Acousto-optic tunable filter using phased-array transducer with linearized RF to optical frequency mapping

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ABSTRACT

We present an optimized design of an acousto-optic tunable filter (AOTF) using a phased-array transducer for a spectrally-multiplexed ultrafast pulse-shaping RF beamformer application. The momentum-space interaction geometry is used to optimize an AOTF using acoustic beam-steering techniques in combination with acoustic anisotropy in order to linearly map the applied RF frequency to the filtered output optical frequency. The appropriate crystal orientation and phased-array transducer design are determined to linearize the RF to optical frequency mapping even in the presence of optical dispersion of the birefringence. After optimizing the phased-array transducer, acoustic anisotropy, and optical anisotropic diffraction geometry, the designed AOTF will compensate for the birefringent dispersion of TeO$_2$ to give a linear modulation of RF frequencies onto the corresponding optical frequencies. This linearized frequency mapped AOTF is required for a squint-compensated, wavelength-multiplexed, optically processed RF imager.

1. INTRODUCTION

Figure 1. a) 1-D phased-array antenna beamformer using AOTFs at each element. Broadband RF signals from far-field sources at specific AOAs illuminate the antenna array and the AOTF at each antenna element spectrally encode the RF signal onto femtosecond pulses. The spectral encoding enables beam-squint compensation by spatial Fourier transform with a single achromatic lens. b) Numerical simulation illustrating that multiple optical spectral components corresponding to a broadband RF signal modulated by an AOTF do indeed focus to a squint-free spot.

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An RF imager or multiple beamformer forms multiple RF beams pointed toward every far-field source illuminating a large, broad-band antenna array, which is used in radio astronomy or surveillance receivers. With current analog RF or digital signal processing technique, it is difficult task to simultaneously process wideband signals from large antenna arrays. Optical signal processing schemes are suitable for this massively parallel signal processing of large antenna arrays since multi-GHz signals can be modulated on an optical carrier, and a Fourier lens based optical processor can form all the beams in parallel. Previous RF photonic phased-array receivers utilized electro-optic modulators (EOM) arranged in a miniaturized topology of the RF antenna array to modulate a CW laser, and the modulated array of optical signals was spatially Fourier transformed by a lens. The position in the focal plane of the Fourier lens is proportional to the sine of the angle-of-arrival (AOA) of the incoming RF signal and is also proportional to the RF frequency, so broadband RF signals lead to frequency-dependent beam steering known as beam-squint.

We have proposed that an acousto-optic tunable filter (AOTF) can replace the EOM at each antenna element, spectrally encoding the received broadband octave RF waveforms onto an octave-spanning femtosecond laser pulse train to enable a spectrally-multiplexed squint-free beamformer. A broadband RF signal from a far-field source illuminates the antenna array with a single angle-of-arrival (AOA). Different RF frequencies of the arriving signal are applied to each AOTF to spectrally encode femtosecond pulses by selectively mapping input RF frequencies to corresponding and, ideally, proportional optical frequencies. The output of the AOTF array in a miniaturized topology of the RF antenna array is spatially Fourier transformed by a conventional achromatic lens, and the wavelength dependence of the lens Fourier kernel exactly compensates for beam-squint. The spatial location of the focused spot indicates the sine of the angle of arrival of a far-field source and the Fourier plane scale factor $\vec{u} = \frac{\lambda}{\lambda_F}$ (which scales with optical frequency) compensates for beam-squint. For demodulation of the received RF signal, a heterodyne detection scheme can be used at each squint-free beam position by adding a reference femtosecond pulse train.

This squint-free RF array beamformer requires AOTFs with a linear proportional mapping between octave spanning RF frequencies and the corresponding optical frequencies in order to have the wavelength dependence of the Fourier transform scale factor of an achromatic lens compensate beam-squint. In this paper, a beam-steering technique using a phased-array transducer in an acoustically anisotropic crystal will be introduced to achieve a linear frequency mapping between RF and optical frequencies. Previously, phased arrays have not been utilized to optimize AOTFs as they have in Bragg cell deflectors, except for the related recent patent by Isomet intended...
to widen the optical bandwidth beyond an octave using just an RF octave. This Isomet design is a related concept but does not require a linearized mapping and was not explained in momentum space.

2. DESIGN ANALYSIS

To achieve beam-squint compensation, the octave RF signal needs to linearly map into the diffracted output optical frequency (an octave somewhere in the 400-1100 nm range detectable with silicon CCDs) from the AOTF. Figure 2 a) shows the momentum space (k-space) interaction geometry of a conventional TeO$_2$ AOTF obeying the parallel-tangent condition to enable large angular aperture. The TeO$_2$ anisotropic acousto-optic interaction can be illustrated in momentum space as geometrical vector summation $\vec{k}_a = \vec{k}_i + \vec{K}_a$ where the acoustic grating vector $\vec{K}_a$ diffracts the incident optical wave $\vec{k}_i$ from the outer extra-ordinary optical momentum surface $(2\pi n_o(\lambda, \theta)/\lambda)$ to the ordinary surface $(2\pi n_e(\lambda)/\lambda)$. The direction of the acoustic momentum is chosen so that the tangents to the incident and diffracted wave vector loci are parallel. In this parallel-tangents geometry, the momentum matching condition is approximately satisfied even with a change in incident light direction, so that the use of an AOTF is not limited to well-collimated light source but it can instead be used to filter natural scenes. The optical dispersion properties of TeO$_2$ affect the phase-matching condition of such an AOTF, resulting in a non-linear relation between the input RF frequency and the output optical frequency as shown in Figure 2 b). The size of the acoustic momentum $|K_a| = \frac{2\pi f}{V_a(\theta_a)}$ linearly varies with applied RF frequency with a specific acoustic velocity ($V_a$). However, the two TeO$_2$ indexes of refraction, $n_o(\lambda)$ and $n_e(\lambda)$ vary with the wavelength (optical frequency) due to material dispersion, which leads to a non-linear mapping of RF to optical frequency.

It has previously been suggested that the range of optical frequencies can be extended beyond an octave while maintaining the limit of an octave of RF by utilizing a phased array geometry. We suggest here that by also harnessing the anisotropic variation of acoustic velocity with beamsteering angle we can approximately linearize the mapping from RF frequency to optical frequency. The acoustic momentum space is represented by the material’s slowness surface (slowness is inverse velocity) which is dependent on the propagation direction of the acoustic wave. TeO$_2$ crystals achieve high diffraction efficiency in AOTF geometries, and in addition the highly anisotropic slow-shear slowness surface of TeO$_2$ crystal is well suited for combining beamsteering with acoustic anisotropy. Typically at RF frequencies acoustic dispersion is negligible, so the length of the acoustic momentum vector is linearly proportional to the RF frequency. To compensate for the optical dispersion, we need to find a geometry that produces a matching acoustic dispersion. The non-linear variation of acoustic momentum $(\vec{K}_a)$ with applied RF frequency can be realized by combining a phased-array transducer with anisotropic acoustics.
A phased-array transducer consists of multiple, appropriately phased RF transducers, whose combined transmitted acoustic wave steers with applied RF frequencies as shown in Figure 3 a). This frequency dependent beam-steering of the acoustic wave adds additional freedom in our design search space for frequency linearization. A 180 degree phase delay between adjacent transducer array elements can be considered to introduce a momentum \( \vec{K}_{bs} \) in the transducer plane whose size, \( |\vec{K}_{bs}| = \frac{2\pi}{2d} \), is related to the transducer periodic spacing \( d \).

In this viewpoint, the total beamsteered momentum is given by the vector sum of the acoustic wave launched normal to the transducer face and the beam steering momentum \( \vec{K}_{bs} \). The net acoustic momentum must self-consistently satisfy the anisotropic slowness relation and moves along a locus perpendicular to the transducer face and offset by \( \vec{K}_{bs} \). Figure 3 b) shows 3 different acoustic momentum surfaces (slowness surface) of TeO\(_2\) slow-shear mode with 3 different RF frequencies.

The beamsteering angle is given by

\[
\sin \theta_{bs} = \frac{|\vec{K}_{bs}|}{|\vec{K}_a|} = \frac{2\pi/2d}{2\pi f/V_a(\theta)} = \frac{V_a(\theta)}{2df} \tag{1}
\]

\( V_a(\theta) \) is anisotropic acoustic velocity of the slow-shear mode of a TeO\(_2\) crystal.

\[
V_a^2(\theta) = (616\text{m/s})^2 \cos^2 \theta + (2104\text{m/s})^2 \sin^2 \theta \tag{2}
\]

Where \( \theta = \theta_a + \theta_{bs} \) as shown in Figure 3 b), \( \theta_a \) is the acoustic rotation angle for the transducer face. And the acoustic momentum \( (K_a(f)) \) is given by

\[
\vec{K}_a(f) = \vec{K}_{bs} + \frac{\hat{n}|\vec{K}_{bs}|}{\tan(\theta_{bs})} \tag{3}
\]

The wavelengths diffracted by the AOTF and the corresponding diffraction angles which satisfy the momentum-matching condition can be calculated by solving the following equations.

\[
\theta = \tan^{-1} \left( \frac{n_e(\lambda, \theta_d) \cos \theta_d - n_o(\lambda) \cos \theta_i}{n_e(\lambda, \theta_d) \sin \theta_d - n_o(\lambda) \sin \theta_i} \right) \tag{4}
\]

\[
|k| = \sqrt{(n_e(\lambda, \theta_d) - n_o(\lambda) \cos \theta_i)^2 + (n_e(\lambda, \theta_d) \sin \theta_d - n_o(\lambda) \sin \theta_i)^2} \tag{5}
\]

where \( \theta_i \) is the angle of incidence for the AOTF, and the extra-ordinary index of refraction \( (n_e(\lambda, \theta_d)) \) is not only a function of wavelength \( (\lambda) \) due to material dispersion but also a function of the diffracted angle \( (\theta_d) \).
either beam near the optical axis, further complications due to optical activity arise, but for this beamsteering AOTF we assume we are far from the optical axis so we can approximate $n_o(\theta_i)$ as a constant and use:

$$n_e(\lambda, \theta_d) = \left(\frac{\cos^2 \theta_d}{n_o^2(\lambda)} + \frac{\sin^2 \theta_d}{n_e^2(\lambda)}\right)^{-1/2}$$  \hspace{1cm} (6)

The required transducer spacing $d$ and acoustic rotation angle $\theta_a$ of the AOTF with a phased array transducer which achieves a linear relation between RF and optical frequency can be determined by numerically calculating the diffracted output optical frequencies (from the diffracted output wavelength) when RF frequencies spanning an octave are applied. The sampled (7 samples) RF frequencies from 50 to 100 MHz gives a set of acoustic momentum $|K_a|$ with beam-steering angles $\theta_{bs}$. With these $|K_a|$ and $\theta_{bs}$, the diffracted output optical frequencies and diffraction angles are numerically calculated with various input angles ($\theta_i$) considering the Sellmeier relation of TeO$_2$.\textsuperscript{12} The non-linear behavior of the diffracted wavelength comes from both the geometry selection (slope-control) and material dispersion (nonlinear curvature). Varying the available design parameters (transducer spacing $d$, acoustic rotation angle $\theta_a$ and input angle $\theta_i$), the optimization routine finds the optimum design specifications which counteract material dispersion and approximately satisfy the linear mapping relation. We use a figure of merit given by

$$\text{Figure of Merit} = \frac{\cos^2 \theta_{rs}}{\sum |\text{error}| \cdot \sqrt[3]{\alpha}},$$  \hspace{1cm} (7)

which is inversely proportional to the mean absolute value deviation from a linear mapping between RF and optical frequencies. The resolution of AOTF is dependent on the angular relation between the diffracted optical wavevector (or more precisely the Poynting vector) and acoustic energy propagation direction as shown in Figure 3 c), because the spectral resolution is proportional to the number of acoustic gratings encountered as the diffracted wave propagate inside the AOTF. Also, the diffraction efficiency of an AOTF which increases with the inverse square of the acoustic velocity and the resolution increases inversely with velocity so inverse acoustic velocity cubed is included in our figure of merit. Figure 4 shows the figure of merit for various transducer spacings.

**Figure 5.** Linearized optical frequency mapping for a beamsteering AOTF with 250 $\mu$m transducer spacing and a 6.5 degree acoustic face rotation at 7 RF frequencies from 50-100 MHz. The solid line shows the linear mapping of RF frequencies versus optical frequencies. The optical frequencies approximately ranges over from 550nm to 1100 nm in wavelength.
spacing and acoustic rotation angles at optimum input angles ($\theta_i$). With 250 µm transducer spacing, 6.5 degree acoustic rotation angle and 16 degree incident input angle, the diffracted light nicely spans over an octave optical frequencies with close to a linear relation between RF and optical frequencies as shown in Figure 5. Also, the diffraction angle is collinear for each wavelength, as required for the heterodyne detection alignment of a beam-squint compensation optical system.

The momentum space\(^{16}\) of the designed AOTF with 3 different filtered optical frequencies (wavelength) as calculated using BRAGGART is as shown in Figure 6 a). Figure 6 b) shows the magnified view of k-space for this anisotropic beamsteering diffraction. The momentum space of this acousto-optic interaction has a distribution of uncertainty given by the Fourier transform of the real space AO interaction region as shown in the Figure 6 b), which is due to the finite nature of the grating ($L$) and the size of incident beam ($A$). When the shape of the transducer and the incident beam are rectangular, the uncertainty distribution can be represented as a sinc box (with 4 dB width of $2\pi/L$ and $2\pi/A$). The uncertainty sinc box has a skew angle which is given by the walk-off angle of the acoustic wave. The spectral resolution can be calculated geometrically by finding the momentum surfaces with different wavelengths which overlap with the uncertainty sinc distribution. Figure 7 a) shows the 4dB optical frequency resolution with 2 mm transducer length. The optical frequency varies from 10-60 nm which indicate relatively poor chromatic resolution. By increasing the length of the transducer to 15 mm (decreasing the uncertainty width), the spectral resolution improves to 1-8 nm as lines as shown in the Figure 7 b).

3. CONCLUSION

A novel AOTF, which linearly and proportionally maps the applied octave spanning RF frequency to the diffracted optical frequency has been designed using a combination of an acoustic phased array transducer
Figure 7. a) With 2 mm transducer length, the spectral resolution ranges from 10 to 60 nm for applied RF frequencies. The poor spectral resolution is represented as 2 distinctive output optical frequencies. b) The spectral resolution can be improved by increasing the size of transducer length to 15 mm. The spectral resolution ranges from 1 to 8 nm, and this improvement is represented as lines with narrow width.

and acoustic anisotropy. A numerical optimization routine found the optimum design specifications for phased-array transducer spacing, acoustic rotation angle and the required input angle. Each diffracted wavelength will collinearly diffract with insignificant amount of deviation which would couple to a single-mode output and simplifies the alignment for heterodyne detection. The diffracted optical frequency from the designed AOTF has a linear mapping relation between the received RF frequencies from antenna array and the modulated optical frequency components of an octave spanning femtosecond laser pulse train. By spatially Fourier transforming the modulated array of optical signals using a single lens, the wavelength dependent Fourier kernel compensate beam-squint giving angle-of-arrival information and the temporal content of any far-field sources illuminating the array.

REFERENCES


