{re}$, ohm, length of ref. plane |
| 1.87,50,0 | ; $\varepsilon_{re}$, ohm, length of ref. plane |
| y | ; Yes for intermediate results |
| 1 | ; Number of segments |
| 10.2E-3,0.55E-3,1.87 | ; Dimensions and ere |
| 2 | ; Number of interconnected ports |
| 1,0.E0,0.E0,0.E0,0.55E-3 | ; Num. of ports & the sec. occupied |
| 1,10.2E-3,0.55E-3,10.2E-3,0.E-3 | ; Num. of ports & the sec. occupied |
| n | ; No for renumbering |
File name: opmczm.in

1,1 ; Number of coupling ports in each comp.
0.0309d0,0.0237d0,0.020d0 ; Dimensions of the cavity
10,8,7 ; Number of terms for modal expansion
1.0027D0,1.0D0 ; \( \varepsilon \)req and \( \mu \)r
0.d0 ; loss tangent as wall loss
10.2E-3,11.85E-3,0.E-3,0.3E-3,3.141592E0 ; x, y, z, w, \( \theta \)
20.7E-3,11.85E-3,0.E0,0.3E-3,0.E0 ; x, y, z, w, \( \theta \)
10.2E-3,11.85E-3,0.E-3,0.3E-3,3.141592E0 ; x, y, z, w, \( \theta \)
20.7E-3,11.85E-3,0.E0,0.3E-3,0.E0 ; x, y, z, w, \( \theta \)
Chapter 7

Example

This chapter gives an example of running the CAP program. The configurations and the parameters used are given Figure 2, Figure 3 and Table 1. An interactive running of the program is followed. Finally, all the files for output are listed with file name but they are edited for demo purpose only.

The problem follows. Suppose the rectangular package shown in Figure 2 has two open-ends. The port numbers are set as 1 and 2 as shown. The microstrip circuit needs to be analyzed in the frequency range 7.8 GHz to 8.2 GHz. The step size of the frequency increment is chosen to be 0.1 GHz so that there are five discrete frequencies for the analysis.

Figure 2: Configuration of the example
Figure 3: geometry of two open-ends

<table>
<thead>
<tr>
<th>Package Dimension</th>
<th>Parameters</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 30.9 mm (along x)</td>
<td>ε_r = 2.2</td>
<td>7.6 &lt; f &lt; 8.0</td>
</tr>
<tr>
<td>b = 23.7 mm (along y)</td>
<td>h (sub. hght) = 0.1 mm</td>
<td></td>
</tr>
<tr>
<td>c = 20.1 mm (along z)</td>
<td>w = 0.3 mm</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Parameters used in the example

The user first prepares a input file like `opmcn.in` and `colfile.in` shown in Chapter 6. Then run EFFDIMS program to prepare input files like `openda.in` and `opendb.in`. Also, using physical dimensions of the circuit and package, build the input file like `opmczm.in`. Now all data files are set to run CAP. Type `cap < opmcn.in` in the command line prompt. The following is an example for the input files shown.
An example of interactive interface with the CAP program

...Enter the number of frequencies = 5
...Enter the starting frequency (Hz)= 7.6d9
...Enter the incremental frequency (Hz)= 0.1d9
...Enter the file name of output S(i,j) = opsmat.out
...Enter the value index i for S(i,j) = 1
...Enter the value index j for S(i,j) = 2
...Enter file name for data file collection = colfile.in
...Enter output file name for [Za] matrix = za.out
ENTER THE NAME TO BE GIVEN TO THE OUTPUT FILE
NOTE: FILE.DAT <= 12 CHARACTERS openda.out
DO YOU HAVE INPUT FILE (Y/N) y
WHAT IS YOUR INPUT FILE NAME(NAME.DAT:<=12 CHAR.) openda.in
ENTER THE NO. OF SEGMENTS TO BE COMBINED 1
ENTER THE LOSS TAN. DELTA,CONDUCTIVITY (SIGMA), SUBSTRATE HEIGHT,AND STRIP THICKNESS
0.0023, 5.8e9, 0.1e-3, 3e-6
ENTER NO. OF EXTERNAL PORTS after SEGMENTATION. 2
ENTER ERE(F), Z0(F), AND THE LENGTH USED TO SHIFT THE REFERENCE PLANE FOR EACH EXTERNAL PORT SEGMENT
1.87,50,0
DO YOU WANT INTERMEDIATE RESULTS (Y/N) y
*******SEGMENTATION PROCESS STARTS*******
THIS PROGRAM COMBINES 2 SEGMENTS AT A TIME, IF THERE ARE MORE THAN 2 SEGMENTS THEN THE FIRST AND THE SECOND SEG. WILL BE COMBINED FIRST THEN THIS COMBINATION IS COMBINED WITH THE THIRD SEG. ETC
------------------------------------------------------------------------
WHICH IS THE TYPE OF SEGMENT 1: 1
1. RECTANGULAR SEGMENT
2. 45-90-45 TRIANGULAR SEG.
3. 30-90-60 TRIANGULAR SEG.
4. 60-60-60 TRIANGULAR SEG.
5. TERMINATION.
6. 2 port TERMINATION.

ENTER DIM. OF SEG1: A(LENGTH),B(WIDTH), & ERE(F)
10.2e-3,0.55e-3,1.87

ENTER # OF SEC. ON SEG1 OCCUPIED BY EXT. PORTS 2
ENTER # EXT.PORTS & THE SEC. THEY OCCUPY(XI,YI,XF,YF)
1,0.e0,0.e0,0.e0,0.55e-3
1,10.2e-3,0.55e-3,10.2e-3,0.e-3

-----SEGMENTATION PROCESS STEP: 1

NEED TO RENUMBER THE PORTS IN ZC (Y/N) n

...Enter output file name for [Zb] matrix = zb.out

ENTER THE NAME TO BE GIVEN TO THE OUTPUT FILE

NOTE: FILE.DAT <= 12 CHARACTERS opendb.out

DO YOU HAVE INPUT FILE (Y/N) y

WHAT IS YOUR INPUT FILE NAME(NAME.DAT:<12 CHAR.) opendb.in

ENTER THE NO. OF SEGMENTS TO BE COMBINED 1

ENTER THE LOSS TAN. DELTA,CONDUCTIVITY (SIGMA), SUBSTRATE HEIGHT,AND STRIP THICKNESS
0.0023,5.8e7,0.1e-3,3e-6

ENTER NO. OF EXTERNAL PORTS after SEGMENTATION. 2

ENTER ERE(F), Z0(F), AND THE LENGTH USED TO SHIFT THE REFERENCE PLANE FOR EACH EXTERNAL PORT SEGMENT
1.87,50.0
1.87,50.0

DO YOU WANT INTERMEDIATE RESULTS (Y/N) y

**********SEGMENTATION PROCESS STARTS**********

THIS PROGRAM COMBINES 2 SEGMENTS AT A TIME, IF THERE ARE MORE THAN 2 SEGMENTS THEN THE FIRST AND THE SECOND SEG. WILL BE COMBINED FIRST THEN THIS COMBINATION IS COMBINED WITH THE THIRD SEG. ETC
WHICH IS THE TYPE OF SEGMENT 1: 1
   1. RECTANGULAR SEGMENT
   2. 45-90-45 TRIANGULAR SEG.
   3. 30-90-60 TRIANGULAR SEG.
   4. 60-60-60 TRIANGULAR SEG.
   5. TERMINATION.
   6. 2 port TERMINATION.
ENTER DIM. OF SEG1: A(LENGTH), B(WIDTH), & ERE(F)
10.2e-3, 0.55e-3, 1.87
ENTER # OF SEC. ON SEG1 OCCUPIED BY EXT. PORTS 2
ENTER # EXT.PORTS & THE SEC. THEY OCCUPY(XI,Y1,XF,YF)
1, 0.e0, 0.e0, 0.e0, 0.55e-3
1, 10.2e-3, 0.55e-3, 10.2e-3, 0.e-3
-----SEGMENTATION PROCESS STEP: 1

NEED TO RENUMBER THE PORTS IN ZC (Y/N) n
...Enter output file name for [Zmc] matrix = zmc.out
...Enter input file name for [Y] computation = opmczm.in
...Enter output file name for [Y] computation = opmczm.out

***Input Param.s: for coupling matrix computation
...cavity size in meter = .0309 .0237 .0200
...number of modes = 10 8 7
...operating freq. in Hz= .76000E+10
...epsilon r and myu r = 1.0027 1.0000
...loss tangent = .00000E+00

...Wait while in computation...
...YIJ( 1 1)= .00000E+00 .79910E-05
...Wait while in computation...
...YIJ( 2 1)= .00000E+00 .63986E-06
...Wait while in computation...
...YIJ( 2 2)= .00000E+00 .79910E-05
...Enter output file name for [Zmcb] matrix = zmcb.out
...Enter output file name for [Zmcab] matrix = zmcab.out

...S( 1 1) = -2.6205E+00  .95148E+00
SMAG( 1 1) = .98690E+00
SANG( 1 1) = -.74604E+02 ANGLE

...S( 1 2) = .60986E-04  .17114E-04
SMAG( 1 2) = .63342E-04
SANG( 1 2) = .15676E+02 ANGLE

...S( 2 1) = .60986E-04  .17114E-04
SMAG( 2 1) = .63342E-04
SANG( 2 1) = .15676E+02 ANGLE

...S( 2 2) = -2.6205E+00  .95148E+00
SMAG( 2 2) = .98690E+00
SANG( 2 2) = -.74604E+02 ANGLE

Note: ^^^The same interface with different frequency^^-^ as many times as the number of frequencies
This example shows only twice of the same interface

...Enter output file name for [Za] matrix =
ENTER THE NAME TO BE GIVEN TO THE OUTPUT FILE
NOTE: FILE.DAT <= 12 CHARACTERS
DO YOU HAVE INPUT FILE (Y/N)
WHAT IS YOUR INPUT FILE NAME(NAME.DAT)<=12 CHAR.)
ENTER THE NO. OF SEGMENTS TO BE COMBINED
ENTER THE LOSS TAN. DELTA, CONDUCTIVITY (SIGMA),
SUBSTRATE HEIGHT, AND STRIP THICKNESS
ENTER NO. OF EXTERNAL PORTS after SEGMENTATION.
ENTER ERE(F), Z0(F), AND THE LENGTH USED TO SHIFT
THE REFERENCE PLANE FOR EACH EXTERNAL PORT SEGMENT
DO YOU WANT INTERMEDIATE RESULTS (Y/N)

**********SEGMENTATION PROCESS STARTS**********

THIS PROGRAM COMBINES 2 SEGMENTS AT A TIME,
IF THERE ARE MORE THAN 2 SEGMENTS THEN THE FIRST
AND THE SECOND SEG. WILL BE COMBINED FIRST THEN
THIS COMBINATION IS COMBINED WITH THE THIRD SEG. ETC

-----------------------------------------------

WHICH IS THE TYPE OF SEGMENT 1:
  1. RECTANGULAR SEGMENT
  2. 45-90-45 TRIANGULAR SEG.
  3. 30-90-60 TRIANGULAR SEG.
  4. 60-60-60 TRIANGULAR SEG.
  5. TERMINATION.
  6. 2 port TERMINATION.

ENTER DIM. OF SEG1: A(LENGTH),B(WIDTH),& ERE(F)

ENTER # OF SEC. ON SEG1 OCCUPIED BY EXT. PORTS

ENTER # EXT.PORTS & THE SEC. THEY OCCUPY(XI,YI,XF,YF)

-----SEGMENTATION PROCESS STEP: 1

NEED TO RENUMBER THE PORTS IN ZC (Y/N)

...Enter output file name for [Zb] matrix =

ENTER THE NAME TO BE GIVEN TO THE OUTPUT FILE

NOTE: FILE.DAT <= 12 CHARACTERS

...before the reading in 2dcap

DO YOU HAVE INPUT FILE (Y/N)

WHAT IS YOUR INPUT FILE NAME(NAME.DAT:=12 CHAR.)

ENTER THE NO. OF SEGMENTS TO BE COMBINED

ENTER THE LOSS TAN. DELTA,CONDUCTIVITY (SIGMA),
SUBSTRATE HEIGHT,AND STRIP THICKNESS

ENTER NO. OF EXTERNAL PORTS after SEGMENTATION.

ENTER ERE(F), Z0(F), AND THE LENGTH USED TO SHIFT
THE REFERENCE PLANE FOR EACH EXTERNAL PORT SEGMENT

DO YOU WANT INTERMEDIATE RESULTS (Y/N)
******SEGMENTATION PROCESS STARTS******

THIS PROGRAM COMBINES 2 SEGMENTS AT A TIME,
IF THERE ARE MORE THAN 2 SEGMENTS THEN THE FIRST
AND THE SECOND SEG. WILL BE COMBINED FIRST THEN
THIS COMBINATION IS COMBINED WITH THE THIRD SEG. ETC

WHICH IS THE TYPE OF SEGMENT 1:
1. RECTANGULAR SEGMENT
2. 45-90-45 TRIANGULAR SEG.
3. 30-90-60 TRIANGULAR SEG.
4. 60-60-60 TRIANGULAR SEG.
5. TERMINATION.
6. 2 port TERMINATION.

ENTER DIM. OF SEG1: A(LENGTH), B(WIDTH), & ERE(F)
ENTER # OF SEC. ON SEG1 OCCUPIED BY EXT. PORTS
ENTER # EXT.PORTS & THE SEC. THEY OCCUPY (XI, YI, XF, YF)

-----SEGMENTATION PROCESS STEP: 1

NEED TO RENUMBER THE PORTS IN ZC (Y/N)
...Enter output file name for [Zmc] matrix =
...Enter input file name for [Y] computation =
...Enter output file name for [Y] computation =

***Input Param.s: for coupling matrix computation
...cavity size in meter = 0.0309 0.0237 0.0200
...number of modes = 10 8 7
...operating freq. in Hz = 770000E+10
...epsilon r and myu r = 1.0027 1.0000
...loss tangent = 0.00000E+00

...Wait while in computation...
...YIJ( 1 1) = 0.00000E+00 84574E-05
...Wait while in computation...
...YIJ( 2 1) = 0.00000E+00  0.93182E-06
...Wait while in computation...
...YIJ( 2 2) = 0.00000E+00  0.84574E-05

...Enter output file name for [Zmc] matrix =
...Enter output file name for [Zmcab] matrix =

...S( 1 1) = 0.20595E+00  0.96509E+00
SMAG( 1 1) = 0.98682E+00
SANG( 1 1) = 0.77956E+02 ANGLE

...S( 1 2) = 0.90104E-04  0.19661E-04
SMAG( 1 2) = 0.92224E-04
SANG( 1 2) = 0.12310E+02 ANGLE

...S( 2 1) = 0.90104E-04  0.19661E-04
SMAG( 2 1) = 0.92224E-04
SANG( 2 1) = 0.12310E+02 ANGLE

...S( 2 2) = 0.20595E+00  0.96509E+00
SMAG( 2 2) = 0.98682E+00
SANG( 2 2) = 0.77956E+02 ANGLE

~~~~~~~~~~~Note:~~~~~~~~~~~

The interface with the program will continue on as many as the number
frequencies. In this example, it will be five times.

........
........
........

File_name: opsmat.out
File_name: opsmat.out

OUTPUT FILE = opsmat.out

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Mag. of $S(1, 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.60000000000000E+009</td>
<td>-83.96615634636802</td>
</tr>
<tr>
<td>7.70000000000000E+009</td>
<td>-80.70312108972299</td>
</tr>
<tr>
<td>7.80000000000000E+009</td>
<td>-75.91271220365474</td>
</tr>
<tr>
<td>7.90000000000000E+009</td>
<td>-66.69196129960533</td>
</tr>
<tr>
<td>8.00000000000000E+009</td>
<td>-62.23596409644073</td>
</tr>
</tbody>
</table>

END OF OUTPUT FILE = opsmat.out
File_name: za.out

*** Find ZA matrix ***

\[ F \text{ (Hz)} = 0.76000 \times 10^1 \]

...Number of External ports in ckt1 = 2

\[ ZA(1 \ 1) = 0.52069 \times 10^2 \quad 0.38057 \times 10^2 \]
\[ ZA(1 \ 2) = -0.16357 \times 10^2 \quad -0.62931 \times 10^2 \]
\[ ZA(2 \ 1) = -0.16357 \times 10^2 \quad -0.62931 \times 10^2 \]
\[ ZA(2 \ 2) = 0.52069 \times 10^2 \quad 0.38057 \times 10^2 \]

*** Find ZA matrix ***

\[ F \text{ (Hz)} = 0.77000 \times 10^1 \]

...Number of External ports in ckt1 = 2

\[ ZA(1 \ 1) = 0.54841 \times 10^2 \quad 0.40418 \times 10^2 \]
\[ ZA(1 \ 2) = -0.19733 \times 10^2 \quad -0.64387 \times 10^2 \]
\[ ZA(2 \ 1) = -0.19733 \times 10^2 \quad -0.64387 \times 10^2 \]
\[ ZA(2 \ 2) = 0.54841 \times 10^2 \quad 0.40418 \times 10^2 \]

*** Find ZA matrix ***
*** Find ZB matrix ***

...Number of External ports in ckt2= 2

...ZB( 1 1)= .52069E+00 .38057E+02
...ZB( 1 2)= -.16357E+00 -.62931E+02

...ZB( 2 1)= -.16357E+00 -.62931E+02
...ZB( 2 2)= .52069E+00 .38057E+02

*** Find ZB matrix ***

...Number of External ports in ckt2= 2

...ZB( 1 1)= .54841E+00 .40418E+02
...ZB( 1 2)= -.19733E+00 -.64387E+02

...ZB( 2 1)= -.19733E+00 -.64387E+02
...ZB( 2 2)= .54841E+00 .40418E+02

*** Find ZB matrix ***
THE INPUT PARAMETERS ARE:
LOS TAN. DELTA = .00000E+00
CONDUCTIVITY(SIGMA)= .58000E+08
OPERATING FREQ. = .80000E+10
SUBSTRATE HEIGHT = .10000E-03
STRIP THICKNESS = .30000E-05

-----SEGMENTATION PROCESS STEP: 1

PORTS COORDINATES OF SEGMENT NO. 1

<table>
<thead>
<tr>
<th>PORTS #</th>
<th>X</th>
<th>Y</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.00000E+00</td>
<td>.27500E-03</td>
<td>.55000E-03</td>
</tr>
<tr>
<td>2</td>
<td>.10200E-01</td>
<td>.27500E-03</td>
<td>.55000E-03</td>
</tr>
</tbody>
</table>

ELEMENTS OF MATRIX ZA

ELEMENTS OF COLUMN# 1

<table>
<thead>
<tr>
<th>ROW#</th>
<th>real*8PART</th>
<th>dimagINARY PT</th>
<th>MAGNITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEG.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.6553574840E+00</td>
<td>.4823836258E+02</td>
<td>.4824281415E+02</td>
</tr>
<tr>
<td>89.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-.3210388820E+00</td>
<td>-.6956229913E+02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.6956303995E+02</td>
<td>-.90.26</td>
<td></td>
</tr>
</tbody>
</table>

ELEMENTS OF COLUMN# 2
THE INPUT PARAMETERS ARE:
LOSS TAN. DELTA = .00000E+00
CONDUCTIVITY(SIGMA)= .58000E+08
OPERATING FREQ. = .80000E+10
SUBSTRATE HEIGHT = .10000E-03
STRIP THICKNESS = .30000E-05

-----SEGMENTATION PROCESS STEP: 1

PORTS COORDINATES OF SEGMENT NO. 1

<table>
<thead>
<tr>
<th>PORTS #</th>
<th>X</th>
<th>Y</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.00000E+00</td>
<td>.27500E-03</td>
<td>.55000E-03</td>
</tr>
<tr>
<td>2</td>
<td>.10200E-01</td>
<td>.27500E-03</td>
<td>.55000E-03</td>
</tr>
</tbody>
</table>

ELEMENTS OF MATRIX ZA

ELEMENTS OF COLUMN# 1

<table>
<thead>
<tr>
<th>ROW#</th>
<th>real*8PART</th>
<th>dimagINARY PT</th>
<th>MAGNITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEG.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.6553574840E+00</td>
<td>.4823836258E+02</td>
<td>.4824281415E+02</td>
</tr>
<tr>
<td>89.22</td>
<td>- .3210388820E+00</td>
<td>-.6956229913E+02</td>
<td>-90.26</td>
</tr>
</tbody>
</table>

ELEMENTS OF COLUMN# 2
***INPUT PARAMETERS USED:
  ...cavity size in meter = .0309 .0237 .0200
  ...number of modes = 10 8 7
  ...operating freq. in Hz = .76000E+10
  ...eps. r(eff) and myu r = 1.0027 1.0000
  ...loss tangent = .00000E+00

***TOTAL H FIELD COMPONENTS AND [Y] MATRIX...

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.THFLDX=</td>
<td>.00000E+00,</td>
<td>.11174E-13</td>
</tr>
<tr>
<td>.THFLDY=</td>
<td>.00000E+00,</td>
<td>.26637E-01</td>
</tr>
<tr>
<td>.YIJ( 1 1)=</td>
<td>.00000E+00</td>
<td>.79910E-05</td>
</tr>
<tr>
<td>.THFLDX=</td>
<td>.00000E+00,</td>
<td>-.38854E-18</td>
</tr>
<tr>
<td>.THFLDY=</td>
<td>.00000E+00,</td>
<td>.21329E-02</td>
</tr>
<tr>
<td>.YIJ( 2 1)=</td>
<td>.00000E+00</td>
<td>.63986E-06</td>
</tr>
<tr>
<td>.THFLDX=</td>
<td>.00000E+00,</td>
<td>-.33797E-18</td>
</tr>
<tr>
<td>.THFLDY=</td>
<td>.00000E+00,</td>
<td>.26637E-01</td>
</tr>
<tr>
<td>.YIJ( 2 2)=</td>
<td>.00000E+00</td>
<td>.79910E-05</td>
</tr>
</tbody>
</table>

***INVERSE MATRIX OF YIJ...

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>.IER=</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>.ZIJ( 1 1)=</td>
<td>.00000E+00</td>
<td>-.12595E+06</td>
</tr>
<tr>
<td>.ZIJ( 1 2)=</td>
<td>.00000E+00</td>
<td>.10085E+05</td>
</tr>
<tr>
<td>.ZIJ( 2 1)=</td>
<td>.00000E+00</td>
<td>.10085E+05</td>
</tr>
<tr>
<td>.ZIJ( 2 2)=</td>
<td>.00000E+00</td>
<td>-.12595E+06</td>
</tr>
</tbody>
</table>
*** Find Zmc ***

...Number of Coupling Ports = 1 1

...ZMC( 1 1)= .00000E+00  -.12595E+06  
...ZMC( 1 2)= .00000E+00  .10085E+05  
...ZMC( 2 1)= .00000E+00  .10085E+05  
...ZMC( 2 2)= .00000E+00  -.12595E+06  

*** Find Zmc ***

...Number of Coupling Ports = 1 1

...ZMC( 1 1)= .00000E+00  -.11969E+06  
...ZMC( 1 2)= .00000E+00  .13188E+05  
...ZMC( 2 1)= .00000E+00  .13188E+05  
...ZMC( 2 2)= .00000E+00  -.11969E+06  

*** Find Zmc ***

...Number of Coupling Ports = 1 1

...ZMC( 1 1)= .00000E+00  -.11055E+06  
...ZMC( 1 2)= .00000E+00  .19177E+05  
...ZMC( 2 1)= .00000E+00  .19177E+05  
...ZMC( 2 2)= .00000E+00  -.11055E+06  

*** Find Zmc ***

...Number of Coupling Ports = 1 1

...ZMC( 1 1)= .00000E+00  -.92245E+05  

29
### Finding Zmcb

<table>
<thead>
<tr>
<th></th>
<th>1 1</th>
<th>1 2</th>
<th>2 1</th>
<th>2 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZMCB</td>
<td>.52086E+00</td>
<td>.38088E+02</td>
<td>.13122E-01</td>
<td>.50406E+01</td>
</tr>
<tr>
<td></td>
<td>.13122E-01</td>
<td>.50406E+01</td>
<td>.33405E-02</td>
<td>-.12514E+06</td>
</tr>
</tbody>
</table>

### Finding Zmcb

<table>
<thead>
<tr>
<th></th>
<th>1 1</th>
<th>1 2</th>
<th>2 1</th>
<th>2 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZMCB</td>
<td>.54863E+00</td>
<td>.40452E+02</td>
<td>.21781E-01</td>
<td>.70964E+01</td>
</tr>
<tr>
<td></td>
<td>.21781E-01</td>
<td>.70964E+01</td>
<td>.66619E-02</td>
<td>-.11824E+06</td>
</tr>
</tbody>
</table>

### Finding Zmcb

<table>
<thead>
<tr>
<th></th>
<th>1 1</th>
<th>1 2</th>
<th>2 1</th>
<th>2 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZMCB</td>
<td>.58002E+00</td>
<td>.42932E+02</td>
<td>.40749E-01</td>
<td>.11448E+02</td>
</tr>
<tr>
<td></td>
<td>.40749E-01</td>
<td>.11448E+02</td>
<td>.17459E-01</td>
<td>-.10722E+06</td>
</tr>
</tbody>
</table>

### Finding Zmcbab

...Number of S ports = 2
ZMCAB( 1 1) = 0.52086E+00 0.38088E+02
ZMCAB( 1 2) = 0.13202E-04 0.25356E-02
ZMCAB( 2 1) = 0.13202E-04 0.25356E-02
ZMCAB( 2 2) = 0.52086E+00 0.38088E+02

*** Finding Zmcab ***

...Number of S ports = 2

ZMCAB( 1 1) = 0.54863E+00 0.40453E+02
ZMCAB( 1 2) = 0.23730E-04 0.38656E-02
ZMCAB( 2 1) = 0.23730E-04 0.38656E-02
ZMCAB( 2 2) = 0.54863E+00 0.40453E+02

*** Finding Zmcab ***

...Number of S ports = 2

ZMCAB( 1 1) = 0.58003E+00 0.42933E+02
ZMCAB( 1 2) = 0.50165E-04 0.70464E-02
ZMCAB( 2 1) = 0.50165E-04 0.70464E-02
ZMCAB( 2 2) = 0.58003E+00 0.42933E+02

*** Finding Zmcab ***
REFERENCES


