Acousto-optic photonic crossbar switch. Part I: design

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We present the design of a $12 \times 12$ photonic crossbar interconnection network constructed using a single three-dimensional acousto-optic crystal. Previous crossbars based on bulk acousto-optic cells require multichannel deflectors with one deflector per optical input; in contrast the design presented here angularly multiplexes these independent deflectors into a single-transducer acousto-optic device. A Fourier-optics analysis of an acoustically lossy Bragg deflector is coupled to a momentum-space analysis that permits the derivation of complete design equations for the switch. As a concrete example, the complete design of a $12 \times 12$ crossbar is presented. Finally, a coupled-mode analysis of the first- and second-order diffractions in the angularly multiplexed Bragg cell reveals the fundamental efficiency bounds of the switching network. © 1996 Optical Society of America

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1. Introduction

The transition from rf coaxial-based to optical-fiber-based communication systems has increased the carrier frequency for information transfer by 5 orders of magnitude. Low-attenuation windows in the fiber limit the useful bandwidth for long-distance transfer to tens of nanometers at several different wavelengths, but the available information rate is still greater than a terahertz. We have yet to exploit this rate fully, chiefly because the electronic systems that generate, switch, and accept the optical data cannot operate at terahertz clock rates. This speed mismatch inspires research into all-optical, or photonic, switching systems. Keeping the switched data in its optical format rather than converting it to electronics can allow the bandwidth of the switch to be as high as that of the fiber itself.

These photonic switches can generally be classified as time division, wavelength division, or space division. Time-division switches insert, extract, modulate, or permute multiple data streams of sequentially interleaved bits transmitted on a single fiber. Wavelength-division switches divide the large fiber bandwidth into multiple-frequency (multiple-wavelength) bands that can be inserted, extracted, or modulated, but usually not permuted. Finally, space-division switches interconnect and permute multiple (spatially distributed) fibers, as illustrated schematically in Fig. 1. If the switch (time, wavelength, or space division) can be reconfigured within a few bit periods, it can be used in a packet-switched network; otherwise it must be used in a circuit-switched environment such as video distribution.

All-optical, space-division interconnects currently under investigation can be further divided into three main categories: crossbars based on spatial masking, multistage interconnection networks based on $2 \times 2$ routing switches, and crossbars based on angular deflectors. The features that distinguish these different approaches are their switching speed, control complexity, optical insertion loss, and the types of interconnection they can implement.

A spatial-masking crossbar of $N$ inputs and $M$ outputs that is based on a classical matrix–vector product architecture is illustrated in Fig. 2. The $N \times M$ switches required to construct a complete crossbar are implemented as binary masks, which implies a control complexity of $N \times M$. To switch $P$-bit words, rather than single-bit channels, this architecture has
to be duplicated—lenses and all—\(P\) times, and the control complexity then is \(N \times M \times P\).

For large crossbars, the optical throughput of this switch is quite inefficient: The fan-out of each fiber to a column of \(M\) pixels results in an initial \(1/M\) power-splitting loss. For the switch to be inserted into a homogeneous network, the mode sizes of the input and output fibers must be the same; as a result, the fan-in of a row of \(N\) pixels causes a further \(1/N^2\) loss. This additional loss occurs because each different pixel within a row needs to be coupled into the output fiber, which means that the entire row must fit within the solid angle of the fiber’s numerical aperture. However, a single pixel fills only \(1/N\) of this acceptance angle, leading to a mode mismatch that produces a focused spot on the output fiber that is \(N\) times larger than the single-mode core, thereby coupling in only \(1/N^2\) of the incident light. Although \(1/N\) of this loss is fundamental because of the constant-radiance theorem, the other \(1/N\) loss can be avoided by the use of a cylindrical lenslet array. Thus the fundamental efficiency bound of the matrix-vector multiplier crossbar switch is \(1/(NM)\).

Switching matrices made from arrays of directional couplers and intersecting waveguide switches are also undergoing intense development for reconfigurable optical-interconnection applications. The directional coupler acts as a primitive \(2 \times 2\) switch, and this switch can be combined into larger networks.

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**Fig. 1.** Space-division crossbar switches: (a) single channel and (b) multiple channel. In this most general interconnect, the inputs fibers (left-hand side) can be broadcast, combined, and arbitrarily permuted to the output fibers on the right-hand side. Bit-serial networks need only single-channel switches, as shown in (a), whereas the interconnection of multiple-bit \((p = 4)\) buses requires either slaved single-channel crossbars for each bit or multiple-channel switches, as shown in (b).

**Fig. 2.** Space-integrating matrix–vector multiplier approach to a reconfigurable optical crossbar interconnection network. Note that the output spot is necessarily \(N\) times larger than the fiber core, as explained in the text. (Figure reprinted with permission from Ref. 3, p. 483).

**Shuffle-Exchange Multistage Interconnection Network**

**Fig. 3.** Diagram of a multistage network based on \(2 \times 2\) switches. (Figure reprinted with permission from Ref. 3, p. 481).
as the $3N \log N$ exchange–bypass modules in a multistage interconnection network, such as the shuffle-exchange network illustrated in Fig. 3. Two of the main disadvantages of the use of these switches in a centralized switching facility is the long interaction length that is typically required and the low bend radius that is a limitation of some waveguide technologies. In LiNbO$_3$ directional couplers, the interaction length is of the order of millimeters, and the permitted bend radius is so large that it would be difficult to fit more than approximately $9 \times 9$ cross points on a single substrate.$^9,10$ The long-range interconnections required of typical multistage interconnect networks require sharp bends and are conceivable only when etched, rather than diffused, guides are used. Although such etched guides are being developed in other materials, such as semiconductors in which coupling lengths can also be decreased owing to large carrier-induced index effects, currently the predominant technology is based on LiNbO$_3$.

These LiNbO$_3$ directional couplers are low loss and can overcome fundamental fan-in and fan-out losses with active amplifiers. However, to build large switching matrices, many $2 \times 2$ devices have to be stitched together, causing the accumulated loss to approach $25 \text{ dB}$ for a $144 \times 144$ switching matrix,$^9$ which is a worse loss than the $1/N$ loss suffered with other techniques. The control complexity is equal to the number of switches, each of which may require a unique, high-voltage signal. In addition, reflections and imperfect switching cause cross talk to build up to such a large extent as to make them unusable for analog applications and to degrade the signal-to-noise ratio (SNR) for digital applications, as well. From a fundamental architectural perspective, these switching matrices could be lossless, but the current technology is far from achieving this goal.$^{14}$

Several approaches to utilizing deflectors for interconnections have been advanced, including approaches based on periodic magnetic domains,$^{15}$ acousto-optic deflectors,$^{16–20}$ and holography.$^{21–23}$ Acousto-optic technology is the most highly developed and offers the highest reconfiguration speed.

Most acousto-optic approaches currently under investigation require one deflector per transmitter, which can be combined into a multichannel Bragg cell, as shown in Fig. 4. An array of fiber inputs uniformly spaced in $x$ are collimated in $y$ and imaged in $x$ onto a multichannel acousto-optic device, so that each acoustic channel is illuminated at the Bragg angle by one of the fiber inputs. This separate illumination can be accomplished with a sphere–cylinder lens pair (as shown). Each of the piezoelectric transducers is driven by a single rf tone, which launches a corresponding periodic acoustic wave that deflects the diffracted light from the fiber in the $y$ direction by an amount proportional to the applied frequency. A Fourier-transform lens is used to convert these diffraction angles into positions in $y$ and to collect the light from each of the input channels onto an array of output fibers positioned along the locus of diffracted spots in the Fourier plane. Each output-fiber position corresponds to a single rf frequency in the Bragg cell, and thus any input fiber can be deflected to a given output fiber by the application of the appropriate frequency. This system implements a crossbar with $N$ acousto-optic devices, but the inherent $N \times M$ crossbar complexity is available within the spatial aperture of the multichannel deflector and is hidden in the $N$-channel drive electronics. Each deflector channel can typically deflect
up to approximately 1000 resolvable angles, allowing up to a few hundred output fibers to be addressed. However, because 32-channel Bragg cells are already at the limit of this technology, these systems will probably be restricted to $32 \times 32$ crossbars.

The reconfiguration time for the acousto-optic deflectors is the aperture fill time, which is typically between 1 and 10 $\mu$s. The required aperture fill time decreases with the number of output fibers; a $32 \times 32$ switch could possibly be reconfigured in as few as 100 ns, at which point the switching time would probably become dominated by the reconfiguration time of the parallel electronic driving circuitry.

The diffraction efficiency of acousto-optic devices can approach 95%, but this level will be difficult to achieve in a multichannel Bragg cell with drive-power and thermal-dissipation limitations, so a 10% to 30% diffraction efficiency is more realistic. Although this deflector-based system has eliminated the fan-out losses when implementing a permutation network, it still suffers fan-in losses when the mode volume of the output fibers is the same as that of the input fibers. This case is illustrated in Fig. 4 by the elliptical focused spot incident on the output-fiber array and results in an overall efficiency of at most $\eta_{AO}/N$, where $\eta_{AO}$ is the acousto-optic (AO) diffraction efficiency and $N$ is the number of input fibers.

It is possible to improve the acousto-optic space-division switch by the use of two crossed, multichannel, acousto-optic devices. This design utilizes active deflectors in $y$ for the transmitters and slaved active-receiving deflectors in $x$ to eliminate the fan-in loss associated with the mode mismatch of the spatial fan-in of the above-described design. One can incorporate active positioning by surrounding each fiber with a quadrant detector array and using error signals to drive the rf voltage-controlled oscillators (VCO’s) when the diffracted spot slightly misses the fiber core. These active-positioning techniques have been demonstrated as slow reconfiguration speeds in a $42 \times 42$ fiber matrix switch that uses piezobimorph-positioned fiber deflectors and receivers. With these techniques, insertion losses of only a few decibels should be obtainable in the acousto-optic crossbar independently of $N$, but this loss is in addition to the cascaded diffraction efficiency $\eta_{AO}^2$, which could be as small as 1–10%, assuming a single-cell efficiency of 10–30%. The requirement for two multichannel Bragg cells and the associated $2N$-channel support electronics makes this solution to optical interconnects both costly and complex for the size of switch that is realizable.

In this paper, we present the design of an alternative acousto-optic crossbar switch that uses an anisotropic acousto-optic interaction in a single-channel Bragg cell. It is shown that this technology can support a $12 \times 12$ crossbar and can even implement a multiple-channel switch [Fig. 1(b)] simply by appropriate positioning of the additional fibers. The switch reduces the $N \times M \times P$ control complexity of a multichannel crossbar switch to one multitone rf signal, with as many tones as there are active inter-connection cross points. Individual tones within this signal can be inserted, deleted, or modified to change a portion of the switching matrix without disturbing the remainder. This flexibility makes it possible to implement advanced switching functions such as latency hiding. Although the light-throughput efficiency of the version of this crossbar presented in this paper is shown in Section 5 to scale to $1/N$, more sophisticated transducer designs permit a fundamentally lossless implementation of a permutation switch.

The $N$-input, $M$-output photonic switch based on this anisotropic acousto-optic interaction is depicted in Fig. 5(a). The optical input is a uniformly spaced array of $N$ single-mode fibers that are widely separated (by a precision V-groove array) and thus very overresolved, by a factor of approximately 80. The $M$ optical outputs are also arranged on a precision V-groove but, as is shown in Section 5, need to be just resolved. Because single-mode fiber cores are of the order of 5–10 $\mu$m and cladings are typically 90 or 125 $\mu$m, an array of lenslets is used as a single-lens imaging system to focus the just-resolved spots on each lenslet onto the smaller fiber cores.

As shown in Fig. 5(b), the overresolved input array is collimated by the first lens [Fig. 5(a), collimation lens], producing a set of beams with a large angular separation that are incident on the Bragg cell. For implementing a permutation of the $N$ inputs, the Bragg cell is driven by a superposition of $N$ widely spaced acoustic frequencies out of the lattice of $N \times M$ possible frequencies, shown in Figure 5(c), each of which is responsible for directing one particular input toward a selected output. The acousto-optic geometry and long transducer allow the propagation directions of the optical inputs to be arranged so that each fiber input is Bragg matched over only a narrow rf bandwidth, and the Bragg-matched rf bands for each input do not overlap. The second lens [Fig. 5(a), focusing lens] focuses the diffracted optical waves onto the destination lenslets, thereby coupling the appropriately permuted input optical signals into the output fibers.

The mismatch of the output-fiber mode profile and the switched optical beam shape that limits the efficiency of the matrix–vector spatial-masking crossbar and multichannel acousto-optic deflector switches does not occur in this design—with proper low-aberration lenses and careful design of the optical train, the insertion loss through this switch can be reduced to nearly the efficiency of the acousto-optic diffraction. A careful analysis of the diffraction through the lenses and the Bragg cell reveals the trade-offs of the various system parameters required to achieve this low-loss performance. This diffraction analysis is presented in Section 2, the results of which are used in the momentum-space design of Section 3. A complete example, including the acousto-optic crystal size and orientation, is presented in Section 4. The efficiency bounds of the switch are derived in Section 5, and several extensions to the basic design are presented in Section 6.
2. Diffraction Analysis

The size of a crossbar switch that may be implemented in an acousto-optic device is fundamentally constrained by the maximum number of degrees of freedom available in that device. This maximum information content, which in a Bragg deflector is equivalent to the number of Rayleigh-resolvable deflection positions, is equal to the change in the number of full acoustic waves that fit within the optical window of the cell when the drive frequency is swept from its lowest to its highest possible value. This well-known space–bandwidth or, equivalently, time–bandwidth product is limited because both the cell width or aperture time and the rf bandwidth are limited.

The bandwidth is constrained to a maximum of an octave (the highest frequency is twice the lowest) so that no inadvertently generated harmonics of any rf drive frequency lie within the bandwidth of operation. The cell width is limited both by the maximum size of the optical-quality crystal that can be grown and by the viscous acoustic loss. Although in some materials, such as LiNbO₃, the acoustic wave propagates across a reasonable-width Bragg cell with negligible damping, in other materials, such as the slow-shear mode of TeO₂, the acoustics decay exponentially at a rate of $18 \text{ dB/}(\mu \text{s GHz}^2)$, roughly a factor of 2 loss in 1 cm at a frequency of 100 MHz.

Because this loss is proportional to the rf frequency squared, increasing the center frequency of an octave-bandwidth Bragg cell in TeO₂ will eventually lead to a decrease in the time–bandwidth product, because the decay of the acoustics will reduce the width of the crystal in which diffraction takes place faster than the bandwidth increases. A commonly used rule of thumb for Bragg deflectors in acoustically lossy materials is to assume that the crystal is being well utilized if, at the highest rf frequency, the acoustics decay by at most a factor of 2 over the width of the cell.

This rule is rather ad hoc and does not yield good results when the center frequency, rf bandwidth, and crystal width are related by other constraints—such as those required to implement a crossbar, as derived.
deflector system. It is assumed that the incident acousto-optic Bragg defectors.

the 1/e contours of the optical intensity. The filled rectangles represent single-mode waveguides, although the analysis is equally valid for single transverse-mode free-space optics.

in Subsection 2.A. However, a simple Fourier-optics analysis of a standard 4-f deflector system reveals the quantitative trade-offs between cell width, acoustic loss, and total diffraction efficiency. This analysis applies equally well to the design of standard acousto-optic Bragg deflectors.

Figure 6 shows the layout of a 4-f acousto-optic deflector system. It is assumed that the incident optical beam has a Gaussian profile with an intensity (1/e) radius \( r_{\text{in}} \) given by

\[
E_{\text{in}}(x_{\text{in}}) = (r_{\text{in}} \sqrt{\pi})^{-1/2} e^{-[1/(2r_{\text{in}}^2)]|x_{\text{in}}|^2},
\]

which is valid for spatially filtered free-space beams and is a reasonably accurate approximation for the mode profile of single-mode guides such as fibers.\(^1\)

The field is normalized by the square root of its 1/e radius of intensity so that the total normalized intensity has a value of \( \int |E_{\text{in}}|^2 \, dx = 1 \).

In the absence of the Fourier-plane perturbations introduced by the Bragg cell, this Gaussian profile is recreated at the Fourier and output planes with the 1/e intensity radii \( r_B \) and \( r_{\text{out}} \), related by

\[
\frac{r_{\text{in}}}{f_{\text{in}}} = \frac{\lambda_0}{2r_B \pi} = \frac{r_{\text{out}}}{f_{\text{out}}},
\]

The relation between \( r_{\text{in}} \) and \( r_{\text{out}} \) is the usual telescopic-magnification formula. It is convenient to normalize quantities in the input, Fourier, and output planes by these radii, as shown by

\[
x_{\text{in}} = x_{\text{in}}/r_{\text{in}},
\]

\[
x_B = x_B/r_B,
\]

\[
x_{\text{out}} = x_{\text{out}}/r_{\text{out}}.
\]

respectively, so that the electric fields in the three planes, in the absence of the Bragg cell, can be written

\[
E_{\text{in}}(x_{\text{in}}) = (r_{\text{in}} \sqrt{\pi})^{-1/2} e^{-[1/(2r_{\text{in}}^2)]x_{\text{in}}^2},
\]

\[
E_B(x_B) = (r_B \sqrt{\pi})^{-1/2} e^{-[1/(2r_B^2)]x_B^2}
\]

\[
E_{\text{out}}(x_{\text{out}}) = (r_{\text{out}} \sqrt{\pi})^{-1/2} e^{-[1/(2r_{\text{out}}^2)]x_{\text{out}}^2},
\]

respectively.

In the low-diffraction-efficiency regime—which is valid in this case as demonstrated by the discussion in Section 5—the effect of the Bragg cell can be modeled as a mask whose transmission is proportional to the acoustic amplitude. This amplitude is an exponentially decaying sinusoid within the window of the Bragg cell and is zero elsewhere. If we assume that the Bragg-matching condition for the thick acousto-optic hologram is satisfied, the sinusoidal carrier of the Bragg cell and is zero elsewhere, and the effective-transmission mask of the Bragg cell \( T_B(x_B) \) may be written as

\[
T_B(x_B) = e^{-\alpha x_B/2 \cdot \text{rect}(x_B/A)},
\]

where \( \alpha \) is the frequency-dependent exponential decay constant of the acoustic intensity, \( \eta_{\text{AO}} \) is the peak acousto-optic diffraction efficiency, and \( A \) is the cell width. The asymmetric rectangular function \( \text{rect}(x) = 1 \) if \( 0 < x < 1 \), and \( \text{rect}(x) = 0 \) otherwise] truncates the diffracted beam to within the width of the acousto-optic crystal, which is assumed to have its transducer at the value \( x_B = 0 \) and to end at the value \( x_B = A \). The peak acousto-optic diffraction efficiency \( \eta_{\text{AO}} \) is evaluated at the transducer face \( (x_B = 0) \) and is the efficiency for an infinite cell without acoustic decay.

The electric field just past the Bragg cell is, therefore,

\[
E_B^+(x_B) = T_B(x_B) E_B(x_B - \tilde{x}_0)
\]

\[
= \frac{\eta_{\text{AO}}}{r_B \sqrt{\pi}} e^{-[(x_B - \tilde{x}_0 - i(k_B/2)])^2/2} \cdot \text{rect}(x_B/A)
\]

\[
= \frac{\eta_{\text{AO}}}{r_B \sqrt{\pi}} e^{(\alpha^2/4) - (\tilde{x}_0^2/2)} e^{-(x_B - \tilde{x}_0 + \alpha^2/2)^2/2}
\]

\[
\times \text{rect}(x_B/A),
\]

where the normalized quantities

\[
\tilde{\alpha} = r_B \alpha,
\]

\[
\tilde{x}_0 = \frac{x_0}{r_B},
\]

\[
\tilde{A} = \frac{A}{r_B},
\]

have been used to simplify the expression. The offset \( x_0 \) has been included to allow the optical beam to
be optimally placed within the Bragg-cell aperture. Equation (6) reveals that the product of a Gaussian and an exponential can be written as a shifted, attenuated Gaussian (by means of the mathematical procedure of completing the square). Note that this procedure does not change the width of the Gaussian, only the amplitude and center position. The Gaussian beam incident on the Bragg cell, the acoustic-loss profile, and the shifted Gaussian after the Bragg cell are shown for a typical set of parameters in Fig. 7.

A. Efficiency Loss Arising from Truncation and Acoustic Loss

Because the incident electric field was normalized to produce a unit intensity, the reduced efficiency of the Bragg cell caused by truncation and acoustic loss (\( \eta_{\text{TL}} \)) can be calculated directly:

\[
\eta_{\text{TL}} = \int_{-\infty}^{\infty} |E_B^{\prime}(\tilde{x}_B)|^2 d\tilde{x}_B
\]

\[
\eta_{\text{TL}} = \frac{\eta_{\text{AO}}}{\sqrt{\pi}} e^{(\alpha/2) - \tilde{x}_0} \int_{\tilde{x}_0}^{A} e^{-(\tilde{x}_B - \tilde{x}_0 + \tilde{\alpha}/2)^2} d\tilde{x}_B
\]

\[
\eta_{\text{TL}} = \frac{\eta_{\text{AO}}}{\sqrt{\pi}} e^{(\alpha/2) - \tilde{x}_0} \left[ \text{erf}(\tilde{A} + \tilde{\alpha}/2 - \tilde{x}_0) - \text{erf}(\tilde{\alpha}/2 - \tilde{x}_0) \right]/2,
\]

where \( \text{erf}(x) = 2\pi^{-1/2} \int_{0}^{x} \exp(-t^2)dt \) is the error function. Equation (8) is easily understood as the product of three terms: the acousto-optic diffraction efficiency in the absence of loss and truncation, the reduction in that efficiency caused by acoustic loss, and the further efficiency reduction caused by truncation at the upper and lower boundaries of the cell. This diffraction efficiency depends on centering the incident optical beam within the cell and can be maximized by the proper choice of \( \tilde{x}_0 \). Obviously, if the crystal is acoustically lossless, the beam should be centered, \( \tilde{x}_0 = \tilde{A}/2 \). However, as the acoustic loss increases, the region of efficient diffraction shifts toward the transducer face, and thus the optimal \( \tilde{x}_0 \) will decrease as well. The optimal position can be found in general by

\[
\frac{d\eta_{\text{TL}}}{d\tilde{x}_0} = 0 = \frac{1}{\sqrt{\pi}} [e^{-(\tilde{\alpha}/2 - \tilde{x}_0)^2} - e^{-(\tilde{A} + \tilde{\alpha}/2 - \tilde{x}_0)^2}] + \frac{\tilde{\alpha}}{2} [\text{erf}(\tilde{\alpha}/2 - \tilde{x}_0) - \text{erf}(\tilde{A} + \tilde{\alpha}/2 - \tilde{x}_0)], \tag{9}
\]

which can be solved with Newton’s method to yield the plot shown in Fig. 8. Using this optimal beam position in Eq. (8) yields the maximum diffraction efficiency versus the crystal width and the acoustic loss, as shown in Fig. 9. The beam position shown in Fig. 7 is chosen to satisfy this maximum-efficiency condition [Eq. (9)], yielding an efficiency of \( \eta_{\text{TL}} = \eta_{\text{AO}} - 2.5 \) dB.

The contours of Fig. 9 reveal that, for a given acoustic loss coefficient and an optimally positioned Gaussian beam, the diffraction efficiency first increases strongly with increasing cell width and then becomes constant. This constant upper limit of efficiency is caused by the fact that the exponential acoustic-loss profile enforces an effective cell width; after the acoustic wave has decayed to nearly zero, additional crystal width does not contribute to the acousto-optic interaction and is wasted. Below the bend in the...
contours (that is, for small width) the diffraction efficiency is a strongly increasing function of $A$ because large amounts of the incident optical beam are being lost past the edges of the cell. Therefore, devices should be designed to operate near the break point of the efficiency contours because wider devices waste crystal material, whereas narrower devices waste incident light. As shown in Fig. 9, the 3-dB rule falls in this optimal region for cells with an efficiency of $\eta_{\text{AO}}/2$, but over- or under-constrains the crystal width for less efficient or more efficient devices, respectively.

\section*{B. Focused Beam Profile}

Having chosen the optimal beam placement by maximizing the diffraction efficiency, we may now Fourier-transform the diffracted field just past the Bragg cell to find the profile of the electric field at the output plane (see Fig. 6). For an acoustically lossless device that is much wider than the incident Gaussian, this output field has a Gaussian profile with a $1/e$ intensity radius $r_{\text{out}}$ [the third of Eqs. (4)]. If this wide cell is used as a scanner for free-space beams, the permissible overlap of these beams determines the number of useful deflection positions. This overlap is a function of the distance between spots $D_{\text{out}}$ and is found by the calculation of the square of the electric-field overlap (ovrlp) integral for two Gaussian spots separated by $D_{\text{out}}$ or, more concretely, the square of the autocorrelation:

$$
\eta_{\text{ovrlp}}(D_{\text{out}}) = \left[ \int_{-\infty}^{\infty} E_{\text{out}}(x_{\text{out}}) E_{\text{out}}(x_{\text{out}} - D_{\text{out}}) \, dx_{\text{out}} \right]^2 = e^{-D_{\text{out}}^2/2},
$$

(10)

where $D_{\text{out}} = D_{\text{out}}/r_{\text{out}}$ is the resolvability of the output spots. Applications that require clearly separated spots require a value for $D_{\text{out}}$ that is much greater than one. Conversely, more spots can be addressed with a smaller separation if a larger overlap is permissible, but the smallest separation usually permitted corresponds to the Rayleigh-resolvability criterion, which (as shown in Subsection 2.C) occurs for a uniformly filled Bragg cell at the value $D_{\text{out}} = 2\pi/\lambda$.

Equation (10) applies equally well when the acousto-optic device is used as a switch between single-mode optical guides. In this case, the amplitude of the guided mode excited by the deflected electric field is calculated by an inner product (overlap integral) of the electric field and the mode profile. With Fresnel reflection losses neglected, the light will couple perfectly into the guide only if the electric-field profile exactly matches the mode shape so that their correlation is unity. As the deflected spot scans past the guide, the square of the correlation decreases, yielding cross talk as a function of guide separation, which is Eq. (10).

When the Bragg cell is of finite width and acoustically lossy, the electric-field profile at the output plane becomes

$$
E_{\text{out}}(x_{\text{out}}) = F[E_{\text{B}}^{+}(x_{\text{B}})]_{x=x_{\text{out}}/\sqrt{\eta_{\text{out}}}}
$$

$$
= \left( \frac{\eta_{\text{AO}}}{r_{\text{out}} \sqrt{\pi}} \right)^{1/2} e^{i\pi^2/4 - \tilde{\alpha}^2/8} e^{-\tilde{x}_{\text{out}}^2/2}
$$

$$
\times \frac{1}{2} \left( \text{erf}(\tilde{A} + \tilde{\alpha}/2 - \tilde{x}_0)/\sqrt{2} + j\tilde{x}_{\text{out}} \right]
$$

$$
- \text{erf}(\tilde{A}/2 - \tilde{x}_0)/\sqrt{2} + j\tilde{x}_{\text{out}} \right].
$$

(11)

The first exponential term on the right-hand side in the second equality is an overall decrease of power, the second exponential term is the nominal focal spot unperturbed by the acousto-optic device in the Fourier plane, and the term in curly braces broadens this ideal spot. This complex-error-function term causes sidelobes as a result of the sharp truncation of the beam in the Fourier plane. Figure 10 shows a typical output-field profile corresponding to the Fourier-plane field rendered in Fig. 7. As shown in Fig. 11, these sidelobes cause the diffracted field to match imperfectly to a Gaussian guided-mode profile, reducing the efficiency of the device by an additional factor,
The correlation of these two profiles, as defined by Eq. (12), is 0.8, or -1 dB.

the output coupling efficiency \( \eta_{OC} \)

\[
\eta_{OC} = \left| \int_{-\infty}^{\infty} E_{\text{out}}(x_{\text{out}}) E_{\text{guide}}(x_{\text{out}}) \, dx_{\text{out}} \right|^2
\]

\[
= \left| \int_{-\infty}^{\infty} E_{\text{out}}(x_{\text{out}}) \pi^{-1/4} e^{-x_{\text{out}}^2/2} \, dx_{\text{out}} \right|^2.
\]

For narrow cells \( \tilde{A} < 3/2 \) the output coupling efficiency \( \eta_{OC} \) can be slightly improved by an adjustment of the width of the guided-mode profile. In this analysis we are interested in Bragg cells with at most 50% truncation losses, which implies values of \( \tilde{A} \geq 3/2 \) (see Fig. 9), and thus we assume the waveguide profile is identical to the unperturbed output electric field.

C. Cross Talk

The sidelobes of the diffracted field in the output plane also increase the cross talk. The amount of cross talk for different waveguide separations is calculated from Eq. (12) with the inclusion of an offset of the guided-mode profile by the normalized guide separation \( D_{\text{out}} \), given by

\[
\eta_{X}(\tilde{D}_{\text{out}}) = \left| \int_{-\infty}^{\infty} E_{\text{out}}(x_{\text{out}}) E_{\text{guide}}(x_{\text{out}} - \tilde{D}_{\text{out}}) \, dx_{\text{out}} \right|^2,
\]

which is the square of the correlation of the output field and the mode profile of the adjacent guide. As shown in Fig. 12, the cross talk decreases most rapidly with guide separation when the sidelobes of the output electric-field profile are small, which occurs in wide cells with little acoustic loss. Note that this calculation of cross talk assumes that only the acousto-optically diffracted optical beam is incident upon the output fibers; surface or bulk scattering can deflect small amounts of the undiffracted optical beam toward the outputs. However, because this acousto-optic interaction is polarization switching, polarization filtering can remove much of this unwanted background.

Figure 12 quantifies how the distance between deflection positions affects the cross talk between those positions. Because the Bragg cell deflects the beam over a finite range of output positions, the total number of those positions is inversely proportional to their separation \( D_{\text{out}} \). In normalized variables, the deflected spot position is

\[
\tilde{x}_{\text{out}} = \tilde{K},
\]

where \( \tilde{K} = r_B K = 2\pi r_B f / V_A \) is the normalized acoustic wave number given an acoustic velocity of \( V_A \). Letting the rf frequency vary over its total bandwidth \( B \) and counting the number of resolvable positions separated by \( D_{\text{out}} \) yields the total number of resolvable positions:

\[
N_{\text{spots}} = \frac{2\pi r_B}{D_{\text{out}}} \frac{B}{V_A}.
\]

This expression is in the familiar form of a time-bandwidth product, and thus the effective aperture of the Gaussian-illuminated, acoustically lossy Bragg cell is

\[
A_{\text{eff}} = \frac{2\pi r_B}{D_{\text{out}}},
\]

which, for a typical output resolvability of the value \( D_{\text{out}} \approx 3 \), is equal to the 1/e intensity diameter of the optical beam incident upon the Bragg cell.
In the limit that the Gaussian optical beam is much wider than the Bragg cell and acoustic loss is negligible, the diffracted field has a rectangular profile of width $A$ and its Fourier transform in the output plane is a sinc function. The Rayleigh-resolvability criterion\(^8\) places the maximum of a spot on the first zero of the adjacent spot, i.e., $D_{\text{out}} = \lambda_0 f_{\text{out}} / A$ or $D_{\text{out}} = 2\pi / A$, which yields

$$N_{\text{spots}} = A \frac{B}{V_A},$$

(17)

and thus $A_{\text{eff}} = A$. This is the familiar Rayleigh-resolved information content of a rectangular deflector, as discussed in the first paragraph of Section 2.

D. Wavelength Tolerance

The dependence of the deflection position on the optical wavelength yields the chromatic tolerance of the switch. A change in wavelength $\Delta \lambda_0$ causes a change in the deflection position, as given by

$$\Delta \bar{x}_{\text{out}} = \bar{K} \frac{\Delta \lambda_0}{\lambda_0}.$$  

(18)

The wavelength change that causes the spot to move a resolvable spot, $\Delta \bar{x}_{\text{out}} = \bar{D}_{\text{out}}$, is

$$\frac{\Delta \lambda_0}{\lambda_0} = \frac{1}{N_{\text{spots}}}. $$

(19)

---

Fig. 12. Coupling efficiency to the neighboring waveguide (cross talk) plotted as a function of the acoustic loss and the cell width for increasing waveguide resolvability. The solid contours are drawn at intervals of 5 dB, and the dotted contours at 2.5-dB intervals.
For a typical TeO$_2$ Bragg cell with of the order of 1000 spots, Eq. (19) reveals a roughly 1-nm wavelength tolerance. This tolerance requires temperature-stabilized laser diodes or feedback control of the rf frequency, as will be discussed in a second part of this paper, now underway. Equation (19) also reveals that the information-bandwidth limit of a switch constructed in a 1000-spot Bragg cell is roughly 100 GHz, which, although much less than the fiber bandwidth, is larger than any fiber-communication system currently demonstrated.

This subsection completes the diffraction analysis of an acoustically lossy, finite-width Bragg deflector. The important results are expressed by Fig. 8, which demonstrate the determination of the optimal centering of the collimated beam in the crystal aperture, and by Fig. 9, which reveals the decrease in efficiency resulting from truncation by the finite cell and the acoustic-loss profile. For situations in which only the total amount of diffracted light is important, these Figs. 7 and 9 relate the tolerable truncation loss to the crystal size and the acoustic attenuation. When the profile of the diffracted light is important, for example, for coupling into single-mode fibers, Fig. 11 shows the additional loss that is due to mode mismatch with the waveguide mode. Finally, the results illustrated in Fig. 12 allow the designer to choose the minimum output-waveguide separation required to achieve a desired level of optical cross talk.

In Section 3, it is assumed that the normalized crystal width $A$, the normalized acoustic loss coefficient $\alpha$, and the fiber resolvability in the output plane $D_{\text{out}}$, have been chosen using the data from Figs. 7, 9, 11, and 12. These fixed values then enter the design equations without reference to the complex calculations that were required to find them. This simplifies the remaining design task to simple geometric calculations in momentum space.
3. Momentum-Space Design

The momentum-space technique (also referred to as \( k \) space or inverse space)\(^{2,20} \) is a common analytic and graphic design method that calculates the diffracted optical wave in the undepleted pump (or first-order Born) limit and is illustrated for the case of the switch presented in this paper in Fig. 13. Although a complete development of the method is beyond the scope of this paper, it can be conceptually understood if one notes that the material polarization field, which is the source term in the electromagnetic-wave equation for the diffracted optical field, is proportional to the product of the acoustic-strain field and the incident optical electric field \( \left[ P(\mathbf{r}) \sim S(\mathbf{r}) E(\mathbf{r}) \right] \). By Fourier transformation of this wave equation and the employment of the orthogonality of the individual plane-wave components, one finds that the amplitude of the diffracted optical field with a particular wavevector \( k_{\text{diff}} \) is proportional to the Fourier transform of the material polarization evaluated at \( k_{\text{diff}} \). Because in real space \( P \) is given by the product of the acoustic and incident optical fields, it is given by the convolution of their transforms in Fourier space. For infinite media this results in the commonly employed vector sum of \( k \) vectors. In finite media, the convolution takes into account all effects of partial Bragg matching that are due to a finite beam width and crystal size through the Fourier transforms of the finite acoustic- and optical-field distributions.

The use of this momentum-space technique in the case of the acousto-optic crossbar is shown in Fig. 13. The full \( k \) space (upper left-hand image) shows the locus of the ordinary (dashed curve) and extraordinary (solid curve) optical wave vectors, as well as the specific \( k \) vectors for the incident and diffracted waves. A detail of the interaction region (lower left-hand image) shows the anisotropic, polarization-switching diffraction by means of the acousto-optic grating. A close-up of the output region (lower right-hand image) shows that each of the extraordinarily polarized output \( k \) vectors lies on the extraordinary \( k \) surface and that angular variation of the diffraction is given by the evaluation of the Fourier polarization distribution on this extraordinary \( k \) surface. The polarization distribution, rendered as a contour plot, is the Fourier transform of the real-space acousto-optic polarization field, shown in the upper right-hand image. In this case, the product of the rectangular, incident, optical beam and the crossed, rectangular, acoustic beam yields a rectangular polarization whose transform is a sinc function in each dimension, as shown in the lower right-hand image. Such plots allow one to quickly understand and calculate the fundamental properties of birefringent Bragg devices, in which the Bragg-matching formulas are more complex.

Birefringent Bragg diffraction in anisotropic crystals has been used for many years to create high-performance acousto-optic deflectors. In these anisotropic media, the directions of the incident and diffracted light strongly influence the behavior of the interaction. A useful and common technique to improve efficiency and bandwidth is tangential matching, in which the acousto-optically generated polarization vector skims tangentially across the diffracted wave-vector surface.\(^{31} \) The diffracted light is therefore Bragg matched over a wide angular range, resulting in a deflector with a large number of resolvable deflection positions.

Our acousto-optic crossbar switch requires exactly the opposite condition. As shown in Fig. 14, we arrange the optical beams such that the central acousto-optic momentum vector is tangential to the incident optical \( k \) surface, rather than to the diffracted wave-vector surface.
fractured $k$ surface, and thus this arrangement is referred to as the antitangential condition. As a result of this antitangential arrangement, the incident optical beams have a wide angular range over which Bragg matching can be obtained because the ordinary (inner) momentum surface is tangential to the acoustic-deflection wave vector, whereas the diffracted optical beams are limited to a much smaller angular range because the acousto-optic momentum vector cuts across the extraordinary (outer) momentum surface at an angle that depends on the optic and acoustic wavelengths. The inputs to the switch can thus be spaced widely enough that, for a sufficiently long acousto-optic crystal, an acoustic grating that is Bragg matched to deflect an input to any output will be Bragg mismatched for neighboring inputs. This is precisely the condition required to implement a crossbar switch: Each input beam responds to only its own narrow band of acoustic frequencies; within this narrow band, individual frequencies deflect that input to any one of the outputs. The multiple Bragg deflectors required to achieve the device shown in Fig. 4 have thus been folded into a single, frequency-multiplexed acousto-optic cell. This multiplexing requires a precise arrangement of the inputs and outputs in relation to the other device parameters—these relations can be discovered with the momentum-space method described above.

We start this analysis by noting that the bulk lenses perform spatial Fourier transforms and thus establish a relation between the fiber spacings and the angles of propagation within the crystal:

$$\Delta k_{x,\text{in}} = \frac{2\pi D_{\text{in}}}{\lambda_0 f_{\text{in}}},$$
$$\Delta k_{x,\text{out}} = \frac{2\pi D_{\text{out}}}{\lambda_0 f_{\text{out}}},$$

(20)

where, as shown in Fig. 6, $f_{\text{in}}$ and $f_{\text{out}}$ are the focal lengths of the input and output lenses, respectively, $D_{\text{in}}$ and $D_{\text{out}}$ are the spacings of the input and output fibers, respectively, and, as shown in Fig. 14, $\Delta k_{x,\text{in}}$ and $\Delta k_{x,\text{out}}$ are the differences between the transverse component of adjacent input and output optical wave vectors both outside of and within the crystal (as the crystal face is normal to the optical axis of the system), respectively. The operation of the switch depends strongly on these separations; the solution for these quantities in this section will fix all of the design parameters.

Using the normalized variables defined in Section 2 [e.g., Eqs. (7)] permits Eqs. (20) to be written in the more revealing forms of

$$\Delta k_{x,\text{in}}r_B = \bar{D}_{\text{in}},$$
$$\Delta k_{x,\text{out}}r_B = \bar{D}_{\text{out}},$$

(21)

which establishes the relation between the angular spacings and resolvabilities of the inputs and out-

puts:

$$\frac{\Delta k_{x,\text{out}}}{\Delta k_{x,\text{in}}} = \frac{\bar{D}_{\text{out}}}{\bar{D}_{\text{in}}}.\quad (22)$$

The angular spacing of the inputs is not arbitrary because, as shown in Fig. 15, the input optical momentum surface is tangential to only the acoustic deflection vector at the center of the angular range. The acousto-optic rf bandwidth is therefore limited by the curvature of the optical wave-vector surface, which Bragg-mismatches inputs at large angles by shifting the acousto-optically generated polarization field in the $k_z$ direction.

Fig. 15. (a) The curvature of the input momentum surface is shown to limit the available angular range by Bragg-mismatching the diffraction as the angle increases. (b) A close-up diagram that illustrates the design constraint that this shift should cause a maximum of a 3-dB decrease in efficiency. (Figure reprinted with permission from Ref. 3, p. 520)
As outlined at the beginning of Section 3, the diffraction efficiency of the acousto-optic interaction will fall by at most 3 dB if the diffracted optical wave vectors lie within the 3-dB contour of the shifted polarization distribution. This 3-dB contour is often referred to as the uncertainty of the polarization because, like all transform pairs, the finite size of the polarization field in real space results in an uncertainty of its location in Fourier space, and thus the angle of Bragg diffraction is a finite range. This Bragg-matching condition, illustrated in Fig. 15(b), implies that any input can be coupled to any output with a maximum efficiency variation of 3 dB across all connections. In terms of the quantities in Fig. 15(b), the following relation must be satisfied:

\[ k_{z,\text{width}} - k_{z,\text{shift}} \geq k_{z,\text{out}}, \]

(23)

where \( k_{z,\text{out}} \) is the extent of the output optical wave vectors in the \( k \) direction, \( k_{z,\text{shift}} \) is the \( k_z \)-directed displacement of the polarization distribution as a result of input curvature, and \( k_{z,\text{width}} \) is the full width at half-maximum of that polarization distribution caused by the finite transducer length (the uncertainty). When the equality [condition (23)] is satisfied, the device is optimally designed to implement a crossbar switch. However, acousto-optic devices that satisfy the inequality of condition (23), for example, with an excessively wide uncertainty \( (k_{z,\text{width}}) \), can also function but usually at the cost of some performance (e.g., the diffraction efficiency). This uncertainty width is determined from the intensity distribution of the acoustic angular spectrum \( P_A(k_z) \) launched from the transducer. For rectangular transducers, the most common transducer shape, the \( k_z \) variation of the uncertainty distribution takes the form

\[ P_A(k_z) = \sin^2 \left( \frac{k_z L}{2\pi} \right), \quad \sin(x) = \frac{\sin(\pi x)}{\pi x}, \]

(24)

where \( k_z \) is the component of acoustic momentum in the [001] direction and \( L \) is the width of the rectangular transducer. Because \( \sin^2 (0.5) \approx -4 \text{ dB} \), a convenient approximation to the value of \( k_{z,\text{width}} \) is

\[ 2\pi/L. \]

The other two terms in condition (23) can be found from Fig. 15. First, the total width occupied by the output optical states, \( k_{z,\text{out}} \), can be derived directly from Fig. 15(b) by the use of the slope of the output momentum surface, which in the isotropic approximation is given by \( \lambda/\Lambda \), yielding

\[ k_{z,\text{out}} = \left( \frac{\lambda}{\Lambda_0} \right) (M - 1) \Delta k_{x,\text{out}}, \]

(25)

where \( \lambda = \lambda_0/n \) is the optical wavelength in the material. Second, the shift in the acousto-optic polarization caused by the curvature of the input optical wave-vector surface, \( k_{z,\text{shift}} \), can be seen in Fig. 15(a). An accurate approximation for the magnitude of this shift can be found by means of a parabolic expansion of the nearly spherical ordinary optical momentum surface; the expansion is valid near the optical axis.

As illustrated in Fig. 15(a), the largest shift occurs between the middle input and those on the edges, yielding

\[ k_{z,\text{shift}} = \left( \frac{N - 1}{2} \right) \frac{\Delta k_{x,\text{in}}^2}{2k}, \]

(26)

where \( k = 2\pi n_0/\lambda_0 \) is the wave vector of the ordinary-polarized light near the \( z \) axis. The distortion of the momentum surface caused by optical activity is neglected here.

Using the quantities from Eqs. (25) and (26) in condition (23) yields the condition on \( L \) for efficient coupling to all output positions:

\[ \frac{2\pi}{L} - \left( \frac{N - 1}{2} \right) \frac{\Delta k_{x,\text{in}}^2}{2k} = \frac{\lambda}{\Lambda_0} (M - 1) \Delta k_{x,\text{out}}. \]

(27)

Equation (27) imposes the constraint that all of the desired connections between inputs and outputs be Bragg matched to within 4 dB.

To create independent connections, however, undesired connections must be Bragg mismatched. As shown in Fig. 16, this is accomplished by spacing the inputs very far apart in relation to the outputs. The narrow angular bandwidth over which the output diffractions can be Bragg matched thus causes the undesired couplings to be inefficient. The strength of these couplings is determined by the magnitude of the Fourier transform of the polarization distribution, sampled on the locus of the diffracted optical \( k \) vector, and thus depletion will be small if the distance from the center of the polarization is far from the nearest point on the \( k \) surface (strong Bragg mismatch). Using Eq. (24), the polarization distribution in \( k \) space, and defining the distance from its center to the nearest point on the diffracted \( k \) surface as \( \Delta k_{x,\text{dep}1} \), shows the efficiency of the undesired depletion to be

\[ \eta_{\text{undesired}} = \eta_{\text{AO}} \sin^2 \left( \frac{k_{x,\text{dep}1} L}{2\pi} \right), \]

(28)

where \( \eta_{\text{AO}} \) is the peak acousto-optic diffraction efficiency. This undesired diffraction can be viewed as a source of insertion loss within the switch: The light diffracted inadvertently from the inputs in this manner falls outside the exit fibers and is lost. The position of this undesired diffraction is sufficiently far from all the output-fiber positions that it is not a source of cross talk. The situation illustrated in Fig. 16 is the worst-case circumstance of neighboring inputs near the edges of the input angular band; the diffraction of the next input by this same grating will be Bragg mismatched by an amount of \( 2k_{x,\text{dep}1} \). We thus choose the quantity \( k_{x,\text{dep}1} = 2\pi/L \), which (at worst) places the output surface on the first zero of the sinc function, hence obtaining the value \( \eta_{\text{undesired}}/\eta_{\text{AO}} \approx -13 \text{ dB} \), which is the maximum of the sinc\(^2 \) function past its first zero. In other words, the worst-case throughput generated by this undesired diffraction is 95%. 

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Substituting both conditions (29) and (30) for the Bragg mismatch of unwanted connections plus the relation between fiber spacings and angles of propagation imposed by the 4-f system [Eq. (22)] into the constraint for efficient coupling of all desired connections [Eq. (27)] yields an expression that can be solved for the input angular separation, which is our goal:

\[
\Delta k_{x,in} = K_0 8^{1/3} - \left( M - 1 \right) \frac{\bar{D}_{out}/\bar{D}_{in}}{\left( N - 1 \right)^2} = K_0 F/(N - 1),
\]

where \( K_0 = 2\pi f_0/V_A \) is the acoustic wave vector at the center-band rf frequency \( f_0 \) and \( F \) is defined by the equation. \( F \) is the fractional bandwidth of the acousto-optic device used for the \( N \times M \) interconnection, defined as the rf bandwidth \( B \) over the center frequency \( f_0 \). The validity of its use in Eq. (31) can be seen by the calculation of the total bandwidth:

\[
B = \Delta K/(2\pi/V_A) = \frac{V_A}{2\pi} (N - 1) \Delta k_{x,in},
\]

where \( \Delta K \) is the difference between the largest and the smallest acoustic wave vectors and approximately equals \( (N - 1) \Delta k_{x,in} \) because \( \Delta k_{x,\text{out}} \ll \Delta k_{x,\text{in}} \). The fractional bandwidth is thus

\[
F = B/f_0 = (N - 1) \Delta k_{x,in}/K_0 = 8^{1/3} - \left( M - 1 \right) \frac{\bar{D}_{out}/\bar{D}_{in}}{\left( N - 1 \right)^2},
\]

which agrees with the definition above. As discussed in Section 1, the fractional bandwidth is limited to a maximum value of 2/3 so that the harmonics of the drive frequencies always lie outside the operational bandwidth. We have now found the proper spacing for the input and output fibers to implement a crossbar interconnect that exhibits efficient coupling of all inputs to all outputs, with a maximum of a 3-dB efficiency ripple, while simultaneously suppressing all undesired depletion by at least \(-13 \text{ dB}\).

The last quantity to be fixed is the rf center-band frequency \( f_0 \). It is advantageous to make the frequency as large as possible because this increases the time bandwidth of the acousto-optic device, yielding a potentially larger switching matrix. Alternatively, given a desired switch size, increasing the frequency of operation linearly decreases all switch dimensions. In TeO\(_2\), the rf frequency is limited by acoustic loss, as discussed in Section 2, for which the maximum normalized acoustic attenuation coefficient \( \bar{\alpha} \) was found for a given truncation loss and coupling efficiency. Because this viscous loss is proportional to the rf frequency squared, we set it equal to the loss at the highest rf-switching frequency:

\[
\alpha = \frac{\bar{\alpha}}{r_B} = \frac{C}{V_A} (f_0 + B/2)^2,
\]

where \( C \) is a material-dependent damping coefficient. In TeO\(_2\), \( C \) has a value of 18 dB/\( \mu \text{s GHz}^2 \), or, in
plexed Bragg cell. In an essentially independent, angularly multi-

crossbar design.

The above-described relations can be used to de-

cally less than the upper limit of $F = 2/3$, however,

Typical TeO$_2$ deflectors have of the order of 1000 resolvable spots; Eq. (37) provides a check on the ability to construct the switch.

Note that this space-division photonic switch can be designed to switch any optical wavelength at which the acousto-optic material is transparent and is therefore not constrained to a particular, narrow optical bandwidth like some other technologies (e.g., self-electro-optic-effect devices$^{32}$). The fractional bandwidth and center frequency are essentially independent of the optical wavelength, and the data listed in Table 1 reveal that the longitudinal, but not the transverse, dimensions of the switch vary inversely with the optical wavelength. For a given acoustic power, the diffraction efficiency decreases like the square of optical wavelength, so devices that operate at long wavelengths may be less efficient.$^{31}$

The above-described relations can be used to design a special-purpose acousto-optic Bragg cell and the associated optical components to implement a crossbar switch. For example, Table 2 lists the parameters for the design of a $12 \times 12$ crossbar in TeO$_2$ operating at a free-space wavelength of 850 nm.

### Table 1. Design Equations for an $N \times M$ Crossbar Switch

<table>
<thead>
<tr>
<th>Design Element</th>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transducer length</td>
<td>$L = L_0 \frac{3N - 1}{2F}$</td>
<td></td>
</tr>
<tr>
<td>Cell aperture width</td>
<td>$\tilde{A} = \tilde{A} \frac{D_{in}N - 1}{K_0 F}$</td>
<td></td>
</tr>
<tr>
<td>Collimation-lens focal length</td>
<td>$f_{in} = \frac{\lambda_0 N - 1}{F}$</td>
<td></td>
</tr>
<tr>
<td>Focus-lens focal length</td>
<td>$f_{out} = \frac{r_{out}}{r_{in}} f_{in}$</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2. Design Parameters for a $12 \times 12$ Crossbar Switch Operating at a Wavelength of $\lambda_0 = 850$ nm in TeO$_2$

<table>
<thead>
<tr>
<th>Design Element Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of inputs $N$</td>
<td>12</td>
</tr>
<tr>
<td>Number of outputs $M$</td>
<td>12</td>
</tr>
<tr>
<td>Wavelength $\lambda_0$</td>
<td>850 nm</td>
</tr>
<tr>
<td>Fiber mode radius $r_{in}$</td>
<td>2.5 (\mu)m</td>
</tr>
<tr>
<td>Truncation loss $\gamma_{TL}$</td>
<td>$-2.5$ dB</td>
</tr>
<tr>
<td>Output coupling loss $\gamma_{OC}$</td>
<td>$-1.0$ dB</td>
</tr>
<tr>
<td>Cross talk $\gamma_{X}$</td>
<td>$-30$ dB</td>
</tr>
<tr>
<td>Normalized aperture $A$</td>
<td>1.92</td>
</tr>
<tr>
<td>Normalized acoustic attenuation $\tilde{\alpha}$</td>
<td>0.433</td>
</tr>
<tr>
<td>Output resolvability $D_{out}$</td>
<td>6.3</td>
</tr>
<tr>
<td>Output-fiber spacing $D_{in}$</td>
<td>250 (\mu)m</td>
</tr>
<tr>
<td>Input resolvability $D_{out}$</td>
<td>300</td>
</tr>
<tr>
<td>Input-fiber spacing $D_{in}$</td>
<td>750 (\mu)m</td>
</tr>
<tr>
<td>Fractional bandwidth $F$</td>
<td>0.317</td>
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<tr>
<td>Center frequency $f_0$</td>
<td>47 MHz</td>
</tr>
<tr>
<td>Transducer length $L$</td>
<td>23.7 mm</td>
</tr>
<tr>
<td>Optical aperture $A$</td>
<td>42.0 mm</td>
</tr>
<tr>
<td>Collimation-lens focal length $f_{in}$</td>
<td>400 mm</td>
</tr>
<tr>
<td>Focus-lens focal length $f_{out}$</td>
<td>6414 mm</td>
</tr>
<tr>
<td>Time bandwidth of cell $T_B$</td>
<td>1100</td>
</tr>
</tbody>
</table>

The 12 x 12 crossbar switch is designed to operate with a total insertion loss equal to the acousto-optic diffraction efficiency minus 3.5 dB and to have a peak cross-talk level of $-30$ dB. The input- and output-fiber spacings are chosen to be multiples of 250 \(\mu\)m, which is a standard for precision silicon V grooves and lenslet arrays.

mks units, $18 \ln(10) \times 10^{-13}$ Np/Hz$^2$. Solving for $f_0$ we find

$$f_0 = \frac{2\pi}{C(N - 1)D_{in}} \frac{F}{(1 + F^2/2)^2},$$

which completes the derivations necessary for the crossbar design.

For a desired switch of size $N \times M$, the fractional bandwidth $F$ is calculated from Eq. (33) and the center-band frequency of operation $f_0$ from Eq. (35). All other switch parameters are determined by these quantities, as summarized in Table 1. The acousto-optic characteristic length, $L_0 = \lambda_0 a^2/\lambda$, has been used in the definition of $L$. This length is the boundary between thin ($L < L_0/2$) and thick ($L > 2L_0$) holographic diffraction.$^{31}$

Table 1 shows that, for an octave-bandwidth device ($F = 2/3$), the transducer length has a value of $L \approx 2(N - 1)L_0$. This reveals how the $N$ distinct Bragg cells of the multiple-channel acousto-optic deflector switch have been multiplexed into a single device: The transducer is $N - 1$ times longer than a standard $2L_0$ deflector, resulting in a Bragg-matched angular bandwidth that is $N - 1$ times smaller. If the $N$ inputs are uniformly spaced by this reduced angular bandwidth (and thus span $N - 1$ bands), each operates in an essentially independent, angularly multiplexed Bragg cell.

This multiplexing of $N$ independent scanners into a single acousto-optic device is limited by the maximum number of degrees of freedom available in the Bragg cell. The number of resolvable spots used by the switch can be found from Eq. (15) to be

$$N_{\text{spots}} = \frac{(N - 1)\Delta k_x \Delta k_{z,\text{out}}}{\Delta k_{z,\text{in}}}$$

Equation (36a) states that the number of deflection positions is equal to the total angular bandwidth over the separation of the outputs, which is to be expected. Equation (36b) states the same result in terms of the physical parameters of the switch. Because the fractional bandwidth determined by Eq. (33) is typi-
The first three entries (i.e., number of inputs and outputs and wavelength) yield the desired coupling and cross-talk performance; these in turn, with the use of the results given in Section 2, yield the normalized aperture, acoustic attenuation, and output resolvability. The electric-field profiles at the Bragg cell and lenslet array for these parameters are illustrated in Figs. 7 and 10. These results, in combination with the physical constraints such as the mode-field diameter of a single-mode fiber at this wavelength, yield the design of the optical system by means of Eqs. (35) and (33) and Table 1. To finish the design, the acoustic propagation, orientation, and cut of the acousto-optic crystal must be fully determined.

4. Crystal Size and Orientation

The results reported in Section 3 give the width of the acousto-optic crystal, the length of the piezoelectric transducer, and its rf center frequency and bandwidth. The complete specification of the acousto-optic device also includes the length and height of the crystal, the height of the transducer, and the 3-D orientation of the device in relation to the TeO$_2$ crystallographic axes. These quantities can be found by an examination of the properties of anisotropic optical and acoustic propagation.

For example, the center frequency of the acousto-optic interaction, as specified by Eq. (35), can be adjusted by the rotation of the plane of the optics about the acoustic wave vector; as shown in Fig. 17, this causes the optical wave-vector surfaces to separate because the gyrotropic splitting of the surfaces at the poles is much smaller than the birefringent splitting of the surfaces at the equator. Thus, rotating the plane of interaction toward the equator increases the acousto-optic center frequency. This optic-rotation technique is well known$^{33,34}$ and can be used in the laboratory to tune the center frequency of a Bragg cell simply by rotation of the cell around the acoustic column.

Conversely, the acoustic-rotation angle is set at the time of fabrication to break the symmetry of the optical wave-vector surfaces.$^{34,35}$ This technique was developed originally for tangentially matched Bragg deflectors that suffer from midband depletion—the grating that diffracts light from the outer, extraordinary polarization to the inner, ordinary surface is also matched to rediffract this light by the same angle to the symmetrical location on the outer surface. The same effect exists in this degenerate antitangential acousto-optic device and leads to symmetric $+1$ and $-1$ orders of diffraction.$^{36}$ This symmetry can be broken by the rotation of the acoustic transducer face away from the [110] axis and toward the [001] axis.

Thus, acoustic rotation is necessary for the antitangential-crossbar arrangement because the negative-order diffraction will cause half the input light to be diffracted away from, rather than toward, the output fibers. Rotating the switch interaction can cause the negative-order diffraction to be Bragg mismatched. Although momentum-space design techniques like those described in Section 3 can be used to derive constraints on this angle, in reality the optic- and acoustic-rotation angles depend so strongly on one another and on the dispersion of the dielectric tensor that an interactive numerical solution is the easiest and most reliable design technique. For the
switch specified by the data listed in Table 2, the required angles are 2.3° of optic rotation and 2.6° of acoustic rotation, which results in a 47-MHz center frequency at 850 nm and no significant symmetrical negative-order diffractions within the 15-MHz switch bandwidth.

Although this acoustic-rotation technique breaks the symmetry by which half the incident light is diffracted to a symmetrical and unwanted position, it also breaks the symmetry by which the direction of the power flow is parallel to the wave vector. This anisotropic power walk-off is severe for the slow-shear mode of TeO₂ because of its extreme anisotropy. As shown in Fig. 18, the walk-off angle \( \theta_{WO} \) for 2.6° of acoustic rotation is 27°. The length of the cell must be extended so that the angled power flow does not cause the acoustics to reflect off the crystal surface, as shown in Fig. 18. The total crystal length \( L_{\text{total}} \) is thus

\[
L_{\text{total}} = L + A \tan(\theta_{WO}) = 45 \text{ mm}. \tag{38}
\]

Although walk-off is caused by the slope of the acoustic wave-vector surface, acoustic diffraction is caused by its curvature. It can be found analytically\(^{31}\) that, along the [110] axis direction, the highly anisotropic slow-shear mode has an excess curvature in the direction of the transducer length of \( b_{zx} = 11.78 \) and in the direction of transducer height of \( b_{xy} = 52.65 \). To maintain a well-collimated acoustic column, the width of the cell must be less than the Rayleigh range \( Z_r \) of the transducer,

\[
Z_r = \frac{D^2}{b\lambda}, \tag{39}
\]

where \( D \) is the transducer height \( H \) or length \( L \). For a device with no acoustic rotation, the Rayleigh range is 12 times closer in the length dimension and 53 times closer in the height dimension than would be the case in an acoustically isotropic material\((b = 1)\). Because \( L \) in this optical crossbar is usually \( \approx \) H and

\[
L = \frac{45}{\tan(27°)} = 850 \text{ mm}.
\]

Fig. 18. (a) Fourier-space and (b) real-space calculations of the acoustic diffraction and highly anisotropic walk-off in an acoustically rotated, TeO₂ acousto-optic cell. The Poynting vector \( s \) is directed normal to the acoustic wave-vector surface, which causes 27° of walk-off when the crystal [110] axis is tilted 2.6° away from the acoustic column.

23.64mm Rectangular Piezoelectric Transducer

Width = 42 mm

Length = 45 mm

Fig. 19. Momentum surface of the slow-shear mode in TeO₂ showing the relevant curvatures for anisotropic diffraction, \( b_{xy} \) and \( b_{zx} \), as the acoustics are rotated from the [110] toward the [001] crystal axis. The curvature of the continuous (solid) ellipse is given by \( b_{zx} \), whereas the three arcs that cross the continuous ellipse show the curvature \( b_{xy} \) at three different points. Curvatures in both directions are highest in the unrotated [110] case and decrease as the acoustic-propagation direction is rotated away from the slow-shear lobe. (Figure reprinted with permission from Ref. 3, p. 528.)
The Rayleigh range in the plane of the interaction is typically much longer than that in the height dimension, and thus only the constraint on transducer height need be considered.

An important question is how these curvatures will vary with acoustic rotation. Figure 19 shows a 3-D plot of the TeO$_2$ slow-shear wave $K$ surface. The solid continuous curve drawn on the image surface shows the locus of the tip of the acoustic $K$ vector as the center direction of acoustic propagation is rotated toward the $z$ axis. At zero rotation, the transducer is placed such that the acoustic wave travels directly in the $[110]$ direction. It is clear that the curvatures $b_{xy}$ and $b_{xz}$ will vary as the acoustics are rotated and that this variation will strongly affect the diffraction of the rotated acoustic wave. The variation of $b_{xz}$ as a function of angle is determined by the curvature of the solid continuous curve, whereas the change in $b_{xy}$ is illustrated by the changing curvature of the three short arcs shown crossing the continuous curve in Fig. 19. The curvatures cannot be found analytically in this case, but a numerical solution is straightforward and agrees with the analytic results at the $[110]$ and $z$ axes. Figure 20 shows that the highly anisotropic diffraction decreases quickly as $K$ rotates away from the $[110]$ direction. For the $12 \times 12$ switch with its $2.6^\circ$ acoustic-rotation angle, this diffraction decrease reduces the required height of the transducer from 9.5 to 6.5 mm, resulting in an acousto-optic diffraction efficiency that is 50% larger for the same applied rf power. Allowing room for the transducer ground plane, this 6.5-mm transducer requires a roughly 10-mm-tall crystal, which fixes the last crystal dimension. The crystal specifications for the example $12 \times 12$ switch are summarized in Table 3.

This transducer height is much smaller than the diameter of the Gaussian beam incident on the Bragg cell. Because increasing the height of the transducer would dramatically decrease the device efficiency, we must instead anamorphically compress the optical beam in the height dimension so it will fit through the cell—a compression by a factor of roughly 7 is required for the $12 \times 12$ switch. This can be accomplished either with matched prism pairs or well-corrected cylindrical telescopes. The latter system is illustrated below in Fig. 23, Section 6.

### Table 3. Crystal Specifications for the $12 \times 12$ Crossbar Switch

<table>
<thead>
<tr>
<th>Crystal Element</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>$A$</td>
<td>42 mm</td>
</tr>
<tr>
<td>Length</td>
<td>$L_{\text{stal}}$</td>
<td>45 mm</td>
</tr>
<tr>
<td>Height</td>
<td>$H_{\text{stal}}$</td>
<td>10 mm</td>
</tr>
<tr>
<td>Transducer Length</td>
<td>$L$</td>
<td>23.7 mm</td>
</tr>
<tr>
<td>Transducer Height</td>
<td>$H$</td>
<td>6.5 mm</td>
</tr>
<tr>
<td>Acoustic rotation</td>
<td>$\theta_{AR}$</td>
<td>2.6°</td>
</tr>
<tr>
<td>Optic rotation</td>
<td>$\theta_{OR}$</td>
<td>2.3°</td>
</tr>
</tbody>
</table>

5. Efficiency Bounds

Tables 2 and 3 specify the complete optical design of a single-crystal acousto-optic crossbar switching matrix. For comparing this device to those described in Section 1 the fundamental efficiency limit for this switch must be derived. An analysis of the unin-
tended second-order couplings in this acousto-optic crossbar reveals a fundamental maximum throughput of exactly 1/N. This can be seen if one solves the linear system of coupled-mode equations for the dominant optical waves within the crystal.

Under the assumption that the switch is well designed and satisfies the constraints derived in Section 3, the only significant unintended acousto-optic diffractions will be satellites of each widely spaced input, as shown in Fig. 21. This assumption limits the number of diffracted waves to be considered, making it possible to find a closed-form solution; without this simplifying assumption, closed-form results are possible for only two rf tones.38 Thus, the only significant optical fields are assumed to be N – 1 unintended second-order depletion waves near each crossbar input that are each coupled to one of the outputs.

If we make the approximation that all the crossbar couplings are perfectly Bragg matched (which is in error at worst by a factor of 2 if the design constraints are satisfied), then there is a single coupling constant for acousto-optic diffraction, $\kappa$. If we further assume that the arbitrary crossbar switch is configured as a permutation so that each of the N inputs ($i$) is directed to one of the N outputs ($o$) and no two inputs are fanned-in to the same output, the coupled-mode equations can be written as

$$\frac{d\xi_{i,o}}{dz} = j\kappa \sum_{l=1}^{N} \xi_{i,o}$$

where $\xi_{i,o}$ ($o = 1, \ldots, N$) are the electric-field amplitudes of the N output waves, and $\xi_{i,o}$ ($i, o = 1, \ldots, N$) are the N bands of N plane waves, each around one of the N inputs. One wave out of each of these bands is the input signal that couples, via an acousto-optic grating, to a selected output; the remaining $N - 1$ waves in each band are the second-order rediffrications from the other $N - 1$ output waves, traveling via the same grating. The specific propagation angles of these $N - 1$ second-order satellites are selected from the adjacent $2N - 1$ positions, spaced by $\Delta k_{x,\text{out}}$ around the input, in a manner that is determined by the particular configuration permutation. Thus, as stated in Eq. (40a), each output is coupled to N waves, one from each of the input bands. Only one of these couplings is intentional, supplying light from the input; the rest are rediffrications that deplete this light and diffract it to useless locations. Equation (40b) states that each of the inputs and its band of $N - 1$ second-order depletion waves is one-to-one coupled to the N outputs by the one acoustical grating intended to direct that input to its selected output. Because each of the unintentional second-order diffracted waves is coupled to one and only one output wave, its only effect is to deplete the output intensity—there is no mixing of outputs that would result in channel cross talk.

Third-order diffractions, however, do cause channel cross talk. The efficiency of these couplings is roughly the cube of the first-order diffractions—and thus small in the low-diffraction-efficiency regime—but there are a large number of third-order diffractions that can all contribute to the same cross talk. This analysis is rather complex and has not been considered here. Additionally, because the acoustic amplitude of each of the N gratings is in the small-diffraction-efficiency regime (for reasonably large N), acoustic nonlinearities will not be significant, and intermodulation products need not be considered.39

The set of linear ordinary differential equations [Eqs. (40)] that describe the second-order couplings can be solved by standard methods. For a particular output wave, $o = \Phi$, the boundary conditions at the point of $z = 0$ are

$$\epsilon_{i,\Phi}(0) = 0,$$

$$\epsilon_{i,\phi}(0) = \begin{cases} \epsilon_{i}(0) & o = \Phi \quad \text{(input)} \\ 0 & o \neq \Phi \quad \text{(rediffractions)} \end{cases},$$

(41)

where $\epsilon_{i}(0)$ is the incident electric-field amplitude of the input that is to be connected to the output $\Phi$. The solutions evaluated at the output $z = L$ are

$$\epsilon_{i,\Phi}(L) = \frac{\epsilon_{i}(0)}{\sqrt{N}} \sin(\sqrt{N}kL),$$

$$\epsilon_{i,\phi}(L) = \frac{\epsilon_{i}(0)}{N} \left[\cos(\sqrt{N}kL) + (N - 1)\right],$$

and

$$\epsilon_{i,o \neq \Phi}(L) = \frac{\epsilon_{i}(0)}{N} \left[\cos(\sqrt{N}kL) - 1\right].$$

In the case of a single-frequency acousto-optic deflector ($N = 1$), these solutions reveal the well-known sinusoidal transfer of intensity with coupling strength. In this case, it is possible to achieve 100% coupling efficiency when $kL = (2n - 1)\pi/2$ ($n = 0, 1, 2, \ldots$), completely deflecting the input to the output. For $N > 1$, at a very low diffraction efficiency (much less than 1), only the desired outputs are produced with an efficiency proportional to ($kL$)$^2$. As the rf drive power is increased, the undesired satellite beams responsible for output depletion grow as ($kL$)$^4/4$. These beams limit the maximum electric-field amplitude of the output wave to $1/\sqrt{N}$ of the input amplitude and thus the maximum intensity to $1/N$. Thus, as stated, this single-transducer acousto-optic crossbar is limited to a maximum efficiency per channel, in permutation mode, of $1/N$. These second-order rediffrications can be suppressed by the use of sophisticated time-delay beam-steering transducer designs, which can be practically implemented only lithographically on surface-acoustic-wave (SAW) devices30 (see Fig. 22).

Whereas the maximum optical throughput of the single-transducer acousto-optic crossbar thus scales as $1/N$, the total rf power required to reach this maximum also scales as $1/N$. This can be seen if one
first notes that the first efficiency maximum occurs at \( \kappa L = \pi/(2\sqrt{N}) \). Because \( \kappa \) is proportional to the acoustic amplitude, the maximum coupling efficiency for a fixed \( L \) occurs at an acoustic intensity \( N \) times smaller than the acoustic intensity required for 100% coupling of a single connection. Thus, for a fixed transducer length, the total acoustic power required to maximize the acousto-optic diffraction efficiency for \( N \) permutations is independent of the size of the crossbar, \( N \). As the length of the transducer is proportional to \( N \) (see Table 1) and the total acousto-optic diffraction efficiency is proportional to \( L \), the total required rf power actually decreases by a factor of \( N \).

6. Extensions to the Basic Design

The \( 1/N \) limit on the coupling efficiency can be avoided by a Bragg mismatch of the second-order rediffractions. We can accomplish this by increasing the length of the transducer, further narrowing the angular bandwidth of the gratings, and using a time-delay, phased-array transducer to Bragg match these highly selective gratings to the desired output waves. Such sophisticated transducers can be realistically fabricated only as interdigitated electrodes on a SAW device, illustrated in Fig. 22.3,40

Although this SAW device is more efficient— as a result of both the suppression of second-order depletion and the longer, more efficient transducer—it sacrifices the parallelization available in the height dimension with the bulk implementation. Figure 23 shows how this Bragg-degenerate dimension can be used to implement a multichannel crossbar [see Figure 1(b)] with the same control and hardware complexity as the single-channel device. The only additional requirements are the two-dimensional (rather than one-dimensional) optical inputs and outputs.

For minimizing aberrations and acoustic rotation within the Bragg cell, the inputs are just resolved in the height dimension. This can be done with an integrated waveguide fan-in, like that shown on the right-hand side of Fig. 22. One such fan-in device is used for each multifiber input signal (such devices are available commercially by special order). The telescopic system formed by the 4-f optical system magnifies this narrow height separation to produce a rectangular array of output spots spaced at 250 \( \mu \)m. Such two-dimensional arrays of fibers, complete with self-aligned, integrated lenslets have recently been fabricated by proton irradiation of poly(methyl methacrylate).41

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Fig. 22. Single integrated-optics device implementation by use of SAW acousto-optic interactions. A waveguide fan-out array is required to match to the spacing of the output-fiber V groove. TIPE IO lenses, titanium in-diffused proton-echange integrated-optics lenses; IDT, interdigital transducer. (Figure reprinted with permission from Ref. 3, p. 489.)

Fig. 23. Parallel-channel acousto-optic crossbar switch implemented with two doublet cylindrical telescopes (compression telescope and expansion telescope). \( N \) input channels of \( P \) bits are switched to \( M \) output channels, also of \( P \) bits. Note that [as illustrated in Fig. 1(b)] no switching takes place in the y direction.
An optical-system analysis of a 32-channel version of the 12 × 12 switch described above was performed to verify the performance of this design. The anamorphic compression required by the 6.5-mm-high transducer is implemented with doublet cylinders in two matched telescopes: Doublets are necessary to maintain the diffraction-limited performance of the optical system. The collimation lens and these telescopes angularly multiplex the columns of input channels in the Bragg-degenerate dimension of the acousto-optic gratings. This optical rotation is less than 0.02 degrees, which is not large enough to significantly shift the diffracted outputs. Therefore, the design of the single-channel crossbar described in Sections 2–5 applies without modification to this multichannel device. A 12 × 12 crossbar of 32-bit words operating at tens of gigawords per second can thus be constructed in a single acousto-optic crystal and controlled by a single rf switching signal.

7. Conclusion
In summary we have fully designed a 12 × 12 space-division crossbar switching network implemented in a single acousto-optic crystal driven by one multitone rf control waveform. The ~10-μs reconfiguration time, minimal control complexity, 1/N fundamental loss, and large format (N = M = 12) compare favorably with other all-optical switching technologies.

The design of the switch is founded on a rigorous diffraction analysis of a finite-width, acoustically lossy Bragg cell in a 4-f optical system. This analysis, appropriate for single-mode-guided or Gaussian free-space optical inputs, derives the optimal placement of the Bragg cell to minimize optical losses deriving from truncation and acoustic decay within the crystal. The electric-field profile of the focused spot in the output plane reveals the coupling efficiency in a single-mode guide centered on this spot or laterally displaced, which is the cross talk. With these results the number of resolvable spots and the effective aperture of the Bragg cell are rigorously defined.

These diffraction results are incorporated into the design of the acousto-optic device by means of the momentum-space technique. This design is founded on the two requirements necessary to construct the crossbar interconnection: (1) all desired acousto-optic deflections are Bragg matched to within 4 dB, and (2) all undesired first-order acousto-optic diffractions are Bragg mismatched with a maximum diffraction efficiency of ~13 dB. Together with the constraints imposed by the 4-f system, these requirements yield a complete set of design equations for the acousto-optic device. The details of the acousto-optic interaction and acoustic propagation in TeO₂ are used to finish the design and include the optical rotation of the crystal (which sets the center frequency), the acoustic-rotation angle (which Bragg-mismatches symmetrical antitangential diffractions), the length of the crystal (which accommodates the significant power walk-off angle caused by the acoustic rotation), and the transducer height (which is required to launch a well-collimated acoustic beam into the highly diffractive acoustic mode).

This momentum-space design is limited to first-order acousto-optic diffractions. One can investigate the combined effects of first- and second-order diffractions by writing the coupled-mode equations in the limit that only the desired crossbar-permutation connections are significant. These equations are solved analytically to reveal that second-order rediffractions limit the peak efficiency to 1/N. However, this decrease in peak efficiency is accompanied by a 1/N decrease in the required rf power.

These second-order rediffractions (and the associated 1/N efficiency limit) can be avoided by the use of an acoustic beam-steering transducer, which can be implemented with a lithographically defined interdigital transducer on a SAW device. Unfortunately, this implementation restricts the switch to two space dimensions; an optical-system analysis shows that fiber connections can be stacked in the third, Bragg-degenerate dimension to create a multichannel crossbar in a single acousto-optic device. The ultrahigh aggregate-data bandwidth of this device is obtained with no increase in control complexity and only moderate additional hardware.

The acousto-optic crossbar switch has minimal control complexity; a single wire is driven with a sum of tones from a look-up table in which each tone is responsible for one crossbar connection. The connections are not restricted in any way—arbitrary space-variant permutation, fan-out, and lossless fan-in are possible. These connections can be reconfigured in roughly 10 μs by a change in one or more of the rf tones in the switching signal. Second-order rediffractions cause the efficiency of the connections to decrease proportionally to the number of permutation interconnects, but the switch does not suffer the excess fan-out and fan-in losses common to many optical crossbars.

References