QPSK Space-Time Turbo Codes

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Abstract— **This paper proposes a class of full space diversity QPSK space-time codes based on parallel concatenated convolutional (turbo) codes. A rank criterion of full space diversity is used in the design. Compared with space-time trellis codes, the simulations show it has robust performance at both quasi-static fading channel and time varying fading channel.**

I. Introduction

One of the major challenges in wireless communication is overcoming channel fading caused by multipath and movement in the radio link. The study of quasi-static fading channel [1] by Foschini and Gans showed the enormous capacity promised by multiple antennas systems in such channels. This presents a new area in the design of channel codes.

A. Space–Time Codes

Recently, there has been a surge of interests, e.g., [2, 3], in the design of so called "space–time" codes which take advantage of both the spatial diversity provided by multiple antennas and the temporal diversity available with time–varying fading. Design criteria [3–5] have been proposed for space-time codes in a variety of environments.

In quasi-static Rayleigh fading channel, each possible code word difference in a linear coded modulation produces a "signal" matrix. Increasing the rank of a "signal" matrix increases the amount of diversity in demodulation and reduces the pairwise error probability [3, 5]. If the code length in time, N_c, is larger than the number of transmit antennas, L_t, the maximum possible rank of a "signal" matrix is L_t. A code is said to achieve full space diversity when the rank of every "signal" matrix corresponding to every code word pair is equal to L_t. While the ranks of the "signal" matrices are defined over the complex number field, traditional code design is usually carried out in finite fields or finite rings. This discrepancy causes a serious obstacle in the design. The paper by Hammons and El Gamal [6] represents an important first step to bridge this discrepancy. They provided a binary rank criteria for binary BPSK codes and $\mathbb{Z}_4$ QPSK codes to ensure full space diversity. More recently, the design of PSK modulated space-time codes is also addressed by Blum [7].

In [8], we provide a theory for the design of space-time codes in quasi-static Rayleigh fading channel with higher order of constellation (2^{2k} QAM), such as QPSK, 16 QAM, 64 QAM, etc. It includes the BPSK binary rank criterion in [6] as a special case. For QPSK constellation, it is applicable to GF(4) codes instead of $\mathbb{Z}_4$ codes. Consequently, many traditional codes and turbo codes can be modified to be QPSK space-time codes.

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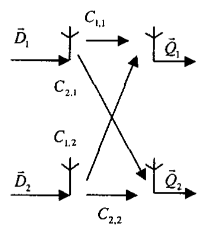

Fig. 1. A system using linear modulation with 2 transmit antennas and 2 receive antennas.

B. Space–Time Turbo Codes

Some space–time trellis codes may have low level of diversity in space–time correlated fading channels [5]. Lower performance is caused by the interaction between the channel structure and the code structure. A "random" coding structure would reduce this interaction. Turbo coding [9] is one of best ways to build decodable "random" codes. The inherent rich structure of turbo codes will likely provide robust performance in a variety of channels that will likely be encountered in wireless practice.

Efforts have been made to adapt turbo codes to multiple transmit antennas environment. A discussion of other groups' work can be found in [8]. In [10], the BPSK binary rank criterion [6] is used to ensure full space diversity of the parallel concatenated turbo codes. Although parallel concatenated QPSK "space-time turbo codes" have been proposed in [11, 12], full space diversity is not guaranteed due to the lack of proper theory. The Σ_o-rank criterion developed in [8] enables us to design full space diversity parallel concatenated QPSK turbo codes.

In Section II, we give the signal models and explain the space-time code design criteria. Section III presents the theory of Σ_o-rank criterion. The design and decoding algorithm of QPSK space-time turbo codes are given in Section IV. Section V presents the performance and design tradeoff. Section VI concludes.

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II. Models and Performance Criteria

A space-time code word $\mathbf{J}$ is an N_c by L_t matrix with elements drawn from a finite alphabet. The code symbol matrix $\mathbf{D}$ is defined as $\mathbf{D} = f(\mathbf{J})$, where $f(\bullet)$ is an element-wise constellation mapping from the finite alphabet to points of a constellation on the complex plane. The element at i^{th} column and j^{th} row of $\mathbf{D}$, $D_i(j)$, is transmitted from i^{th} antenna at time j. Note that, in this paper, the variable in the subscript is used to indicate the column of a matrix if not explicitly specified.

Let L_r be the number of receive antennas. The signals transmitted from each antenna experience spatial independent quasi-static Rayleigh fading. The matched filter output of the received signal at k^{th} receive antenna from time 1 to N_c is

$$\vec{Q}_k = \mathbf{D}\vec{C}_k + \vec{W}_k, \tag{1}$$

where $\vec{Q}_k$ is an N_c by 1 vector, $\vec{W}_k$ is N_c by 1 complex Additive White Gaussian Noise (AWGN) vector with one sided power spectrum N_0, and $\vec{C}_k$ is an L_t by 1 fading coefficient vector. Let $C_{k,l}$ denote the l^{th} element of $\vec{C}_k$. $C_{k,l}$ models a Rayleigh fading channel common in wireless communications by being a zero mean complex Gaussian random variable. $C_{k,l}$'s are assumed independent for different k or l and known to the receiver. Figure 1 shows an example of such system with 2 transmit antennas and 2 receive antennas.

Considering two code symbol matrices, $\mathbf{D}^\alpha$ and $\mathbf{D}^\beta$, the code symbol difference matrix $\mathbf{Z}$ is defined as $\mathbf{Z} = \mathbf{D}^\alpha - \mathbf{D}^\beta$. With a goal of minimizing the pair-wise error probability, the quasi-static fading channel design criteria [3,5] are to maximize the rank of code symbol difference matrix $\mathbf{Z}$ and maximize the product of nonzero eigenvalues of $\mathbf{Z}^H\mathbf{Z}$ for all pairs of code words.

III. Theory of Rank Criteria

In [8], sufficient conditions are provided to ensure full space diversity of a code using QAM Modulation. The main results are presented here without proof.

A. Preliminary Definitions

The full space diversity rank criteria developed in [8] are for codes defined on the ring $\mathbb{Z}_{2^k}(j)$. In the sequel, $\oplus_n$ and $\ominus_n$ are used to denote the modulo n addition and subtraction, and subscript n is dropped if the context is clear. Subscripts I and Q will be used to indicate the real part and imaginary part of a complex number or a matrix.

Definition 1: **(Ring $\mathbb{Z}_{2^k}(j)$)** The ring $\mathbb{Z}_{2^k}(j)$ is a finite set and k is a positive integer. Each element V has the form,

$$V = V_I \oplus jV_Q,$$

where the real part V_I and imaginary part V_Q are integers in $\mathbb{Z}_{2^k}$ and $j^2 = -1 = -1 + 2^k$. In this paper, nonnegative integers in $\{0, 1, \cdots, 2^k - 1\}$ are used to label elements in $\mathbb{Z}_{2^k}$. The addition and multiplication in this ring are the addition and multiplication in complex number field followed by modulo 2^k operation on the real part and the imaginary part.

Definition 2: **(Linear $\mathbb{Z}_{2^k}(j)$ Code with Translation Mapping)** A linear $\mathbb{Z}_{2^k}(j)$ code $\mathcal{C}$ is a set of code words which form an additive group. Each code word $\mathbf{J}$ is an N_c by L_t matrix with elements in the ring $\mathbb{Z}_{2^k}(j)$. The linearity implies that if $\mathbf{J}^\alpha, \mathbf{J}^\beta \in \mathcal{C}$, then $\mathbf{J}^\alpha \oplus \mathbf{J}^\beta \in \mathcal{C}$. Each code word matrix $\mathbf{J}$ is mapped to a complex code symbol matrix $\mathbf{D}$ by the translation, $\mathbf{D} = \mathbf{J} - ((2^k - 1)/2 + j(2^k - 1)/2)$. It results in a 2^{2k} QAM constellation.

A linear $\mathbb{Z}_{2^k}(j)$ code can be represented as a linear transformation from information sequence to code word:

$$\begin{aligned} \mathbf{J} &= [\ \vec{J}_1 \ \ \vec{J}_2 \ \cdots \ \vec{J}_{L_t}\ ] & (2)\\ &= [\ \mathbf{G}_1\vec{I} \ \ \mathbf{G}_2\vec{I} \ \cdots \ \mathbf{G}_{L_t}\vec{I}\ ], & (3)\end{aligned}$$

where $\vec{I}$ is an $\mathbb{Z}_{2^k}$ input information sequence, $\vec{J}_i$ denotes the i^{th} column of the code word matrix, and $\mathbf{G}_i$ is the $\mathbb{Z}_{2^k}(j)$ generator matrix for i^{th} antenna.

Definition 3: **(Σ_o-Coefficients)** Coefficients, $\alpha_1, \alpha_2, \ldots, \alpha_L$, in $\mathbb{Z}_{2^k}(j)$ are said to be Σ_o-coefficients if there exists i^* such that $a_{i^*} + b_{i^*}$ is odd, where $a_{i^*} \oplus jb_{i^*} = \alpha_{i^*}$.

Definition 4: **(Column Σ_o-Rank)** A matrix $\mathbf{V}$ over the ring $\mathbb{Z}_{2^k}(j)$ has column Σ_o-rank L if L is the maximum number of column vectors of $\mathbf{V}$, such that

$$\exists \mathcal{V} = \{\vec{V}_{i_1}, \ldots, \vec{V}_{i_L}\},\ \bigoplus_{l=1}^{L} \alpha_l \vec{V}_{i_l} \neq \vec{0},$$

for any Σ_o-coefficients, $\alpha_1, \alpha_2, \ldots, \alpha_L$.

The row Σ_o-rank can be similarly defined. Since column Σ_o-rank is equal to row Σ_o-rank [8], they are called Σ_o-rank.

Definition 5: **(Full Σ_o-Rank)** An m by n matrix $\mathbf{V}$ over ring $\mathbb{Z}_{2^k}(j)$ is said to be of full Σ_o-rank if it has Σ_o-rank equal to the minimum of m and n.

B. Σ_o-Rank Criterion

The sufficient conditions on code words are given first.

Theorem 1: **(Σ_o-Rank Criterion)** Let $\mathcal{C}$ be a linear $\mathbb{Z}_{2^k}(j)$ code with translation mapping to 2^{2k} QAM constellation. If every nonzero code word $\mathbf{J} \in \mathcal{C}$ has full Σ_o-rank, then $\mathcal{C}$ achieves full space diversity.

For linear codes, the conditions can be translated into the conditions on the generator matrices.

Theorem 2: Let $\mathcal{C}$ be a linear $\mathbb{Z}_{2^k}(j)$ code. The i^{th} column of the code word matrix is defined as

$$\vec{J}_i = \mathbf{G}_i\vec{I}, \tag{4}$$

where $\vec{I}$ is the information sequence in $\mathbb{Z}_{2^k}(j)$, $\mathbf{G}_i$ is the generator matrix for i^{th} antenna. If for all Σ_o-coefficients, $\alpha_1, \alpha_2, \ldots, \alpha_{L_t}$, and for all nonzero information sequence, $\vec{I}$,

$$\left(\bigoplus_{i=1}^{L_t} \alpha_i \mathbf{G}_i\right) \vec{I} \neq \vec{0}, \tag{5}$$

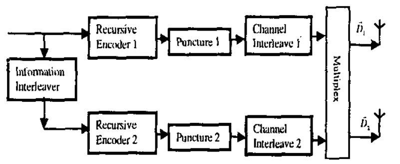

Fig. 2. Space-time turbo code encoder for 2 transmit antenna system.

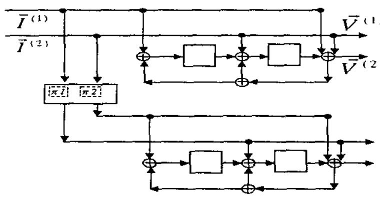

Fig. 3. A rate 2/4 turbo code in [13].

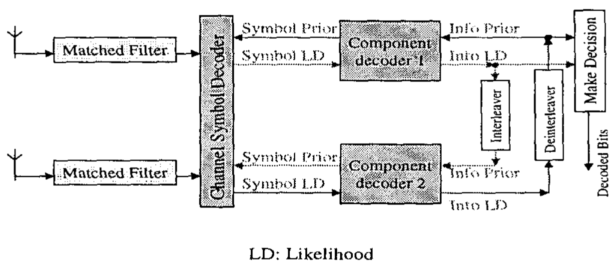

Fig. 4. Iterative Decoder for the case of 2 receive antennas.

then $\forall$ nonzero $\mathbf{J} \in \mathcal{C}$, $\mathbf{J}$ is of full Σ_o-rank. Thus, the code achieves full space diversity.

IV. Space–Time Turbo Codes

The analysis of the existing space-time codes and construction of new space-time codes using the Σ_o-rank criterion are detailed in [8]. In this section, we present QPSK full space diversity turbo code design.

A. Encoder Design

The encoder of the proposed space–time turbo code [11] is composed of a turbo code followed by the operation of puncturing, channel interleaving and multiplexing (Figure 2). Puncturing allows flexible code rates and channel interleaving and multiplexing take advantage of both time diversity and space diversity. As an example, we consider a rate 2/4 turbo code [13] with 2 transmit antenna and QPSK modulation. Figure 3 illustrates the structure of the turbo code. Let $\mathbf{H}_{ij}$ be the matrix corresponding to the transfer function from $\vec{I}^{(i)}$ to $\vec{V}^{(j)}$, where $\vec{I}^{(i)}$ and $\vec{V}^{(j)}$ are the binary input and output of the component code of the turbo code. If $(\vec{V}^{(1)}, \vec{V}^{(2)})$ are mapped to QPSK symbols using the Gray mapping, then the space-time turbo code can be viewed as a linear $\mathbb{Z}_2(j)$ code with i^{th} column of a code word as

$$\begin{aligned} \vec{J}_i &= \mathbf{G}_i \vec{I} \\ &= \mathbf{M}_i \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{21} \\ \mathbf{H}_{11}\mathbf{P} & \mathbf{H}_{21}\mathbf{P} \end{bmatrix} \begin{bmatrix} \vec{I}^{(1)} \\ \vec{I}^{(2)} \end{bmatrix} + \\ & j\mathbf{M}_i \begin{bmatrix} \mathbf{H}_{12} & \mathbf{H}_{22} \\ \mathbf{H}_{12}\mathbf{P} & \mathbf{H}_{22}\mathbf{P} \end{bmatrix} \begin{bmatrix} \vec{I}^{(1)} \\ \vec{I}^{(2)} \end{bmatrix}, \end{aligned}$$

where matrix $\mathbf{P}$ corresponds to the information interleaver and matrix $\mathbf{M}_i$ corresponds to the operation of puncturing, channel interleaving and multiplexing.

It is difficult to find a systematic method to construct the matrix $\mathbf{P}$ and $\mathbf{M}_i$ so that $\mathbf{G}_1$ and $\mathbf{G}_2$ satisfy Theorem 2. However, the chance is high that a randomly picked information interleaver and channel interleaver result in a full space diversity code.

B. Decoder

The optimal decoding of a space–time turbo code is complicated for the following reasons. First, the received signal is a linear combination of symbols transmitted from different transmit antennas and the noise. Secondly, the inputs of the component encoders are the interleaved versions of the same information bits sequence as in turbo codes. However, we can use a sub–optimal iterative algorithm like that for turbo decoding to decode the code.

The decoder diagram is shown in Figure 4 for the case of two receive antennas. The fading coefficients are assumed known to the receiver. The arrows in Figure 4 form two circles. The iterations are performed simultaneously along them. The matched filter output is used to calculate the likelihood of the channel symbols. In the first iteration, channel symbols from different transmit antenna are assumed to have equal a priori probability. The turbo decoder uses the likelihood information to do one iteration of standard turbo decoding and produces a priori probability of the channel symbols. The channel symbol decoder uses the "new" a priori probability to refine the likelihood information about the channel symbols which is the start of the second iteration. After several iterations, hard decisions on information bits are made based on the soft information provided by the turbo decoder.

V. Simulation Results

The performance of the example code without puncture mentioned in Section IV-A, whose turbo encoder is shown in Figure 3, is evaluated by simulation. In the simulation, each frame corresponds to 260 information bits. The rate of the code is 2 bits/symbol.

The performance with one receive antenna in quasi-static fading channel is shown in Figure 5. There are three cases. In the first case, the information interleaver and the channel interleaver are randomly chosen. The outputs of each component code of the turbo code are multiplexed to two transmit antennas. The code does not satisfy the Σ_o-rank criterion and rank deficient error event did happen in the simulation. In the second case, information interleavers and channel interleavers were examined

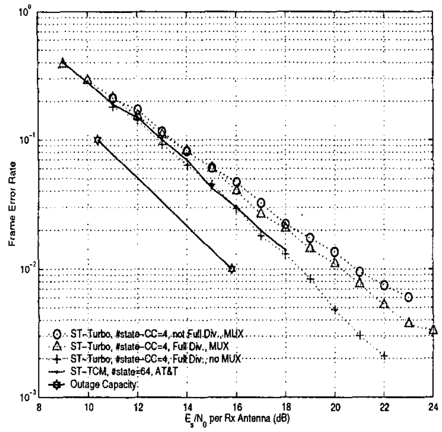

Fig. 5. Performance in quasi-static fading channel for 3 different cases.

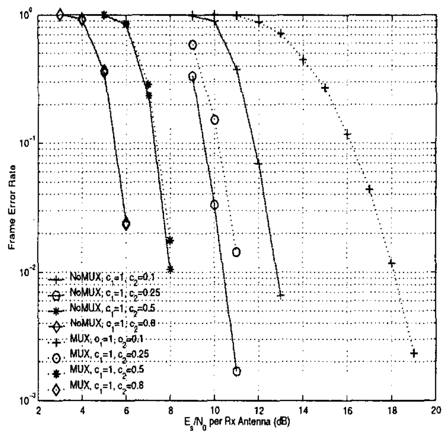

Fig. 6. Performance comparison of cases with and without multiplexing for different fading coefficients with genie decoder.

until the generator matrices satisfied the Σ_o-rank criterion. The resulting code is a guaranteed full space diversity code. The third case is also a full space diversity code. But what differs from the first and the second case is that there is no multiplexing, which means that all the outputs from one component code are transmitted only on one antenna. For comparison, the performance of a 64 state space-time trellis code in [3] and the outage capacity [1] are also shown in the plot. The third case has the same or better performance than the 64 state trellis code and is 2.5 dB away from the outage capacity at frame error rate 0.01. This indicates that the space-time turbo code is full space diversity and can achieve good performance with simple component codes even if the size of the frame is short.

At low SNR, all the three cases have performances close to the 64 state trellis code. When the SNR increases, the frame error rates of the first two cases do not decrease as much as that of the 64 state code. The performance of the second case showed steeper slope than the first case since it is a full diversity code. The third case performs better than the second case. This is counter intuitive since in the second case, with multiplexing, the outputs of one component code experience spatial independent fading, which should result in better performance.

To explain this counter intuitive phenomenon, one needs to realize that the code rate of each component code of the turbo code in Figure 3 is 1. If part of the outputs of one component encoder experience deep fades, the MAP decoder of turbo decoding for this component code will produce unreliable soft information about the information bits. If both of the component encoders have a portion of outputs experience deep fades, then the iterative decoding can have difficulty converging to correct estimates. This is what happens for case 2 when one of the fading coefficients associated with one of the transmit antenna has small amplitude. When the code rate is lower, this phenomenon may vanish and multiplexing may bring benefits due to the additional diversity.

This argument is demonstrated by simulation results in Figure 6. In order to exclude the sub-optimal effect of channel symbol decoder, a genie decoder is used, i.e., the channel symbol decoder has perfect knowledge about symbols transmitted from other antenna when it calculates the likelihood of a symbol. The fading coefficient for the first transmit antenna is set to 1. When the second fading coefficient varies from 0.8 to 0.1, case 2 performs progressively worse than case 3.

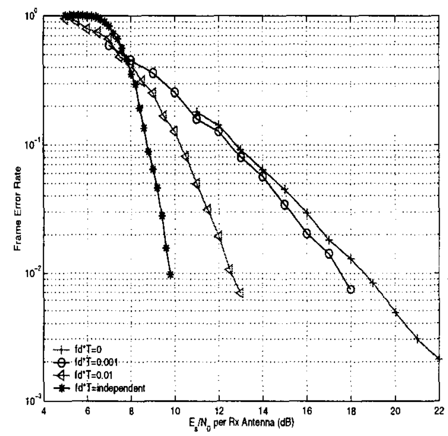

Fig. 7. Robustness of performance in slow to fast fading channel.

Figure 7 shows the performance of case 3 with different Doppler spread. The performance improved significantly when the temporal diversity is available. At frame error rate 0.01, the performance corresponds to the independent fading has a gain of 8 dB over the quasi-static fading case.

The performance of case 2 is also compared with the performance of a 32 state space-time trellis code [14] at different Doppler spreads. At frame error rate 0.01, the performance gain in terms of E_s/N_0 per receive antenna is shown in Figure 8. The gain increases with the Doppler spread. In independent fading channel, the gain is as much as 6.4 dB.

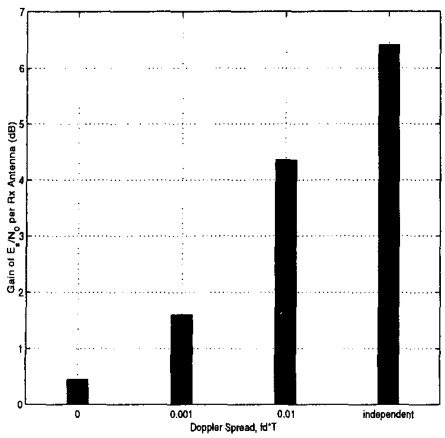

Fig. 8. Performance gain over a 32 state space-time trellis code [14] at different Doppler spread.

These figures show that the space-time turbo code has a significantly rich structure to take advantage of both the temporal and spatial diversity.

VI. Conclusions

We proposed a class of space–time parallel concatenated trellis codes, or space–time turbo codes. Methods based on rank theory [8] are given for full space diversity design. The structure of the code is flexible in that the code rate and the number of transmitter antennas can be easily adjusted according to design requirement.

The simulation results of an example code demonstrate that the code has full space diversity and that the performances are within 2.5 dB of the outage capacity in quasi-static fading channel. In fast fading channels, the code performance has 8 dB significant gain with the additional time diversity at frame error rate 0.01. The example code shows that we can use very simple component codes, e.g. 4 states, to obtain the same or better performance than previously proposed space–time codes with 64 states [3].

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