Optics of finite size

Introduction

• Up to now, all optics have been infinite in transverse extent. Now we’ll change that.

• Types of apertures: edges of lenses, intermediate apertures (“stops”).

• Two primary questions to answer:
  – What is the angular extent (NA) of the light that can get through the system. “Aperture” = largest possible angle for object of zero height. Depends on where the object is located.
  – What is the largest object that can get through the system = “Field”. At the edge of the field, the angular transmittance is one half of the on-axis value.

• Will find each of these with two particular rays, one for each of above.

• Will find two specific stops, one of which limits aperture and one which limits field.

• The conjugates to these stops in object and image space are important and get their own names.

• Finally, this will allow us to understand the total power efficiency of the system.
The aperture stop and the paraxial marginal ray.

- Launch an axial ray, the *paraxial marginal ray*, from the object.
- Increase the ray angle until it just hits some aperture.
- This aperture is the *aperture stop*.
- The sin of $\alpha$ is the *numerical aperture*.
- The aperture stop determines the system resolution, light transmission efficiency and the depth of field/focus.
Pupils
The images of the aperture stop

The entrance (exit) pupil is the image of the aperture stop in object (image) space.

- Axial rays at the object (image) appear to enter (exit) the system entrance (exit) pupil.
- When you look into a camera lens, it is the pupil you see (the image of the stop).
- Both pupils and the aperture stop are conjugates.
- A ray launched into the exit pupil will make it through the aperture stop – this is a major reason the pupil is a useful concept.
Windows
Images of the field stop

• Launch an axial ray, the *chief ray*, from the aperture stop.
• Increase the ray angle until it just hits some aperture.
• This aperture is the *field stop*.
• The angle of the chief ray in object space is the *angular field of view*.
• The height of the chief ray at the object is the *field height*.
• The chief ray determines the spatial extent of the object which can be viewed. When that object is very far away, it is convenient to use an angular field of view.
• When the field stop is not conjugate to the object, *vignetting* occurs, cutting off ~half the light at the edge of the field.
Field stops and windows

Definitions

Entrance Window

Aperture stop

Field stop & Exit Window

Chief ray

M&M refer to the chief ray as the paraxial pupil ray (PPR)

The entrance (exit) window is the image of the field stop in object (image) space.
**Numerical aperture**

The measure of angular bandwidth

\[ \text{NA} \equiv n \sin \alpha \]

Definition of numerical aperture. Note inclusion of \( n \) in this expression.

\[ = \frac{D_{pupil}}{2F} \]

Paraxial approximation

\[ F\# \equiv \frac{1}{2NA} \]

Definition of F-number

\[ = \frac{F}{D_{pupil}} \]

Paraxial approximation

\[ r_0 = 0.6 \frac{\lambda}{NA} \approx \lambda F\# \]

Radius of Airy disk

NA is the conserved quantity in Snell’s Law because it represents the transverse periodicity of the wave:

\[ k_x = 2\pi f_x = \frac{2\pi}{\lambda_0} n \sin \alpha = \frac{2\pi}{\lambda_0} NA \]

Therefore \( NA/\lambda_0 \) equals the largest spatial frequency that can be transmitted by the system.

Note that NA is a property of cones of light, *not* lenses.
Effective F#

Lens speed depends on its use

Infinite conjugate condition

\[ F\# = \frac{1}{2\,NA} \]

\[ = \frac{f}{D} \]

Finite conjugate condition

\[ F^i\# = \frac{t'}{D} = \frac{f(1-M)}{D} \]

\[ = (1-M)F\# \]

\[ F^o\# = \frac{t}{D} = \frac{f(1-rac{1}{M})}{D} \]

\[ = (1-\frac{1}{M})F\# \]

or

\[ NA^i = NA/(1-M) \]

\[ NA^o = NA/(1-\frac{1}{M}) \]
Depth of focus

Dependence on F#

Your detection system has a finite resolution of interest (e.g. a digital pixel size) = ρ

\[ \delta z = 2 \rho F^* \# = \frac{\rho}{NA^*} \]

\[ = 4 \lambda (F^* \#)^2 = 1.2 \frac{\lambda}{(NA^*)^2} \]

First eq. is “detector limited”, second is “diffraction limited”

Thus as we increase the power of the system (NA increases) the depth of field decreases linearly for a fixed resolvable spot size ρ or quadratically for the diffraction limit r₀.
Hyperfocal distance
Important for fixed-focus systems

The object distance which is perfectly in focus on the detector is defines the nominal system focus plane.

When this plane is positioned such that the far point goes to infinity, then the system is in the “hyperfocal” condition and the nominal focus plane is called the “hyperfocal distance”.

This is very handy for fixed focus cameras – you can take a portrait shot (if the person is beyond the near point) out to a landscape and not notice the defocus.
Vignetting

Losing light apertures or stops

Take a cross-section through the optical system at the plane of any aperture. Launch a bundle of rays off-axis and see how they get through this aperture:

Unvignetted

\[ a \geq |y| + |\bar{y}| \]

Fully vignetted

\[ a \leq |\bar{y}| - |y| \]
Exit pupil of telescope (image of A.S. in image space) matches entrance pupil of following instrument (eye).

Note that the entrance pupil in the system above will appear to be ellipsoidal for off-axis points. Thus even in this well-designed, aberration-free case, off-axis points will not be identical on axis.

Vignetting: Further, we might find extreme rays at off-axis points are terminated. This loses light (bad) but also loses rays at extreme angles, which might limit aberrations (good).
In a telecentric system either the EP or the XP is located at infinity. The system shown above is doubly telecentric since both the EP and the XP are at infinity. All doubly telecentric system are afocal.

When the stop is at the front focal plane (just lens f2 above) the XP is at infinity and slight motions of the image plane will not change the image height.

When the stop is at the back focal plane (just lens f1 above) the EP is at infinity and small changes in the distance to the object will not change the height at the image plane.

Telecentricity is used in many metrology systems.
Analyzing an optical system

1. Specify the system via distances, component locations, focal length, and clear apertures.
2. Run a marginal ray using a y-u trace
3. Find the aperture stop (calculate the ratio of clear aperture to ray height (smallest r/y_k is aperture stop)
4. Run a chief ray trace
5. Calculate as in step 3 above to find field stop
6. Find locations of pupils and windows via thin lens equations if small # of elements or yu/matrix methods if large #.
7. Sizes of pupils can be determined by taking the slope angle of the chief ray at the aperture stop and dividing by the image and object space slope angles of that ray to get the magnification of the aperture stop in image and object space, respectively. Multiplying the aperture stop size by the magnification gives the exit and entrance pupil sizes. (Optical invariant is the basis).

A full report of a optical system contains: image location and magnification, the location of stops, pupils, and windows, the effective focal length of the system, F/#, and its angular field of view.
Detailed example

Holographic data storage

We’re going to design the imaging path
1 to 1 imaging
Aperture in Fourier plane

SLM

40 mm

Lens

20 mm

Aperture

20 mm

CCD

Material

D_{SLM} = 10 \text{ mm}

D_{Lens} = 10 \text{ mm}

D_{Ap} = 2 \text{ mm}

D_{CCD} = 10 \text{ mm}

p = 10 \mu\text{m}

f = 20 \text{ mm}

Fourier diffraction calculation for a random pixel pattern.
Finding the aperture stop

Trace the paraxial marginal ray (PMR)

\[
\begin{bmatrix}
y_k \\
u'_k
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
-\phi_k & 1
\end{bmatrix}
\begin{bmatrix}
y_k \\
u_k
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_{k+1} \\
u_{k+1}
\end{bmatrix}
= \begin{bmatrix}
1 & t'_k \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
y_k \\
u_k
\end{bmatrix}
\]

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Aperture stop
What does the aperture stop do?
Limits the NA of the system

\[ \text{NA}^* = \frac{D/2}{f(1-M)} = \frac{5}{20(-2)} = \frac{1}{8} \]

However we have stopped the system down to \( \text{NA} = 2/40 = 1/20 \)

Pixel size = 10 \( \mu \text{m} \)

\[ \text{NA}_{\text{pix}} = \frac{\lambda}{p} = \frac{0.5}{10} = \frac{1}{20} \]

The aperture stop thus:
1. Determines the system field of view
2. Controls the radiometric efficiency
3. Limits the depth of focus
4. Impacts the aberrations
5. Sets the diffraction-limited resolution
Find the entrance pupil
(image of aperture stop in object space)

\[-\frac{1}{t_0} + \frac{1}{t_1} = \frac{1}{f}\]  \[\Rightarrow \frac{1}{20} + \frac{1}{t_1} = \frac{1}{20}\]  \[\Rightarrow t_1 = \infty\]

What’s *that* mean? If the entrance pupil is at infinity, then every point on the object radiates into the same cone:
Find the field stop

Trace the chief ray (PPR)

\[
\begin{bmatrix}
y_k \\
u'_k
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\phi_k & 1 \end{bmatrix} \begin{bmatrix} y_k \\
u_k
\end{bmatrix}
\]

\[
\begin{bmatrix}
y_{k+1} \\
u'_{k+1}
\end{bmatrix} = \begin{bmatrix} 1 & t'_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_k \\
u'_k
\end{bmatrix}
\]

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<tr>
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<td>5</td>
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Entrance and exit windows at field stop (since it is lens).
Required lens diameter
And why windows should be at images

Object vignetted at edges

Note that the extreme marginal ray makes the same angle with the chief ray (PPR) as the paraxial marginal ray (PMR) makes with the axis. Thus to fit the extreme marginal ray through an aperture, we require:

\[ D \geq 2\left(|y| + |\bar{y}|\right) = 2(2 + 5) = 12 \]

- Now the camera is the field stop and the entrance window is the SLM.
- The optical system now captures the same light from each SLM pixel and it can be adjusted for all pixels via the aperture stop.
- Independently, we can adjust the field size.
Thicken the lenses

Gaussian design

Design of ideal imaging systems with geometrical optics

Summary
Optical invariant
aka Lagrange or Helmholtz invariant

Write the paraxial refraction equations for the marginal ray (PMR) and chief or pupil ray (PPR):

\[ n'u' - nu = -\phi y \]
\[ n'u' - n\bar{u} = -\phi \bar{y} \]

With a bit of algebra:

\[ n'(u' \bar{y} - \bar{u}' y) = n(u \bar{y} - u y) \]

So

\[ H = n(u \bar{y} - u y) \]

is conserved.

At the object (or image) of limited field diameter \( L \):

\[ y = 0, \quad \bar{y} = \text{edge of field}, \quad u = \text{maximum ray angle} \]

\[ H = nu \bar{y} = NA \frac{L}{2} = .6 \frac{\lambda}{2} \frac{L}{r_0} \approx \frac{\lambda}{2} N_{\text{spots}} \]

Thus we have found the information capacity of the optical system, aka the space-bandwidth product:

\[ N_{\text{spots}} = \frac{2}{\lambda} H \]

Where “spots” here are separated by their diameter, \( 2r_0 \)
Typical number of resolvable spots
A survey of >3000 optical designs

\[ N_{\text{spots}} \equiv \frac{\text{Image field diameter}}{\text{Spot diameter}} \]

So, for a well-optimized system up to ~ 4 elements, we have
1200 spots/element, monochromatic systems with all-spherical elements
1600 spots/element, monochromatic systems with some aspheric elements
1100 spots/element, polychromatic systems with all-spherical elements
1100 spots/element, polychromatic systems with some aspheric elements
Optical invariant for the holographic storage example

\[ N_{\text{spots}} \approx \frac{2H}{\lambda_0} = \frac{2\bar{y}_0 NA_0}{\lambda_0} \]

Definition of optical invariant

\[ = \frac{D_{\text{SLM}} \lambda_0 / p}{\lambda_0} = \frac{D_{\text{SLM}}}{p} = N_{\text{pix}} \]

Stopped-down NA

So we have correctly designed this system to transmit the proper number of resolvable spots.
Counting resolvable waves in the pupil

Remove quadratic phase of converging wavefront. Remainder is rect with linear phase (=tilt) whose FT is a shifted sinc in the image plane.

Wavefronts in AS

Phase in AS

E at image plane

K-space (what is \( \theta \) in AS?)

Spectrum in AS

One \( \lambda \) of tilt in AS is equivalent to shift of DL radius in image plane.