Material figures of merit for spatial soliton interactions in the presence of absorption

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The effects of linear and two-photon absorption on bright spatial soliton propagation are studied. A spatial soliton switch that achieves gain through the novel mechanism of colliding, dragging, or trapping of two fundamental solitons of different widths is proposed. Figures of merit for use in evaluating the suitability of absorbing nonlinear media for soliton switching applications are presented. The main effect of linear absorption is to limit the propagation distance, which places an upper bound on the width of the soliton in order to fit sufficient characteristic soliton propagation lengths within the device. The optical limiting nature of two-photon absorption places an upper bound on the gain that an interaction can achieve. The combined effects of linear and two-photon absorption are to reduce the gain upper bound imposed by two-photon absorption alone with the addition of the soliton width constraint. A maximized gain upper bound is determined solely by material parameters and is compared among three promising nonlinear materials. It is shown numerically that the spatial soliton dragging interaction requires shorter propagation distances and achieves greater gain than the collision interaction and that both are tolerant to the presence of absorption and can provide, with high contrast, gains of three or greater using measured material parameters. These results warrant pursuing the implementation of spatial soliton-based logic gates. © 1996 Optical Society of America.

1. INTRODUCTION

Strongly nonlinear optical materials invariably suffer from the presence of absorption. Resonant enhancement of the nonlinearity directly leads to an increase in the linear or two-photon absorption and usually increases the response time; both effects are detrimental to ultrafast all-optical switching. It is well known that linear absorption places fundamental restrictions on the operation of nonlinear devices such as bistable Fabry–Perot étalons\(^1\) and directional couplers.\(^2\) These studies have produced a nonlinear Fabry–Perot material figure of merit \(3n_2/\alpha\lambda\) (Ref. 1), where \(n(I) = n_0 + n_2l\) defines the nonlinear Kerr index \(n_2\), \(\alpha\) is the linear absorption constant, \(\lambda\) is the material wavelength, and \(I\) is the intensity, and a nonlinear directional coupler material figure of merit \(\Delta \beta_{\text{abs}}/\alpha \lambda_0 \gg 1\)\(^3\), where \(\Delta \beta_{\text{abs}}\) is the maximum nonlinear change in the effective index and \(\lambda_0\) is the free-space wavelength. In order to minimize linear absorption, a promising regime of operation in nonlinear optics is near half band gap in semiconductors\(^4\) or in the transparency regions of glasses and organics. These nonlinearities are essentially instantaneous with very small linear absorption, due mainly to material or waveguide scattering, but with some nonzero two-photon absorption. The effects of two-photon absorption on nonlinear directional couplers\(^5\) have been studied, and a switching criterion has been defined as \(T = 2\beta_2\lambda_0 n_2 \ll 1\)\(^,\)\(^6\),\(^7\) where \(\beta_2\) is the two-photon absorption coefficient defined by the total absorption \(\alpha(I) = \alpha + \beta_2I\).

Bright soliton propagation in the presence of linear absorption was first studied to determine the implications for temporal soliton transmission in fiber.\(^8\) More recent work has focused on the effects of nonlinear absorption on spatial soliton propagation. The effects of two-photon absorption on bright spatial soliton propagation were observed experimentally in glass waveguide experiments\(^9\) and then were studied analytically.\(^10\) Beam propagation has been used successfully to model soliton breakup due to two-photon absorption.\(^10\)\textendash\(^12\) Stationary propagation of dark solitons was predicted in the presence of both two-photon absorption and linear amplification,\(^13\) and the effects of two-photon absorption on soliton nonlinear interface switches\(^14\) and dark soliton steering\(^15\) have also been studied. Linear absorption alone has been considered in the operation of a switch based on the phase-sensitive spatial soliton repulsion interaction.\(^16\)

In this paper we look at the separate and the combined effects of linear and two-photon absorption on bright spatial soliton propagation and the implications for spatial soliton switching. We define figures of merit in order to evaluate the suitability of a given material for use in soliton switching applications and numerically demonstrate the operation of spatial soliton dragging\(^17\) and small-angle collision\(^18\) in the presence of absorption using realistic material parameters.

2. SPATIAL SOLITON DRAGGING AND COLLISION INTERACTIONS

The class of interactions we are concerned with here are those that produce an inversion operation, which can be cascaded to implement logically complete, multi-input \textsc{nor} gates, as originally suggested for temporal soliton interactions\(^19\) and in this paper considered in an analogous spatial soliton interaction geometry\(^17,\)\(^20\) in order to achieve high-density, low-latency switching. For the spatial soliton switching considered here, a soliton that we call the pump propagates by itself the length of the
gate and passes through an output aperture to provide the high-output state. It is important to note that only this undeviated pump is used to switch subsequent stages, providing true three-terminal input–output isolation. When present, a signal soliton interacts with the pump, changing its angle of propagation, so that neither passes through the output aperture, thus providing the low-output state. It is this angle change, coupled with the width of the solitons and the output aperture, that determines the minimum gate length; hence the larger the angle change, the shorter the gate. Therefore each type of interaction (i.e., dragging or collision) has some lower bound on the gate length in order to produce this inversion operation. The pump soliton must carry more power through the output aperture than the signal at the input to provide gain. In order to get a pump stronger than the signal, the pump can be a higher-order soliton of the same width as the fundamental signal, or the pump can also be a fundamental soliton of narrower width than the signal, and hence for a one-dimensional (1-D) soliton, carry more power. Absorption over the length of the gate will decrease the gain and is the main effect we are concerned with here. The gain should be at least unity to avoid the use of external amplification and at least two or more to provide fan-out.

We have defined previously \(^{17}\) the confocal distance
\[
Z_0 = \frac{\pi w_0^2}{\lambda}
\]
and a normalized interaction angle
\[
k = 5.6 w_0 \sin \theta \lambda,
\]
where \(w_0\) is the fundamental soliton width parameter, \(\lambda = \lambda_0/\eta_0\) is the material wavelength, and \(\theta\) is the initial relative angle between the beams. The confocal distance is twice the distance over which the beam will broaden its intensity full width at half-maximum by a factor \(\sqrt{2}\) in linear propagation. The normalized interaction angle is defined so that, for \(k = 1\), the two beams of initial sech \((x/\omega_0)\) profiles will be just resolvable in linear propagation (i.e., the angular spectra overlap at the half-power points). In this paper both the pump and the signal will be fundamental solitons, with the pump having a narrower width in order to carry more power. This leads to stable propagation without periodic overfocusing of the strong pump even in nonsaturating materials. The normalized parameters \(Z_0\) and \(k\) will both be in reference to the signal soliton width \(w_0\), and we will denote the length of a switching gate in terms of the signal’s confocal distance, or \(z = d Z_0\), where \(z\) is the gate length. If \(P_0\) is the initial power of the signal, then \(P_p = r P_0\) is the initial power of the pump. The pump soliton width is given by \(w_p = w_0/r\), where \(r\) is the initial pump-to-signal power ratio; therefore, the confocal distance of the pump is
\[
Z_p = \frac{\pi^2 w_p^2 \lambda}{Z_0} = \frac{Z_0}{r^2}.
\]
Because for \(r > 1\) the linear diffraction of the pump is stronger than that of the signal, as evidenced by the shorter confocal distance of the pump, interactions with \(r > 1\) will behave differently than interactions with \(r = 1\) for \(d\) and \(k\) constant with respect to \(r\). As the pump-to-signal ratio \(r\) increases, \(k\) must increase in order for the beams to be resolvable in linear propagation; we therefore expect that

![Fig. 1. Spatial soliton dragging over a gate length of 10Z₀ with a normalized initial relative propagation angle of κ = 1.7. The top shows the signal (thin lines) and the pump (thick lines) with no cross-phase modulation (Δ = 0). The bottom shows the dragging interaction with cross-phase modulation (Δ = 2/3, appropriate for orthogonal linear polarizations). The initial beam ratio is r = 3.0. The contrast of the gate is 6565, and the gain is 3.](image)

![Fig. 2. Spatial soliton collision over a gate length of 10Z₀ with a normalized initial relative propagation angle of κ = 0.7. The top shows the signal (thin lines) and the pump (thick lines) with no cross-phase modulation (Δ = 0). The bottom shows the collision interaction with cross-phase modulation (Δ = 2/3). The initial beam ratio is r = 3.0. The contrast of the gate is 79 and the gain is 3, but the reflected signal partially transmits through the aperture in this case. If a polarizer is used at the output to block the signal, the contrast of the gate rises to 4368.](image)
lager values of $\kappa$ are needed for large $r$ than for small $r$. Results of numerical simulation verify this prediction, but we have also found that, for large $r$, further increases of $\kappa$ beyond a certain point provide little or no benefit.

The pump output of a high-gain gate can be used as a signal input to subsequent stages by simply flipping the polarization and tilting the angle, but the width is narrower than the assumed signal width. When the narrow-width high-power output pump is fanned out to several gate inputs, its width will not be consistent with the resulting soliton eigenmode unless it is magnified by the fan-out factor. This can be achieved with a lenslet-based imaging system or simply through diffractive beam spreading in free space, allowing fully cascadable soliton interactions. In a cascaded NOR gate, two signals interact with a single pump in two successive stages, so that either one can switch the output to low. In this case the pump output of one stage is used as the pump input to the subsequent stage, and the beam profile is correct and does not need to be modified. The initial beam ratio of the second stage is given by the gain of the first stage, which may then limit the overall gain of the cascaded NOR gate.

In a system it is essential to avoid or reduce phase dependence; therefore we will only consider interactions between solitons of orthogonal polarizations, in which cross-phase modulation (more exactly, cross focusing) is responsible for the interaction between the two polarizations. Phase dependence can be completely eliminated in the case of pseudo 1-D soliton interactions in bulk\(^{21}\) with orthogonal circular polarizations, and it can be reduced for orthogonal linear polarizations by use of waveguide or material birefringence to mismatch the phase-dependent terms in the induced nonlinear material polarization. We do not consider phase-dependent collision (which produces a small spatial shift) or attraction/repulsion\(^{22}\) (which produces a small angle change) between copolarized solitons because they cannot provide large gain; therefore our prototype interactions will be spatial soliton dragging and small-angle collision between solitons of orthogonal linear polarizations.

A. Soliton Dragging Interaction

The phase-independent dragging interaction occurs when two orthogonally polarized solitons are launched with different angles of propagation into a nonlinear medium such that their profiles initially overlap at the linear/nonlinear interface.\(^{17}\) The pump and signal mutually attract through positive cross-phase modulation such that, when complete trapping occurs, they copropagate at the weighted-mean angle, and neither passes through the output aperture. Normalized interaction angles near $\kappa = 1.1$ produce optimal gate contrast for solitons of the same width, i.e., $w_p = w_0$. For larger values of $r$ we choose the appropriate value of $\kappa$ based on numerical optimization of the gate contrast.

A simulation of soliton dragging is shown in Fig. 1 with $\kappa = 1.7$ and $r = 3$ for a gate length of $10Z_0$ in the absence of absorption. The aperture width is $3.5w_0$, with more detailed discussion of the optimal aperture width deferred to a later section. Notice that a small portion of the signal escapes the attraction of the pump with a larger portion of the signal being trapped by the pump, indicating nearly complete trapping. If the angle is too small, even though the pump and the signal effectively trap each other, their weighted mean propagation angle requires a long distance to drag out of the aperture. For larger angles, no trapping occurs, and the signal simply deflects the pump by giving up some of its transverse momentum, without trapping, with the amount of deflection decreasing with increasing angle. Soliton dragging works well for distances as short as $2Z_0$, but we will use gate lengths up to $10Z_0$ in order to tolerate the weakened interaction due to the presence of absorption.

B. Soliton Collision Interaction

Soliton collision results when two solitons are initially well separated and propagate toward each other until they meet somewhere within the nonlinear medium. At large angles the collision produces a spatial shift in the propagation of each soliton, with no change in propagation angle, and has been suggested as a mechanism for a photonic switch.\(^{23-24}\) Between solitons of orthogonal polarizations, with cross-phase modulation equal to two thirds of the self-phase modulation, the small-angle collision (or slow collision) results in a much more useful deflection interaction such that the two solitons bounce off each other\(^{19,25}\) with mutual changes in propagation angles. Between copolarized solitons, collision becomes increasingly phase-dependent with decreasing angle, so we consider only the phase-independent collision interaction between orthogonal linear polarizations.

Collision requires somewhat longer propagation distances than dragging because the solitons are nonoverlapping at the interface and smaller angles are optimal, $\kappa \approx 0.45$ for solitons of the same width, and degrades more quickly as the initial beam ratio $r$ is increased. This interaction is shown in Fig. 2 with $\kappa = 0.7$, $r = 3.0$ and an initial separation of $3.5w_0$ for a gate length of $10Z_0$. Notice that with collision, after interacting with the pump, part of the signal may pass through the output aperture, thus requiring a polarizer at the output to achieve input-output isolation. In contrast, the signal always moves away from the aperture in the dragging interaction, and no polarizer is required. We will assume that a polarizer is always present at the output for the collision interaction, and from now on we will just refer to this small-angle collision interaction as simply the collision interaction.

3. EFFECTS OF ABSORPTION ON SOLITON PROPAGATION

We start by considering the effects of absorption on the propagation of a single soliton, which will result in simple relationships that place upper bounds on the gain that an interaction can provide. The effects of absorption on the soliton dragging and collision interactions will be studied numerically. The nonlinear Schrödinger equation with the addition of linear and two-photon absorption becomes

$$2ik \frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + 2k^2 \frac{n_2}{n_0} (1 + iK) |E|^2 E + i k \alpha E = 0,$$

(1)
where \( k \) is the material propagation constant, \( K = \beta_2/2k_0n_2 \), \( \beta_2 \) is the two-photon absorption coefficient, \( n_2 \) is the nonlinear Kerr index, \( k_0 \) is the free-space propagation constant, and \( \alpha \) is the linear absorption constant.

The equation of motion for the 1-D integrated power of the nonlinear wave, with the boundary conditions \( E(x = \pm\infty) \to 0 \), is

\[
\frac{\partial}{\partial z} P(z) = \frac{\partial}{\partial z} \int_{-\infty}^{x} |E|^2 dx = -\alpha \int_{-\infty}^{x} |E|^2 dx - \beta_2 \int_{-\infty}^{x} |E|^4 dx, \quad (2)
\]

where \( \beta_2 = 2kK_n/n_0 \). Equation (2) shows that the decrease in integrated power with respect to propagation is linearly proportional to the power by the constant \( \alpha \) and nonlinearly proportional to the two-photon absorption constant \( \beta_2 \). In the absence of absorption the change in integrated power is zero, and Eq. (2) corresponds to the conservation of energy.

We assume the soliton maintains the characteristic sech shape during propagation. Therefore, in order to include the effects of absorption on soliton propagation, we make use of the ansatz

\[
|E(x, z)| = \frac{1}{kw(z)} \sqrt{\frac{n_0}{n_2}} \operatorname{sech} \left[ \frac{x}{w(z)} \right], \quad (3)
\]

where the width parameter \( w(z) \) is the only soliton parameter and is assumed to vary adiabatically with propagation, \( w_0 = w(0) \) is the initial fundamental soliton width, and the initial power \( P_0 = \int |E|^2 dx = 2n_0/k^2n_2w_0 \). Substituting Eq. (3) for the fundamental soliton into the equation of motion (2), we obtain the following expressions for the soliton width and power:

\[
w^2(z) = \frac{4K}{3\alpha k} \left[ \exp(2\alpha z) - 1 \right] + w_0^2 \exp(2\alpha z), \quad (4)
\]

\[
P^2(z) = \frac{P_0^2}{\left( k^3n_2^2K^2P_0^4/3an_0^2\right) \left[ \exp(2\alpha z) - 1 \right] + \exp(2\alpha z)}. \quad (5)
\]

In order to determine when the ansatz is valid, we calculate the percentage error between Eq. (5) and numerical simulation versus the linear attenuation parameter \( s = \alpha z \) and normalized two-photon absorption constant \( K \) for \( z = 10Z_0 \). The numerical simulation makes no use of the ansatz and therefore provides a good reference point to determine in what range of \( s \) and \( K \) the ansatz is valid. Figure 3 shows a contour plot of the error surface with each contour representing a constant percentage error. Notice that the ansatz is valid to within \( \sim 5\% \) for \( s < 1.0 \) and \( K < 1.0 \), which is in reality not a large restriction because any material of interest will more than satisfy these criteria. Before considering the general case of combined linear and two-photon absorption, we now examine two special cases of Eqs. (4) and (5): linear and two-photon absorption in isolation.

### A. Linear Absorption Only

The first case is when \( \beta_2 = 0 \), corresponding to negligible two-photon absorption. Then we have, for the evolution of the soliton width and power,

\[
w(z) = w_0 \exp(\alpha z), \quad P(z) = P_0 \exp(-\alpha z), \quad (6)
\]

where the power attenuation is the same as for linear plane waves, as expected. If we assume a propagation distance of \( z = dZ_0 \) and attenuation over that distance of \( \exp(-s) \), the relationship \( s = adZ_0 \) gives us a constraint on the initial soliton width

\[
w_0(s, d) = \mathcal{F} \lambda, \quad P_0(s, d) = \frac{\lambda_0}{2\pi^2n_2\mathcal{F} \lambda}, \quad (8)
\]

in terms of the linear absorption figure of merit \( \mathcal{F} \), defined as

\[
\mathcal{F}(s, d) = \sqrt{\frac{s}{\pi^2d\lambda}}, \quad (9)
\]

We require \( \mathcal{F} \) to be greater than a material wavelength in order for the paraxial approximation to remain valid. For \( w_0 = \lambda \) the linear angular divergence is approximately \( \pm 10^\circ \), so the paraxial approximation is at the edge of validity; clearly for \( \mathcal{F} < 1 \) the paraxial approximation begins to break down.

![Fig. 3. Contour plot of the percentage error between Eq. (5) and numerical simulation results versus \( s = \alpha z \) and \( K \), where \( z = 10Z_0 \). The range of validity of the ansatz used to derive Eq. (5) (in which the soliton maintains its sech shape but changes its width and amplitude self-consistently during propagation) is \( s < 1.0 \) and \( K < 0.1 \) to within \( \sim 5\% \) error of the simulation.](blair-figure3.png)
In the absence of two-photon absorption we can choose narrower fundamental solitons or higher-order solitons to obtain a pump with more power than the signal. If a higher-order soliton is used, then index saturation may be needed to stabilize the propagation of the pump to obtain an effective interaction with the signal. The pump is a fundamental of narrower width than the signal (i.e., \( w_p = w_0/r = F \lambda/r \)), as is the case considered in this paper. The maximum initial ratio of the pump to the signal is given by \( F \) so that \( w_p \gg \lambda \), and an upper bound to gain that an interaction can provide is \( F \exp(-s) \). In this case the largest gain upper bound of 0.1365\( \sqrt{\Delta \alpha \lambda} \) occurs when \( s = 0.5 \).

Figure 4 shows the propagation over \( 10Z_0 \) of a fundamental signal, an \( r = 3 \) fundamental pump, and an \( r = 4 \) higher-order pump (for later comparison with propagation in a two-photon absorbing medium) in a linearly absorbing material in which \( s = 0.5 \). Notice the adiabatic increase in the width of each fundamental beam during propagation.

### B. Two-Photon Absorption Only

The other special case is when the material has negligible linear absorption, but high enough intensities are used such that two-photon absorption becomes significant. Then \( \alpha = 0 \), and we arrive at the expressions for the width and the power:

\[
\begin{align*}
\omega^2(z) &= \omega_0^2 + \frac{8Kz}{3k}, \\
P^2(z) &= \frac{P_0^2}{(2k^3n_2^2K/3n_0^2)^2P_0^2 + 1},
\end{align*}
\]

which were first obtained in normalized form.\(^{10}\) After a propagation distance of \( z = dZ_0 \), where \( Z_0 = \pi^2w_0^2\lambda \) is the confocal distance of the signal soliton, the power of the signal can be written

\[
P^2(d) = \frac{P_0^2}{dT/K + 1},
\]

where \( T = 2\beta_2\lambda_0/n_2 = 8\pi K \) is the two-photon absorption material figure of merit for a nonlinear directional coupler.\(^7\) Notice that the attenuation of the signal is independent of its width \( w_0 \) and therefore initial intensity, which is a result of the confocal distance (and absolute gate length) growing as the square of the width.

In the presence of two-photon absorption, higher-order solitons have been shown to break up during propagation,\(^{10-12}\) thereby not providing a cascadable high-output state of our inverter. We therefore use a fundamental pump soliton of narrower width than the fundamental signal that has an integrated power given by \( P_p = rP_0 \), where \( r \) is the ratio of pump power to signal power and the width of the pump is \( r \) times smaller than the width of the signal, \( w_p = w_0/r \). The power of the pump after propagating the gate length \( dZ_0 \) is then

\[
P^2_p(d) = \frac{r^2P_0^2}{(dT/6)r^2 + 1}.
\]

The maximum pump power (as \( r \to \infty \)) at the output of the gate can be written as

\[
P_p(d)_{\text{max}} = \sqrt{\frac{6}{dT}} P_0.
\]

We define the two-photon absorption figure of merit for soliton interactions as the maximum ratio of the pump at the output \( P_p(d)_{\text{max}} \) to the signal at the input \( P_0 \),

\[
\mathcal{F}_{\text{TPA}}(d) = \frac{P_p(d)_{\text{max}}}{P_0} = \sqrt{\frac{6}{dT}}.
\]

and we want \( \mathcal{F}_{\text{TPA}} > 1 \). \( \mathcal{F}_{\text{TPA}} \) in the absence of linear absorption provides an upper bound on the gain that an interaction can provide and is a result of the optical limiting nature of two-photon absorption. Equation (13) for the power of the pump at some propagation distance \( z = dZ_0 \) can be rewritten in terms of the figure of merit and the initial ratio as

\[
P_p(d) = \frac{r\mathcal{F}_{\text{TPA}}(d)P_0}{\sqrt{r^2 + \mathcal{F}_{\text{TPA}}^2(d)}}.
\]

Figure 5 shows a plot of this relationship versus \( d \) and parameterized by the initial beam ratio \( r \) for a material in which \( \mathcal{F}_{\text{TPA}}(10) = 2.9 \), corresponding to lead-doped silicate (discussed later). It is clear that the majority of the absorption for high pump-to-signal beam ratios takes place at the beginning of propagation, so longer propagation distances do not pose as much of an additional penalty as with linear absorption. It is also clear that, for large \( r \), the power of the pump at the output asymptotically approaches the maximum given by \( \mathcal{F}_{\text{TPA}}(d) \). For \( \mathcal{F}_{\text{TPA}} \gg 1 \) the attenuation at the signal level (\( r = 1 \)) becomes negligible.

Figure 6 shows the propagation over \( 15Z_0 \) of a fundamental signal, a narrower \( r = 3 \) fundamental pump, and an \( r = 4 \) higher-order pump with the same width as the signal for \( K = 0.005 \) (or \( T = 0.13 \)). The signal power at some normalized gate length \( d \) follows Eq. (12); therefore
The attenuation of the signal is negligible, as shown in the figure, when $dT < 6$, as is the case here for $d = 15$ and $dT = 2.0$. The fundamental pump is absorbed rapidly over the first $5Z_0$ with much less attenuation over the next $10Z_0$. The width of both fundamental beams adiabatically increases with propagation. The longer propagation distance was chosen so that the splitting of the $r = 4$ higher-order pump would be clearly shown. It is clear from this figure that a higher-order pump cannot be used for cascadable logic in a material with two-photon absorption.

**C. Two-Photon Absorption and Plane Waves**

The nonlinear directional coupler figure of merit $T$ was derived with a plane-wave analysis ignoring the effect of attenuation on the accumulated nonlinearity induced phase shift. The power (in some finite area $A_{\text{eff}}$) of a plane wave that evolves due to two-photon absorption is given by

$$P_{\text{plane}}(z) = rac{P_0}{1 + \beta_{\text{TPA}} I_0 z},$$

where $I_0 = P_0/A_{\text{eff}}$. Using the plane-wave model and ignoring any transverse variation, we find that, for efficient switching in a nonlinear directional coupler, $T < 0.7$, fully accounting for the decreasing rate of accumulated nonlinear phase due to absorption throughout the device. Bright spatial soliton propagation in the presence of two-photon absorption should include the transverse variation of the field that accounts for the square-root relationship to the $T$ parameter shown in Eq. (15). For a 1-D soliton the peak intensity decreases as a result of two effects: the attenuation that is due to two-photon absorption and the adiabatic broadening of the beam that is due to the tendency to maintain the characteristic soliton shape. The smaller the peak intensity, the less attenuation; therefore, as a result of the beam broadening, the power attenuation for the same initial peak intensity $I_0$ over a given distance is then less for a soliton than for a plane wave, as evident from a comparison of Eqs. (11) and (17), where Eq. (11) can be rewritten as

$$P_{\text{soliton}}(z) = \frac{P_0}{\sqrt{1 + (4/3) \beta_{\text{TPA}} I_0 z}},$$

which asymptotically approaches zero with increasing distance much more slowly than the plane-wave propagation given by Eq. (17).

**D. Simultaneous Linear and Two-Photon Absorption**

In many cases, even operating in a transparency region of a material, there is some linear absorption due to material or waveguide scattering (typically $<1 \text{ cm}^{-1}$). Considering both linear and two-photon absorption, the power of the pump at some distance $dZ_0$ is given by

$$P_p(s, d) = \frac{r \sqrt{2s} \mathcal{F}_{\text{TPA}}(d)P_0}{\sqrt{r^2[\exp(2s) - 1] + 2s \mathcal{F}_{\text{TPA}}^2(d)\exp(2s)}},$$

which reduces to Eq. (16) for $s \ll 1$. Now the initial pump-to-signal beam ratio $r$ is limited to $\mathcal{F}_a$, so the upper bound on the gain becomes

$$\mathcal{F}(s, d) = \frac{\sqrt{2s} \mathcal{F}_a(s, d) \mathcal{F}_{\text{TPA}}(d)}{\sqrt{\mathcal{F}_a^2(s, d)[\exp(2s) - 1] + 2s \mathcal{F}_{\text{TPA}}^2(d)\exp(2s)}},$$

which reduces to $\mathcal{F}(s, d) = \mathcal{F}_{\text{TPA}}(d)^{2s/\exp(2s) - 1}$ for large $\mathcal{F}_a$ and approaches $\mathcal{F}_{\text{TPA}}$ for small $s$. The gain upper bound given by Eq. (20) can be maximized by the value of $s$ that satisfies
\[
\exp\left(-2s_{\text{opt}}\right) = \frac{1}{1 - 2s_{\text{opt}}} = \left(\frac{12\pi^2 a\lambda}{T} + 1\right),
\]

which depends only on material parameters. Notice that \(0 \approx s_{\text{opt}} < 0.5\) for \(T > 0\), and \(s_{\text{opt}} = 0.5\) when \(T = 0\), as previously shown with linear absorption in isolation.

4. MATERIAL EXAMPLES

As an example of the figures of merit in the case of both linear and two-photon absorption, we consider silicate glass containing 39% cation lead, where for \(\lambda_0 = 1.064\) \(\mu\)m: \(n_0 = 1.774\), \(n_2 = 2.2 \times 10^{-15}\) \(\text{cm}^2\text{W}^{-1}\), \(\beta_2 = 7.2 \times 10^{-13}\) \(\text{cm}^2\text{W}^{-1}\), and we assume \(\alpha = 0.1\) \(\text{cm}^{-1}\) due to scattering in a slab waveguide since the material absorption is negligible (2 dB/m measured loss in fiber). These material constants give \(T = 0.069\) and \(s_{\text{TPA}}(0) = 2.9\), as introduced previously, and \(s_{\text{TPA}}(6) = 3.8\). The upper bound on the ratio of the undetected pump at the output to the signal at the input, or gain, is then determined by the value of the linear absorption constant \(s\) chosen, and hence the linear absorption figure of merit \(F_s\). The maximum value of \(s\) that is tolerable is determined by the interaction (note that the interaction degradation due to two-photon absorption must also be considered), but the optimal value of \(s\) in terms of maximizing the gain upper bound given by Eq. (21) is determined only by the material parameters. For our silicate example, Fig. 7 plots Eq. (9) for \(F_s\) and Eq. (20) for the gain upper bound, versus \(s\) and parameterized by two normalized propagation distances, \(d = 6\) and \(d = 10\). For both device lengths the optimal value is \(s = 0.068\) as given by Eq. (21), providing a gain upper bound of \(F_s(0.068, 6) = 3.5\) for \(r = F_s(0.068, 6) = 14\) at \(d = 6\) and \(F_s(0.068, 10) = 2.7\) for \(r = F_s(0.068, 10) = 11\) at \(d = 10\).

Table 1 shows calculated figures of merit, where \(F_s\) and \(s\) are calculated for \(s_{\text{opt}}\), for three promising nonlinear materials for use in spatial soliton switching applications. Since the gain upper bounds are greater than unity, any of these three materials should be able to support multilevel, cascaded spatial soliton-based switching. Even though silicate has large figures of merit, such high powers may be necessary to create a single soliton due to its small nonlinear coefficient that the implementation of practical, low power systems may be impossible. As a result, it appears that p-toluene sulfonate (PTS) could be more useful because of its similarly large figures of merit and three-orders-of-magnitude larger nonresonant nonlinearity.

Operating below half band gap in semiconductors is promising for nonlinear optics because it is expected that two-photon absorption is zero in this regime. Because of defect states, however, there can be the equivalent of an Urbach tail in the two-photon absorption that extends below half band gap. This is the case in the first example for AlGaAs at \(\lambda_0 = 1.545\) \(\mu\)m, where linear and two-photon absorption limit the gain to \(~1.6\). The second AlGaAs example, where \(\lambda = 1.55\) \(\mu\)m, is for a sample of sufficient purity in which two-photon absorption is, for all practical purposes, zero. In this case where linear absorption is dominant, we chose the optimal value \(s = 0.5\), resulting in \(F_s(0.5, 10) = 27\) with a gain upper bound of 17. The large linear absorption figure of merit indicates we have a lot of headroom to reduce \(s\) and still maintain \(F_s(0.5, 10) > 1\), but we must note that the soliton power and peak intensity increase with decreasing \(F_s\).

Since two-photon absorption is nearly zero, we might need to consider the effects of nonzero three-photon absorption. To estimate the magnitude of this effect, we calculate the three-photon absorption figure of merit \(F_{\text{TPA}}(6) = 1.774, 2\pi k n_3^2 F_{\text{TPA}}(6)\), which should be much smaller than unity, where for \(F_{\text{TPA}}(0.5, 10) = 27, V = 0.07\), indicating that three-photon absorption can probably be neglected. Lowering \(s\) and hence lowering \(F_s\) will increase \(V\).

The material examples we have presented do not make use of one-photon resonant enhancement. We will now briefly discuss the implications of the linear absorption figure of merit and the maximum gain figure to one-photon resonant enhancement. At the very minimum, a logic gate needs unity gain. This requirement, taken with the expression for the gain upper bound in a linearly absorbing material \(F_{\text{opt}}(0.5, d) \exp(-0.5) = 1.365/\sqrt{\Delta a\lambda}\), places the restriction on the linear absorption constant \(\alpha \approx 0.0186/\Delta a\lambda\). As will be shown in Section 5, we can set the normalized gate length to \(d = 5\) for the dragging interaction with \(s = 0.5\). It is also unreasonable to assume a material wavelength of \(\lambda = \lambda_0 / n_0 = 0.3\) \(\mu\)m. This means that to obtain a soliton dragging logic gate with at least unity gain requires a material with linear absorption constant \(\alpha \approx 125\) \(\text{cm}^{-1}\) and, for a gain of two, \(\alpha \approx 31\) \(\text{cm}^{-1}\). This clearly indicates the importance of experimental determination of the linear absorption spectrum and the nonlinear dispersion curve in order to achieve the maximum nonlinearity within the absorption constraint. Thus the trade-off between low absorption and large nonlinearity will have to be made in favor of tuning far out on the wings of the resonant absorption peak.

![Fig. 7. Plot of the linear-absorption figure of merit \(F_{\text{opt}}\) (thin curves) given by Eq. (9) and gain upper bound \(F_s\) (heavy curves) given by Eq. (20) versus the linear absorption parameter \(s\) parameterized by normalized gate lengths \(d = 6\) and \(d = 10\). The material is 39% lead silicate glass, and the optimum gain upper bound occurs in both cases at \(s_{\text{opt}} = 0.068\) as given by Eq. (21). For \(d = 6\) the upper bound on gain is 3.5, whereas for \(d = 10\) it is 2.7.](image-url)
5. NUMERICAL SIMULATIONS OF DRAGGING AND COLLISION

Lastly, we numerically demonstrate the operation of the spatial soliton dragging and soliton collision interactions in the presence of absorption. The phase-insensitive interaction between orthogonally polarized solitons is modeled by the coupled nonlinear Schrödinger equations

\[
2ik \frac{\partial E_x}{\partial z} + \frac{\partial^2 E_x}{\partial x^2} + 2k^2 \frac{n_a}{n_0} (1 + ik)(|E_y|^2 + \Delta|E_x|^2)E_x + ikaE_x = 0, \quad (22)
\]

\[
2ik \frac{\partial E_y}{\partial z} + \frac{\partial^2 E_y}{\partial x^2} + 2k^2 \frac{n_a}{n_0} (1 + ik)(|E_y|^2 + \Delta|E_x|^2)E_y + ikaE_y = 0, \quad (23)
\]

where we neglect any material or form birefringence and the phase-dependent term in the nonlinear polarization; we use \( \Delta = 2/3 \) for an isotropic material and orthogonal linear polarizations. Note that, for the frequency-degenerate case of self and cross-focusing and isotropic symmetry class, the third-order susceptibility \( \chi^{(3)} \) has two independent elements. Kleinman symmetry can be used to reduce the number of independent elements to one. However, in our case, Kleinman symmetry is not strictly valid because of the possible proximity to a two-photon resonance in order to obtain large refractive nonlinearity. But, since we want to avoid strong one- and two-photon resonance even at the expense of the refractive part of the nonlinearity, we will assume that Kleinman symmetry is approximately valid, allowing us to use just the one independent element of \( \chi^{(3)} \).

The two-photon absorption for the x polarization consists of two parts: the simultaneous absorption of two x-polarized photons and the cross absorption of one x-polarized and one y-polarized photon. Since the real and imaginary parts of \( \chi^{(3)} \) are subject to the same symmetries, \( \Delta \) is the same for both cross-phase modulation and cross-two-photon absorption. This additional term in the nonlinear absorption is illustrated by writing the analog of Eq. (2) for the total integrated power of each polarization of the coupled system

\[
\frac{\partial}{\partial z} P_x(z) = -\alpha \int_{-\infty}^{\infty} |E_x|^2 \, dx - \beta_2 \int_{-\infty}^{\infty} (|E_x|^4 + \Delta|E_y|^2|E_x|^2) \, dx, \quad (24)
\]

where \( P_x \) is the power contained in the x polarization and the coupled equation for \( P_y \) is similar. The cross-two-photon absorption of a probe beam interacting with a stronger pump was used recently to measure the two-photon absorption anisotropy in cubic GaAs and CdTe. The same effect occurs here in the isotropic case in that the pump (signal) will absorb more strongly in the presence of the signal (pump) than without, but because \( \Delta = 2/3 \) for an isotropic material when orthogonal linear polarizations are used, the induced absorption caused by an orthogonally polarized beam of the same intensity is slightly weaker than the self-induced absorption.

Using both split-step Fourier and Crank–Nicholson finite-difference numerical techniques and obtaining similar results, we simulated spatial soliton dragging and collision for gate lengths up to 10\( Z_0 \), or \( d = 10 \). We define the gate contrast as the initial power of the signal divided by the power passing through the aperture in the low state so that a contrast greater than unity indicates there is not enough power at the output to launch a signal-like soliton in the succeeding stage; contrasts in excess of 5 are desirable and will be our baseline. Therefore an interaction does not provide useful gain until the contrast exceeds 5. The contrast of a gate will increase for decreasing aperture width, but the gain will decrease because of decreased throughput of the pump; for a fundamental soliton of width \( w_0 \), more than 95% of the power will pass through an aperture of width 3.5\( w_0 \). The width of the fundamental pump soliton is given by \( w_p = w_p / r \), where \( r \) is the initial beam power ratio, and the optimal case is to base the aperture width on the width of the pump at the end of the gate, or 3.5\( w_p(d) \). For simplicity we use an aperture width of 3.5\( w_0 \), where \( w_0 \) is the width of the fundamental signal soliton.

A. No Absorption

We begin with a comparison of the interactions in the absence of absorption in order to determine the change in interaction behavior with absorption. For a given \( r \) the value of \( \kappa \) and the aperture width determine the minimum gate length; a value of \( \kappa \) too small or too large causes a smaller change in the pump’s propagation angle, requiring longer distance for the pump to be moved out of the aperture. The combination of properly varying \( \kappa \) and aperture width with \( r \) results in a nearly uniform minimum gate length; the combination of optimal \( \kappa \) and aperture width determines the minimum length.

A contour plot of minimum desirable contrast (>5) for the optimized soliton dragging and collision interactions

### Table 1. Calculated Figures of Merit for Promising Nonlinear Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>( \lambda_0 ) (( \mu m ))</th>
<th>( n_0 )</th>
<th>( a ) (cm(^{-1} ))</th>
<th>( n_2 ) (cm(^2/W ))</th>
<th>( \beta_2 ) (cm/W)</th>
<th>( \xi_{TPA}^{(10)} )</th>
<th>( \xi_{s_{opt}}^{(10)} )</th>
<th>( G(s_{opt}, 10) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>39% Pb silicate(^{26})</td>
<td>1.064</td>
<td>1.774</td>
<td>0.10</td>
<td>2.2 \times 10^{-15}</td>
<td>7.2 \times 10^{-13}</td>
<td>2.9</td>
<td>0.068</td>
<td>11</td>
</tr>
<tr>
<td>p-toluene sulfonate(^{28})</td>
<td>1.6</td>
<td>1.8</td>
<td>0.30(^n)</td>
<td>2.2</td>
<td>10^{-12}</td>
<td>&lt;5.0 \times 10^{-10}</td>
<td>2.9</td>
<td>0.13</td>
</tr>
<tr>
<td>Al(<em>{0.18})Ga(</em>{0.82})As(^{29})</td>
<td>1.545</td>
<td>3.4</td>
<td>0.20</td>
<td>1.1 \times 10^{-13}</td>
<td>8.0 \times 10^{-11}</td>
<td>1.6</td>
<td>0.047</td>
<td>7.3</td>
</tr>
<tr>
<td>Al(<em>{0.18})Ga(</em>{0.82})As(^{20})</td>
<td>1.55</td>
<td>3.4</td>
<td>0.15</td>
<td>1.2 \times 10^{-13}</td>
<td>0.055</td>
<td>10^{-13}</td>
<td>0.055</td>
<td>10^{-13}</td>
</tr>
</tbody>
</table>

\(^n\)Three-photon absorption is estimated at \( \beta_2 = 0.055 \times 10^{-13} \) cm/W.

\(^n\)Given by \( \xi_{s} \) exp(\( -s \)) for a fundamental soliton pump.
is shown in Fig. 8, in which $\kappa$ was chosen for every value of $r$ for each interaction, and the aperture width is $3.5w_p = 3.5w_0/r$. The contrast rises steeply to the right (with increasing gate length) of the contours, so we display only the contrast 5 contour (so that less than one fifth of the energy necessary to form the fundamental signal soliton passes through the output aperture). Notice that, for initial ratios greater than $r = 2$, the minimum gate length is $\sim 2Z_0$ for dragging and $\sim 4.5Z_0$ for collision and that, in the absence of absorption, the gain is given by the initial beam ratio $r$. The minimum gate lengths remain approximately flat with beam-power ratio even above $r = 11$, with the limit on $r$ imposed only by the paraxial approximation: $w_p = w_0/r \gg \lambda$. The figure illustrates two important differences between the dragging and the collision interactions. The first difference is that the minimum gate length for collision is longer than dragging, between 4 and $5Z_0$ for $r > 2$. The second difference is that collision does not work well for $r \approx 1$ because the solitons tend to pass through each other instead of deflecting.

As mentioned previously, there are two reasons why soliton collision requires a longer gate length than soliton dragging. The first reason is that, with collision, the solitons are not overlapping at the input, with the actual interaction occurring $\sim 2Z_0$ inside the material. We could eliminate this extra $2Z_0$ by decreasing the initial separation at the input such that there is some overlap (but not complete overlap as in the dragging case), but then it becomes difficult to distinguish between the collision and the dragging interactions. In fact, this geometry can be used to demonstrate the tolerance to spatial misalignments that each interaction possesses. The second reason that collision requires a longer gate length is that smaller interaction angles are optimal for the collision interaction. This means that, when compared with dragging with the same aperture width, the smaller induced angle of propagation of the pump requires a longer distance to achieve a fixed transverse displacement given by the aperture width.

In an optimized gate, as illustrated in the previous example, the normalized interaction angle $\kappa$ is based on the initial beam ratio $r$ at the input of each gate, and the aperture width is based on the width of the pump at the output. It is not practical in a system to change $\kappa$ for each value of $r$; therefore, for each interaction we will use two fixed values of $\kappa$, one suitable for small ratios and one for large ratios, to illustrate how the interactions behave near the extremes. Ultimately, $r \gg 1$ is the case in which we are interested in order to provide large gain. In the absence of absorption the width of the pump at the output is the same as the width at the input, but in the presence of absorption, however, the width of the pump at the output will be wider than at the input and often will be close to that of the signal at the input. Therefore the optimal output aperture width is different when $r = 3$, for example, in each of the cases of no absorption, linear absorption only, and both linear and two-photon absorption, and even depends on the material in the latter two cases. For simplicity and comparison purposes, in all further simulations we will fix the aperture width at $3.5w_0$, even though optimizing the aperture width may account for an improvement in gain at any fixed contrast threshold.

Figure 9 shows a contour plot of contrast and gain versus gate length and initial beam ratio for soliton dragging and collision in the absence of absorption with fixed output aperture size ($3.5w_0$) and two fixed values of $\kappa$ for each interaction. Each contrast contour represents a contrast of 5 for a single simulation. The enclosed regions inside each of these contours are the regions in which the gate operates with sufficient contrast (typically much greater than 5) to define and compare gain. The four contrast contours are for the dragging and the collision interactions superimposed with gain contours (thin dotted lines) versus gate length and initial beam ratio in a material with no absorption. The optimal normalized interaction angles are approximated by the expressions $\kappa(r) = 2.2r/(r + 1)$ for dragging (heavy) and $\kappa(r) = 0.9r/(r + 1)$ for collision (medium), and the aperture width is given by $3.5w_0 = 3.5w_0/r$. 

![Fig. 8. Regions exceeding contrast of 5 (thick curves) for optimized spatial soliton dragging and collision interactions superimposed with gain contours (thin dotted lines) versus gate length and initial beam ratio in a material with no absorption. The optimal normalized interaction angles are approximated by the expressions $\kappa(r) = 2.2r/(r + 1)$ for dragging (heavy) and $\kappa(r) = 0.9r/(r + 1)$ for collision (medium), and the aperture width is given by $3.5w_0 = 3.5w_0/r$.](image1)

![Fig. 9. Regions exceeding contrast of 5 (thick curves) for optimized spatial soliton dragging and collision interactions superimposed with gain contours (thin dotted lines) versus gate length and initial beam ratio in a material with no absorption. The optimal normalized interaction angles are approximated by the expressions $\kappa(r) = 2.2r/(r + 1)$ for dragging (heavy) and $\kappa(r) = 0.9r/(r + 1)$ for collision (medium), and the aperture width is fixed at $3.5w_0$.](image2)
sion interactions, each with two values of normalized interaction angle: $\kappa = 1.5$ (small $r$) and $\kappa = 2.0$ (large $r$) for dragging, and $\kappa = 0.6$ and $\kappa = 0.8$ with an initial soliton separation of $3.5w_0$ for collision. These values of $\kappa$ were chosen to minimize the required gate lengths for small and large $r$, respectively. The maximum gain of 8.4 for soliton dragging occurs for a gate length of $10Z_0$, but, in the absence of absorption, the gain continues to increase for longer propagation distances. When $\kappa = 1.5$, the minimum gate length is $3.0Z_0$ with a gain of $-1.5$, whereas for $\kappa = 2.0$ the minimum gate length is $4.0Z_0$ with a gain of $-2.5$. For collision the maximum gain is 7.0 with a gate length of $10Z_0$. The minimum gate length is $6.8Z_0$ with a gain of 2.0 for $\kappa = 0.6$ and $7.2Z_0$ with a gain of 3.5 for $\kappa = 0.8$. Note that dragging only requires a gate length of $4.4Z_0$ to provide a gain of 3.5. Because of the shorter gate length required by dragging, it is expected that dragging will be able to provide larger gain in the presence of absorption. The contour plots for both interactions, especially in the case of collision, clearly indicate that, as expected, higher gain for small gate lengths can be achieved with the larger values of $\kappa = 2.0$ for dragging ($\kappa = 0.8$ for collision) over $\kappa = 1.5$ ($\kappa = 0.6$), but further increases of $\kappa$ provide little or no improvement.

Table 2 provides a summary of the important data for comparing the dragging and the collision interactions by using the three material parameter cases discussed in this section: no absorption, linear absorption only, and both linear and two-photon absorption. The table provides entries for the maximum gain (with the associated gate length) and the minimum gate length (with the associated gain) for each interaction. The maximum gain and the minimum gate length will be our comparison metrics to evaluate the performance of the interactions in the presence of absorption.

### B. Linear Absorption

We now look at the linear absorption case with the optimized linear absorption parameter $s_{\text{opt}} = 0.5$, thereby providing the largest gain upper bound as determined in Subsection 3.A. Note that we maintain $s = 0.5$ for each normalized gate length $d$, so for each gate length the total linear absorption over that length is $\exp(-0.5)$. We again used gate lengths up to $10Z_0$ with initial beam power ratios varying from unity to 11, where the maximum obtainable gain at $10Z_0$ is 6.7 for $r = 11$. The simulations were performed with $F_0(0.5, d) = 11$ to stay within the paraxial approximation. These contour plots are material independent in the sense that dragging and collision will behave the same in any material for $s = 0.5$ in paraxial propagation. The material dependence enters in through the linear absorption figure of merit $F_a$, where the maximum initial ratio $r$ is given by $F_a(0.5, d)$. For soliton dragging, as shown in Fig. 10, the maximum gain is 5.5 with a ratio of $r = 9$ at $d = 10$, which means that a gain of 5.5 should be obtainable in any linearly absorbing material with $F_a(0.5, 10) \geq 9$. Similarly, at $d = 6.5$ a gain of 4.0 should be obtainable for any material with $F_a(0.5, 6.5) \geq 6.6$. Notice that the minimum gate lengths for the dragging interaction have increased slightly over the case with no absorption. For soliton collision an initial ratio of $r = 7.2$ provides the maximum gain of 4.4 at $10Z_0$. The minimum gate lengths for collision with linear absorption have increased by a larger proportion relative to collision with no absorption (8.7/7.2 = 1.2 for $\kappa = 0.8$) than the corresponding proportion for dragging (4.3/4.0 = 1.1 for $\kappa = 2.0$), indicating that soliton collision is not quite as tolerant to linear absorption as dragging.

The reason soliton dragging can better tolerate linear absorption is that the interaction takes place at the beginning of the nonlinear material before significant absorption of the solitons occur. Soliton collision, on the other hand, takes place somewhere within the material such that absorption has already attenuated the pump and the signal, with the power ratio between the two remaining constant, causing their widths to increase. Because the widths of the pump and the signal at the point of interaction are greater than at the input, the interaction will require a longer distance to achieve high contrast due to the scaling of the confocal distance (and therefore the gate length).

Maximized gain can be calculated for a material based on single soliton propagation; however, this does not mean that a switching interaction can achieve that gain or that the maximum gain that an interaction can provide in that material occurs for $s = 0.5$, which gives the maximum possible ratio of the pump at the output to the signal at the input in a linearly absorbing material. We illustrate this point by reducing the amount of absorption over the gate length by using $s = 0.25$ and hence lowering $F_a$, resulting in narrower solitons carrying more power as given by Eqs. (8). In this case for $d = 10$, soliton dragging with $\kappa = 2.0$ can provide a gain of 6.7 with

<table>
<thead>
<tr>
<th>Interaction</th>
<th>No Absorption</th>
<th>Linear Absorption ($s = 0.5$)</th>
<th>Both ($s = 0.068$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max. Gain</td>
<td>Min. Gate</td>
<td>Max. Gain</td>
</tr>
<tr>
<td>Dragging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 1.5$</td>
<td>8.0/10</td>
<td>1.5/3.0</td>
<td>5.1/10</td>
</tr>
<tr>
<td>$\kappa = 2.0$</td>
<td>8.4/10</td>
<td>2.5/4.0</td>
<td>5.5/10</td>
</tr>
<tr>
<td>Collision</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 0.6$</td>
<td>5.5/10</td>
<td>2.0/6.8</td>
<td>3.0/10</td>
</tr>
<tr>
<td>$\kappa = 0.8$</td>
<td>7.0/10</td>
<td>3.5/7.2</td>
<td>4.4/10</td>
</tr>
</tbody>
</table>

*The first number in each table entry is the gain, and the second number is the corresponding gate length.
Fig. 10. Plot of minimum desired contrast (thick curves) and gain contours (thin dotted lines) versus gate length and initial beam ratio for spatial soliton dragging and collision in a linearly absorbing material for \( s = 0.5 \). The normalized interaction angles for dragging are \( k = 1.5 \) (heavy dashed) and \( k = 2.0 \) (heavy solid) and for collision \( k = 0.6 \) (medium dashed) and \( k = 0.8 \) (medium solid), and the aperture width is \( 3.5w_0 \). The maximum gain is limited by the linear absorption material figure of merit \( F_a(0.5, d) \), which determines the maximum initial ratio \( r \).

an initial ratio of 8.6, and collision with \( k = 0.8 \) can provide a gain of 5.5 with an initial ratio of 7.1.

In order to reduce the power required to launch a fundamental soliton given by Eq. (8), we may want to keep \( s \) larger than the optimal value, thereby increasing \( F_a \). Using \( s = 1 \) only reduces the maximum gain upper bound to 0.86 times the optimal value when \( s = 0.5 \) and allows the signal power to be reduced by a factor \( \sqrt{2} \), increasing the gate length by a factor of 2. For this case, dragging can achieve a gain of 3.4 with an initial ratio of \( r = 9.3 \), but collision is unable to provide gain with contrast greater than 5.

C. Simultaneous Linear and Two-Photon Absorption

We do not consider two-photon absorption in isolation because there will always be some residual linear absorption or scattering, although in some cases, the linear absorption is small enough that two-photon absorption dominates.

We now look at the performance of the interactions in the presence of both linear and nonlinear absorption, using the lead-doped silicate material parameters. Figure 11 shows the soliton dragging interaction over a distance of \( 10Z_0 \) for \( s = 0.068 \) with an initial ratio of \( r = 3.0 \) (with \( w_p = 2.15 \mu m \)) with \( k = 1.7 \), achieving a gain of 2.0 with very high contrast. Figure 12 shows the soliton collision interaction with \( r = 3 \), an initial soliton displacement of \( 3.5w_0 \), and a normalized relative angle of propagation of \( k = 0.7 \), also achieving a gain of 2.0 with high contrast.

From Fig. 11 the effects of cross-two-photon absorption on the trapped signal when \( \Delta = 2/3 \) are evident when compared with \( \Delta = 0 \), in which the signal is largely unaffected by self-two-photon absorption. Since the effect of the signal on the pump occurs near the input to the nonlinear medium, it is expected that the additional attenuation of the signal due to the presence of the pump does not significantly degrade the dragging interaction.

Figure 13 shows gain and contrast versus gate length and initial beam ratio, where \( s = 0.068 \) for each value of \( d \). At \( d = 10 \), the upper bound on gain of 2.7 can be obtained by both dragging and collision with an initial ratio of \( r = 11 \) \( (w_p = \lambda = 0.6 \mu m) \). It is useful to note also that a slightly smaller gain of 2.5 can be obtained with an initial ratio of only 5.7; the fact that a large reduction in the initial ratio results in a small reduction in the gain illustrates the diminishing returns due to two-photon absorption. For both interactions the minimum gate lengths increase by only 5%–7% over the case with no absorption, so both interactions tolerate two-photon absorption nearly equally. The maximum gain provided by the dragging interaction occurs near \( d = 6 \), where \( F_a(0.068, 6) = 14 \); for an initial beam ratio of \( r = 6.0 \) (less than the maximum of 14), a gain of nearly 3.1 can be achieved, a little less than the optimal value of 3.5 shown in Fig. 7. The collision interaction is able to reach the gain upper bound for gate lengths in the range 8.6 to 10\( Z_0 \), with the maximum gain of 3.0 at 8.6\( Z_0 \) for an initial ratio of \( r = F_a(0.068, 8.6) = 12 \). Again it is clear that the main disadvantage to the collision interaction is the longer gate lengths necessary to achieve a given gain criterion and hence requiring stronger pumps at the input and therefore more total input energy per switch and dis-

Fig. 11. Spatial soliton dragging with the parameters of 39% lead-doped silicate glass, where \( F_a(0.068, 10) = 11 \) and \( F_{TPA}(10) = 2.9 \) with an initial relative propagation angle of \( k = 1.7 \). The top shows the signal (thin lines) and the pump (thick lines) with \( \Delta = 0 \), whereas the bottom shows the dragging interaction with \( \Delta = 2/3 \). The initial beam ratio is 3.0, and the gain is 2.0 with a contrast of 6637 without the use of a polarizer at the output. The material parameters place an upper bound of 2.7 on the gain at 10\( Z_0 \). Note that cross-two-photon absorption suppresses the trapped signal beam (when \( \Delta = 2/3 \)) in comparison with the case when \( \Delta = 0 \).
sipating more energy as heat. For example, to achieve a gain of 3.0 with the glass parameters, collision requires a gate length of $8.6Z_0$ with $r = 12$, and dragging requires a gate length of $5.3Z_0$ with a smaller initial ratio of 5.1; therefore the collision gate must dissipate 113% more power (given by the ratio of total input powers for switching) than the dragging gate for the same gain.

The reason that the collision interaction does not degrade as quickly in the presence of two-photon absorption as with linear absorption is that, in the former case, for large $\gamma_{\text{TPA}}$, the signal is not significantly attenuated, as shown in Fig. 6. Therefore, with two-photon absorption, at the point of interaction the signal is nearly the same width as at the input (but with a much smaller power ratio than at the input), and the gate length in terms of the signal width at the input is approximately correct.

6. CONCLUSIONS

In conclusion, we have presented soliton interaction material figures of merit for both linear and two-photon absorption that are valid within the paraxial approximation. These figures are useful guidelines to evaluate the suitability of a given nonlinear material for spatial soliton switching applications between fundamental solitons of different widths. Linear absorption alone constrains both the widths of the solitons (and hence the maximum input power ratio) and the maximum gain achievable by an interaction, and it also places an upper bound of $\alpha = 30 \text{ cm}^{-1}$ on the linear absorption constant of a material in order to support a gain of at least 2. Two-photon absorption alone constrains only the maximum gain that an interaction can achieve, and it places no constraints on the widths, and hence the maximum initial ratio is unlimited. In combination, linear and two-photon absorption constrain the soliton widths and the maximum gain. In all three cases, material parameters can be used to calculate an optimized gain upper bound, and we have identified three potentially useful material candidates.

We also demonstrated numerically the operation of the spatial soliton dragging and collision interactions using material parameters with no absorption, linear absorption only, and both linear and two-photon absorption. In general, the dragging interaction requires a shorter gate length and can provide higher gain than collision. In a linearly absorbing material with a sufficiently large linear absorption figure of merit (such as AlGaAs) the dragging interaction can provide gain as high as 5.5 with an exponential absorption length $s = 0.5$ or a gain of 6.7 with $s = 0.25$. When both linear and two-photon absorption are present, such as 39% Pb silicate, dragging can provide gain of just over 3.0. Both interactions tolerate the presence of absorption and can provide reasonable gain and contrast that should allow the implementation of spatial soliton gates for logic and photonic switching.

REFERENCES

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