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Decorrelated State Estimation for Distributed Tracking of Interacting Targets in Cluttered Environments *

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Abstract

This paper develops a process to construct decorrelated state estimates when tracking in cluttered environments using a distributed fusion architecture. This construction removes correlation with previous state estimates from the current updates in order to use the updates as measurements in the Kalman filter at the global processor. The effects of correlation with other interacting targets are also investigated for multiple target tracking. The algorithm to construct the decorrelated sequences is presented and applied on a particular distributed architecture where each processor receives measurements from only one sensor.

1. Introduction

The ability to track multiple targets in cluttered environments is needed in many applications such as military surveillance, air traffic control, and mobile robots. Development of fusion algorithms assuming a centralized processing architecture shows that tracking performance can improve significantly when using multiple sensors [8]. While centralized fusion yields better performance, distributed processing architectures are more practical due to considerations such as reliability, survivability, communication bandwidth, and computational resources [5, 9, 10]. However, the merging of state estimates is more difficult in distributed tracking due to the loss of information inherent in forming the track estimates at the local processors.

The general distributed fusion architecture of Fig. 1 consists of several local processors and one global processor. Each local processor independently tracks targets in its surveillance region with its own sensors and implements centralized algorithms for tracking targets in clutter such as Nearest Neighbor (NN) [3], Joint Probabilistic Data Association (JPDA) [3], or Mixture Reduction (MR) [8, 11]. The target state estimates from each local processor are passed to a global processor and possibly other local processors. At the global processor, a distributed fusion algorithm employs track fusion to combine the local tracks to form global tracks of targets in the entire surveillance region.

Compared with using measurement fusion in a centralized architecture, using track fusion at the global processor of a distributed architecture is a more complex problem.

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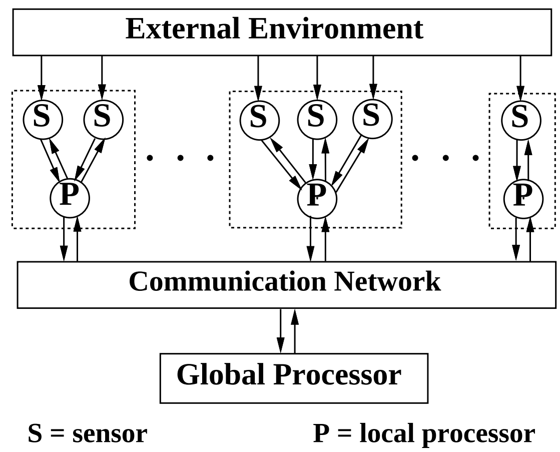

Fig. 1: Distributed sensor fusion architecture.

One of the major issues is accounting for the correlation between different local processor estimates for a common target [1, 4]. Although the measurement errors are independent across sensors, the local track estimates are correlated among local processors because of the common process noises in the target dynamics. To appropriately account for this correlation in the estimation process, the cross covariances between different track estimates must be computed. Because the calculation of these cross covariances is quite involved and not practical, other approaches to distributed fusion are usually necessary. If the local processors are employing Kalman filters and the covariance estimates from these filters are provided to the global processor as part of the track information, then optimal estimates can be formed without computing the cross covariances [2]. However, this approach is applicable only for filtering without including any data association algorithm.

For a particular distributed architecture where each local processor has a single sensor supplying it measurement data, the measurement reconstruction approach [9] or the construction of tracklets [6, 7] can bypass the correlation problem by providing reconstructed measurements or tracklets for the global processor. The major advantages of the tracklet approach are that its statistics are known and easier to compute, and its errors are independent among each other. Like track estimates, tracklets consist of target state vectors and their corresponding error covariance matrices. Tracklets are passed to the global processor, where they serve as measurement inputs to a Kalman filter. The tracklet approach is implemented to remove cross correlation between local and global target estimates. Tracklets are constructed so that their errors are not cross correlated with those of any other data in the system for the same target.

Several techniques to formulate tracklets have been introduced [6, 7]. One method that has a similar behavior with the measurement reconstruction approach [9] is the decorrelation of state estimates. Because the errors of decorrelated sequences are independent, the distributed fusion architecture overcomes the correlation problem and can use a centralized fusion algorithm for each processing stage. The decorrelated sequence method allows the global processor to use the same processing algorithm to obtain the global estimates of targets as does the measurement reconstruction method.

Though previous work has demonstrated how to construct the decorrelated sequences [6, 7], the development has thus far been only for tracking in environments without clutter with specific fusion architectures. This paper explores the statistics of the decorrelated sequences when a data association algorithm such as JPDA is employed. Further, an algorithm for ensuring that the sequences remain uncorrelated even in cluttered environments will be developed and evaluated.

This paper is organized as follows. The Kalman filter and JPDA algorithm are reviewed in Section 2. The characteristics of decorrelated sequences and their construction process for tracking non-interacting targets in clutter are described in Section 3. The process of decorrelation when tracking interacting targets is presented in Section 4. Section 5 describes the integration of the methods of Sections 3 and 4 to obtain an overall decorrelation algorithm for tracking interacting targets in cluttered environments, and Section 6 explains the global filtering process. Finally, concluding remarks are given in Section 7.

2. Kalman Filter and JPDA

Suppose the target and measurement dynamics are determined by known matrices $F^t(k)$, $G^t(k)$, and $H^t(k)$ as

$$x^t(k) = F^t(k)x^t(k-1) + G^t(k)w^t(k) \tag{2.1}$$

$$z^t(k) = H^t(k)x^t(k) + v^t(k) \tag{2.2}$$

where $x^t(k)$ and $z^t(k)$ are the state and measurement of target t, respectively, at the kth time interval. $w^t(k)$ and $v^t(k)$ are independent Gaussian random noise vectors with $\mathcal{N}[0, Q^t(k)]$ and $\mathcal{N}[0, R^t(k)]$ distributions, respectively. The predicted state and measurement are

$$\hat{x}^t(k|k-1) = F^t(k)\hat{x}^t(k-1|k-1), \tag{2.3}$$

$$\hat{z}^t(k|k-1) = H^t(k)\hat{x}^t(k|k-1), \tag{2.4}$$

and

$$\nu^t(k) = z^t(k) - H^t(k)\hat{x}^t(k|k-1) \tag{2.5}$$

is known as the innovation. The covariance of the state and the innovation predictions are

$$P^t(k|k-1) = F^t(k)P^t(k-1|k-1)\big(F^t(k)\big)' + G^t(k)Q^t(k)\big(G^t(k)\big)' \tag{2.6}$$

$$S^t(k|k-1) = H^t(k)P^t(k|k-1)\big(H^t(k)\big)' + R^t(k). \tag{2.7}$$

The state estimate and error covariance updates are

$$\hat{x}^t(k|k) = \hat{x}^t(k|k-1) + K^t(k)\nu^t(k) \tag{2.8}$$

$$P^t(k|k) = \left[I - K^t(k)H^t(k)\right] P^t(k|k-1) \tag{2.9}$$

$$K^t(k) = P^t(k|k-1)\left(H^t(k)\right)'\left[S^t(k|k-1)\right]^{-1} \tag{2.10}$$

where $K^t(k)$ is the Kalman gain. Once the target state and covariance estimates have been updated, they are fed back into the algorithm and the process is repeated for the new measurements at the next time step.

When tracking in cluttered environments and the origin of measurements is not known, a data association algorithm such as the JPDA [3] method is needed. Clutter refers to detections or returns from nearby objects, clouds, electromagnetic interference, acoustic anomalies, false alarms, *etc*. These additional detections lead to the occurrence of several measurements in the validation region of each target. A common mathematical model for such interference is a uniform distribution in the measurement space. In the JPDA algorithm, the combined measurement

$$z^t(k) = \sum_{j=0}^{m_k} \beta_j^t(k)z_j(k) \tag{2.11}$$

is used in (2.5) where $z_j(k)$ is the jth measurement at time k, $\beta_j^t(k)$ is the probability that $z_j(k)$ is the measurement originating from target t, and m_k is the number of gated measurements at time k. $j = 0$ denotes the possibility that there are no target originated measurements, with $z_0(k) = \hat{z}^t(k \mid k-1)$. The updated state covariance is

$$\begin{aligned} P^t(k|k) &= \beta_0^t(k)P^t(k|k-1) \\ &\quad + \left[1 - \beta_0^t(k)\right] P^C(k|k) + \tilde{P}(k) \\ P^C(k|k) &= \left[I - K^t(k)H^t(k)\right] P^t(k|k-1) \\ \tilde{P}(k) &= K^t(k)\Bigg\{\sum_{j=1}^{m_k} \beta_j^t(k)\nu_j^t(k)\left(\nu_j^t(k)\right)' \\ &\quad - \nu^t(k)\left(\nu^t(k)\right)'\Bigg\}\left(K^t(k)\right)'. \end{aligned} \tag{2.12}$$

3. Decorrelated State Estimates

The decorrelated process is achieved as depicted in Fig. 2, by removing correlation between any 2 sequences to form the uncorrelated sequences. If the input sequences have statistics that are jointly normal distributions, the Gauss-Markov theorem can be used to describe this decorrelation process [12]. When tracking a single target in environments without clutter, these 2 correlated inputs are usually the predicted and updated state estimates of the target, and their statistics are jointly normal distributions. In contrast, the statistics of the predicted and updated target state estimates when tracking in cluttered environments are mixtures between jointly normal distributions from actual measurements

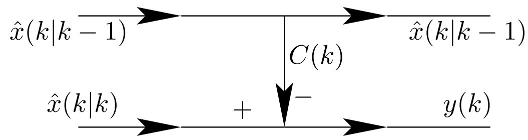

Fig. 2: Using channel model analysis, two correlated input signals are converted into uncorrelated outputs.

and uniform distributions from clutter. However, the decorrelation process is still applicable for both tracking environments if the cross correlation between the 2 input sequences is correctly determined.

For single target tracking, the cross correlation between local and global tracks can be removed by compensating the local target estimates. This decorrelated state estimate $y(k)$ is computed as [6, 7]:

$$y(k) = \hat{x}(k|k) - C(k)\hat{x}(k|k-1) \tag{3.1}$$

$$C(k) = P(k|k)P^{-1}(k|k-1) \tag{3.2}$$

$$Y(k) = P(k|k) - P(k|k)P^{-1}(k|k-1)P(k|k) \tag{3.3}$$

where $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$ are local updated and predicted target state estimates with corresponding error covariances $P(k|k)$ and $P(k|k-1)$, respectively. $C(k)$ is a decorrelation matrix and $Y(k)$ is the corresponding covariance matrix of $y(k)$. The errors of $y(k)$ are not cross correlated with the estimation errors of the corresponding track of the global processor.

Several observations can be made based on the properties of the decorrelated sequence $y(k)$. First, $y(k)$ is also uncorrelated with updated estimates of the previous time, $\hat{x}(k-1|k-1)$, since $\hat{x}(k|k-1) = F(k)\hat{x}(k-1|k-1)$. Also, the decorrelated sequences $\{y(k)\}$ are uncorrelated to each other, or cov$[y(\ell), y(n)] = 0$ for $\ell \neq n$.

3.1. Constructions

In this section, we show that the decorrelated state estimates $y(k)$ can be constructed from the updated estimates $\hat{x}(k|k)$ and predicted states $\hat{x}(k|k-1)$ such that $y(k)$ and $\hat{x}(k|k-1)$ are uncorrelated. The derivations begin by expressing $y(k)$ in terms of the true state $x(k)$ and the error $\tilde{y}(k)$:

$$y(k) = \hat{x}(k|k) - C(k)\hat{x}(k|k-1) \tag{3.4a}$$

$$= B(k)x(k) - \tilde{y}(k), \tag{3.4b}$$

where

$$\tilde{y}(k) = B(k)x(k) + C(k)\hat{x}(k|k-1) - \hat{x}(k|k). \tag{3.5}$$

Using the estimate errors $\tilde{x}(k|k) = x(k) - \hat{x}(k|k)$ and $\tilde{x}(k|k-1) = x(k) - \hat{x}(k|k-1)$, we have

$$\tilde{y}(k) = [B(k) + C(k) - I]\,x(k) + \tilde{x}(k|k) - C(k)\tilde{x}(k|k-1). \tag{3.6}$$

We also assume that all of the errors have zero mean, $E\{\tilde{y}(k)\} = 0$, $E\{\tilde{x}(k|k)\} = 0$, and $E\{\tilde{x}(k|k-1)\} = 0$. These assumptions hold if the coefficient of the true state $x(k)$ vanishes. Therefore, we must have

$$B(k) = I - C(k). \tag{3.7}$$

Using (3.7), $\tilde{y}(k)$ in (3.6) can be rewritten as

$$\tilde{y}(k) = \tilde{x}(k|k) - C(k)\tilde{x}(k|k-1). \tag{3.8}$$

The error covariances of $y(k)$ can be computed as

$$\begin{aligned} Y(k) &= P(k|k) - \text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] C'(k) \\ &\quad - C(k)\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] \\ &\quad + C(k)P(k|k-1)C'(k). \end{aligned} \tag{3.9}$$

To determine $C(k)$, we enforce that $y(k)$ is uncorrelated with $\hat{x}(k|k-1)$:

$$\begin{aligned} \text{cov}[y(k), \hat{x}(k|k-1)] &= E\left\{\tilde{y}(k)\tilde{x}'(k|k-1)\right\} \\ &= E\left\{\tilde{x}(k|k)\tilde{x}'(k|k-1)\right\} - C(k)P(k|k-1) = 0, \end{aligned}$$

or

$$\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] = C(k)P(k|k-1) \tag{3.10}$$

where $\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] \triangleq E\left\{\tilde{x}(k|k)\tilde{x}'(k|k-1)\right\}$.

In order to compute the cross covariance of $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$, $\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right]$, we consider updated state estimates $\hat{x}(k|k)$ derived from environments with and without clutter.

3.2. Environments without Clutter

Only the actual measurement, which is used in the Kalman filter as outlined in (2.3) – (2.10) in Section 2, contributes to the updated estimates when there is no clutter present. It can be shown that

$$\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] = P(k|k). \tag{3.11}$$

Substituting (3.11) in (3.10) to determine $C(k)$ and using (3.7) to compute $B(k)$ gives

$$C(k) = P(k|k)P^{-1}(k|k-1) \tag{3.12}$$

$$B(k) = I - P(k|k)P^{-1}(k|k-1). \tag{3.13}$$

Now using (3.11), (3.12), and (3.13) in (3.9), we have

$$Y(k) = P(k|k) - C(k)P(k|k) = B(k)P(k|k). \tag{3.14}$$

3.3. Environments with Clutter

We initially assume the targets are non-interacting, so the algorithm tracks each target individually and the undesirable measurements constitute random interference for each target. Let $\theta_j(k)$ be the event that $z_j(k)$ originates from a target. Denote $z(k) = \{z_j(k)\}_{j=1}^{m_k}$ as the set of measurements in the target's validation region at time k, and $\mathcal{Z}^k = \{z(1), \ldots, z(k)\}$ as the collection of measurements up to time k. A combined measurement is computed as in (2.11) and used in calculating the innovation in (2.5) as outlined for the JPDA algorithm in Section 2. The covariance of $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$ is

$$\begin{aligned}\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] &= \sum_{j=0}^{m_k} \beta_j(k) E\Big\{\left[x(k) - \hat{x}(k|k)\right] \\ &\quad \cdot \left[x(k) - \hat{x}(k|k-1)\right]' \mid \theta_j(k), \mathcal{Z}^k\Big\} \\ &= T_1 + T_2 + T_3 + T_4. \end{aligned} \tag{3.15}$$

Since we can express $x(k)$ in terms of the estimate and error based on event $\theta_j(k)$ as $x(k) = \hat{x}_j(k|k) + \tilde{x}_j(k|k)$,

$$\begin{aligned} T_1 &= \sum_{j=0}^{m_k} \beta_j(k) E\left\{ x(k)x'(k) | \; \theta_j(k), \mathcal{Z}^k \right\} \\ &= \sum_{j=0}^{m_k} \beta_j(k) E\left\{ \hat{x}_j(k|k)\hat{x}_j'(k|k) + \hat{x}_j(k|k)\tilde{x}_j'(k|k) \right. \\ &\quad \left. + \tilde{x}_j(k|k)\hat{x}_j'(k|k) + \tilde{x}_j(k|k)\tilde{x}_j'(k|k) | \; \theta_j(k), \mathcal{Z}^k \right\}. \end{aligned}$$

Because we assume that $E\left\{\tilde{x}_j(k|k)\right\} = 0$, the second and third terms vanish. Consequently, we obtain

$$T_1 = \sum_{j=0}^{m_k} \beta_j(k) \left[\hat{x}_j(k|k)\hat{x}_j'(k|k) + P_j(k|k) \right] \tag{3.16}$$

where $P_j(k|k) \triangleq E\left\{\tilde{x}_j(k|k)\tilde{x}_j'(k|k) | \; \theta_j(k), \mathcal{Z}^k\right\}$ for $j = 0, 1, \ldots, m_k$. When $j = 0$, $\hat{x}_0(k|k) = \hat{x}(k|k-1)$ and $P_0(k|k) = P(k|k-1)$. For $j = 1, 2, \ldots, m_k$, we have $P_j(k|k) = P^C(k|k)$ where $P^C(k|k)$ is as defined in (2.12). Thus, (3.16) becomes

$$\begin{aligned} T_1 &= \beta_0(k)P(k|k-1) + \left[1 - \beta_0(k)\right] P^C(k|k) \\ &\quad + \sum_{j=0}^{m_k} \beta_j(k) \left[\hat{x}_j(k|k)\hat{x}_j'(k|k) \right]. \end{aligned} \tag{3.17}$$

Similarly, we can obtain T_2, T_3, and T_4 as

$$\begin{aligned} T_2 &= -\sum_{j=0}^{m_k} \beta_j(k) E\left\{ x(k)\hat{x}'(k|k-1) | \; \theta_j(k), \mathcal{Z}^k \right\} \\ &= -\hat{x}(k|k)\hat{x}'(k|k-1) \end{aligned} \tag{3.18}$$

$$\begin{aligned} T_3 &= -\sum_{j=0}^{m_k} \beta_j(k) E\left\{ \hat{x}(k|k)x'(k) | \; \theta_j(k), \mathcal{Z}^k \right\} \\ &= -\hat{x}(k|k)\hat{x}'(k|k) \end{aligned} \tag{3.19}$$

$$\begin{aligned} T_4 &= \sum_{j=0}^{m_k} \beta_j(k) E\left\{ \hat{x}(k|k)\hat{x}'(k|k-1) | \; \theta_j(k), \mathcal{Z}^k \right\} \\ &= \hat{x}(k|k)\hat{x}'(k|k-1) \quad = -T_2. \end{aligned} \tag{3.20}$$

Now, substitute (3.17) – (3.20) in (3.15), and after some cancelations, we obtain

$$\begin{aligned} &\text{cov}\left[\hat{x}(k|k), \hat{x}(k|k-1)\right] \\ &= \beta_0(k)P(k|k-1) + \left[1 - \beta_0(k)\right] P^C(k|k) + \tilde{P}(k) \\ &\triangleq P(k|k) \end{aligned} \tag{3.21}$$

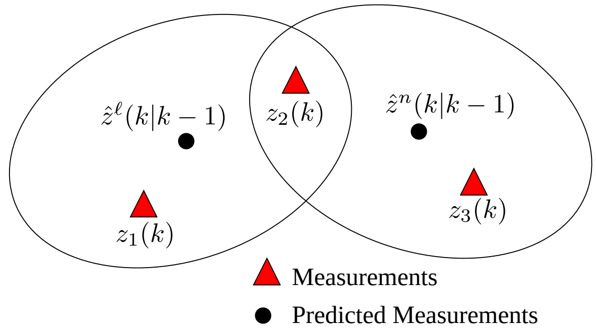

Fig. 3: Tracking multiple targets in the same vicinity may cause overlapping validation regions and create sharing of measurements. Here, $z_2(k)$ is gated by both targets ℓ and n.

where $P^C(k|k)$ and $\tilde{P}(k)$ are defined in (2.12).

Note that $P(k|k)$ represents the covariances between $\hat{x}(k|k)$ and $\hat{x}(k|k-1)$ for environments with and without clutter. However, their computations are different as shown in (2.9) and (3.21), respectively. In either case, we can determine a decorrelated matrix $C(k)$ by using (2.9) or (3.21) in (3.10) to obtain

$$C(k) = P(k|k)P^{-1}(k|k-1). \tag{3.22}$$

Since both $P(k|k)$ and $P^{-1}(k|k-1)$ are computed in the filtering process, the computation of $C(k)$ is easily obtained and does not significantly increase the complexity of the algorithm.

4. Interacting Targets

Up to this point, the decorrelation process only accounts for correlations from one particular target, or self correlations. While the JPDA filter can track multiple targets, thus far, our construction of these decorrelated sequences assumes that validation regions for different targets do not overlap. However, this assumption is not practical in general, since overlapping regions can easily occur in dense target environments. Recall that the JPDA algorithm associates the origins of the measurements to either targets or undesirable clutter. Consequently, target estimates that have overlapping validation regions may share some of the same measurements and hence are correlated with each other.

This section focuses on how to remove this cross-target correlation from the decorrelated sequences, and it is organized into two parts. The cross-target correlations and their effects between interfering targets are first investigated, and then the algorithm to construct the decorrelated sequences is presented.

4.1. Cross-Target Correlations

In the JPDA filter, each gated measurement contributes to the final estimate of a target. When multiple targets are in close vicinity of each other, several common measurements may be gated by T targets. Consequently, these shared measurements affect the estimates of all involved targets. For simplicity, we will first investigate the effect of the cross-target correlation of an individual measurement in an overlapping gate region. In Fig. 3, there are 3 measurements in the surveillance region, but only the $z_2(k)$ measurement is gated by both targets (targets ℓ and n).

Let $\theta_j^t(k)$ denote the event that measurement $z_j(k)$ originates from target t; and let $\beta_j^t \triangleq P\left\{\theta_j^t(k) | \mathcal{Z}^k\right\}$ be the probability of the event being true, where the association of any measurement to clutter is denoted with $t = 0$. The cross correlation between targets ℓ and n based on the $z_j(k)$ measurement can be defined as

$$
\begin{aligned}
R_j^{\ell,n}(k) &\triangleq E\left\{\tilde{x}_j^\ell(k|k)\left(\tilde{x}_j^n(k|k)\right)' \mid \mathcal{Z}^k\right\} \\
&= \sum_{t=0}^{T} \beta_j^t(k)\, E\Big\{\left[\tilde{x}^\ell(k|k-1) - K^\ell(k)\nu_j^\ell(k)\right] \\
&\quad \cdot \left[\tilde{x}^n(k|k-1) - K^n(k)\nu_j^n(k)\right]' \mid \theta_j^t(k),\ \mathcal{Z}^k\Big\}.
\end{aligned}
$$

Since $E\left\{\tilde{x}^\ell(k|k-1)\left(\tilde{x}^n(k|k-1)\right)' \mid \theta_j^t(k),\ \mathcal{Z}^k\right\} = 0$, the correlation $R_j^{\ell,n}(k)$ simplifies to

$$
\begin{aligned}
R_j^{\ell,n}(k) = K^\ell(k)\Bigg[\sum_{t=0}^{T}\beta_j^t(k) E\Big\{\nu_j^\ell(k) \\
\cdot \left(\nu_j^n(k)\right)' \mid \theta_j^t(k),\ \mathcal{Z}^k\Big\}\Bigg]\left(K^n(k)\right)'. \qquad (4.1)
\end{aligned}
$$

Define the mean of $z_j(k)$ associated with target t as

$$
\begin{aligned}
\mu_t &\triangleq E\left\{z_j(k) | \theta_j^t(k),\ \mathcal{Z}^k\right\} \\
&= \begin{cases} \dfrac{1}{T}\displaystyle\sum_{t=1}^{T}\hat{z}^t(k|k-1) & \text{for } t = 0, \\ \hat{z}^t(k|k-1) & \text{for } t = 1, 2, \ldots, T. \end{cases}
\end{aligned}
$$

The expectation of (4.1) can be simplified as

$$
\begin{aligned}
\Gamma_t^{\ell,n}(k) &\triangleq E\left\{\nu_j^\ell(k)\left(\nu_j^n(k)\right)' \mid \theta_j^t(k),\ \mathcal{Z}^k\right\} \\
&= S^t(k|k-1) + \left[\hat{z}^\ell(k|k-1) - \mu_t\right] \\
&\quad \cdot \left[\hat{z}^n(k|k-1) - \mu_t\right]'. \qquad (4.2)
\end{aligned}
$$

Substituting (4.2) in (4.1), we have

$$
R_j^{\ell,n}(k) = K^\ell(k)\left[\sum_{t=0}^{T}\beta_j^t(k)\Gamma_t^{\ell,n}(k)\right]\left(K^n(k)\right)'. \qquad (4.3)
$$

Now, we can extend Fig. 3 to the general case where more measurements are gated by both targets. Let $\mathcal{U}^{\ell,n}(k)$ be the set of measurements that are gated by both targets ℓ and n at time k. Since $\hat{x}^t(k|k) = \sum_{j=0}^{m_k}\beta_j^t(k)\hat{x}_j^t(k|k)$, the total effect of the shared measurements on the cross-target correlation is

$$
\begin{aligned}
R^{\ell,n}(k) &= \sum_j \beta_j^\ell(k)\beta_j^n(k)R_j^{\ell,n}(k) \qquad (4.4) \\
&= K^\ell(k)\left[\sum_{t=0}^{T}\alpha_t^{\ell,n}(k)\Gamma_t^{\ell,n}(k)\right]\left(K^n(k)\right)' \\
\alpha_t^{\ell,n}(k) &\triangleq \sum_j \beta_j^\ell(k)\beta_j^n(k)\beta_j^t(k). \qquad (4.5)
\end{aligned}
$$

The sums are over those indices j where $z_j(k) \in \mathcal{U}^{\ell,n}(k)$, and the effect of the shared measurements between targets ℓ and n due to the association from target t is denoted as $\alpha_t^{\ell,n}(k)$. Since $0 \le \beta_j^t(k) \le 1$, $\alpha_t^{\ell,n}(k) \ll 1$ for all $t = 0, 1, \ldots, T$. Thus, the $R^{\ell,n}(k)$ matrix may be negligible compared to the error covariances $P^\ell(k|k)$ or $P^n(k|k)$ of the state updates of either targets.

4.2. Constructions

In this section, we describe the process to construct decorrelated sequences from two correlated estimates of different targets. The cross-target correlations between targets must be removed when the measurement validation regions are overlapping so that the decorrelated sequences are completely independent.

Suppose that the two interacting targets are targets ℓ and n. The construction can be described as

$$
y^\ell(k) = \hat{x}^\ell(k|k) - C^{\ell,n}(k)\hat{x}^n(k|k) \qquad (4.6)
$$

where superscripts represent target identities. The matrix $C^{\ell,n}(k)$ denotes the decorrelation coefficient between targets ℓ and n. We can also write $y^\ell(k)$ as $y^\ell(k) = B^\ell(k)x^\ell(k) + B^n(k)x^n(k) - \tilde{y}^\ell(k)$, where the error $\tilde{y}^\ell(k)$ is

$$
\begin{aligned}
\tilde{y}^\ell(k) &= B^\ell(k)x^\ell(k) + B^n(k)x^n(k) - y^\ell(k) \\
&= \left[B^\ell(k) - I\right]x^\ell(k) + \left[B^n(k) + C^{\ell,n}(k)\right]x^n(k) \\
&\quad + \tilde{x}^\ell(k|k) - C^{\ell,n}(k)\tilde{x}^n(k|k) \qquad (4.7)
\end{aligned}
$$

where $\tilde{x}^t(k|k) = x^t(k) - \hat{x}^t(k|k)$ for $t = \ell$ or n. Since we assume that all errors have zero means, the coefficients of $x^\ell(k)$ and $x^n(k)$ must vanish. Thus,

$$
B^\ell(k) = I \quad \text{and} \quad B^n(k) = -C^{\ell,n}(k). \qquad (4.8)
$$

Using (4.8), the error $\tilde{y}^\ell(k)$ in (4.7) can be rewritten as $\tilde{y}^\ell(k) = \tilde{x}^\ell(k|k) - C^{\ell,n}(k)\tilde{x}^n(k|k)$. To compute $C^{\ell,n}(k)$, we enforce $y^\ell(k)$ and $\hat{x}^n(k|k)$ to be uncorrelated to obtain

$$C^{\ell,n}(k) = R^{\ell,n}(k)\left(P^n(k|k)\right)^{-1}, \tag{4.9}$$

and the error covariance of $y^m(k)$ is then

$$Y^\ell(k) = P^\ell(k|k) - C^{\ell,n}(k)\left(R^{\ell,n}(k)\right)'. \tag{4.10}$$

5. Overall Decorrelated Process

We can now incorporate the decorrelation processes described in Sections 3 and 4 such that the decorrelated sequence is uncorrelated with both previous state estimates and information from interacting targets. The algorithm to construct $y^\ell(k)$ from $\hat{x}^\ell(k|k)$, $\hat{x}^\ell(k|k-1)$, and $\{\hat{x}^n(k|k)\}_{n=1}^T$ where $n \neq \ell$ can be summarized as

$$y^\ell(k) = x^\ell(k|k) - C^{\ell,\ell}(k)\hat{x}^\ell(k|k-1) - \sum_{n=1,n\neq\ell}^{T} C^{\ell,n}(k)\hat{x}^n(k|k) \tag{5.1}$$

$$C^{\ell,n}(k) = \begin{cases} P^\ell(k|k)\left[P^\ell(k|k-1)\right]^{-1}, & n = \ell, \\ R^{\ell,n}(k)\left[P^n(k|k)\right]^{-1}, & n \neq \ell \end{cases} \tag{5.2}$$

$$B^{\ell,n}(k) = \begin{cases} I - C^{\ell,\ell}(k), & n = \ell, \\ -C^{\ell,n}(k), & n \neq \ell \end{cases} \tag{5.3}$$

$$Y^\ell(k) = B^{\ell,\ell}(k)P^\ell(k|k) + \sum_{n=1,n\neq\ell}^{T} B^{\ell,n}(k)\left(R^{\ell,n}(k)\right)'. \tag{5.4}$$

The derivations of the above algorithm follow similar procedures as discussed in Sections 3 and 4.

6. Filtering Process at Global Level

Suppose that the distributed fusion architecture of Fig. 1 consists of N processors at the local level, where each local processor receives measurements from a single sensor in order to maintain target tracks. If the JPDA filter is implemented at the local level, each processor will send decorrelated sequences $y_p^\ell(k)$, its covariances $Y_p^\ell(k)$, and its corresponding measurement matrices $B_p^\ell(k)$ for $m = 1, \ldots, T$ and $p = 1, \ldots, N$ to the communication network of the system.

To maintain the same implementation at the global processor of the data association and filtering algorithm as at the local level, each decorrelated sequence $y_p^\ell(k)$ is treated as a measurement $z_{G,j}(k)$ at the global processor. Further, its covariances $Y_p^\ell(k)$ replace the sensor noise covariances $R_{G,j}(k)$, and $B_p^\ell(k)$ takes the role of the measurement dynamics $H_{G,j}(k)$. In other words, the JPDA filter for the global level uses the following substitutions

$$z_{G,j}(k) \Leftarrow y_p^\ell(k), \quad R_{G,j}(k) \Leftarrow Y_p^\ell(k), \quad H_{G,j}(k) \Leftarrow B_p^\ell(k)$$

where $B_p^\ell(k) = \sum_{n=1}^{T} B_p^{\ell,n}(k)$, and j denotes the index of measurements at the global processor. The JPDA filter at the global level will yield the global target estimates $\hat{x}_G^\ell(k|k)$ and corresponding covariances $P_G^\ell(k|k)$.

7. Conclusion and Future work

The characteristics of decorrelated state estimates have been investigated when tracking interacting targets in cluttered environments. The uncorrelated property of these sequences allows distributed fusion algorithms to use them as measurements at the global processor, where existing centralized fusion algorithms such as the JPDA filter can thus be used. The construction of these decorrelated state estimates removes correlation from previous state estimates as well as correlation due to other interacting targets from the current updated estimates when tracking multiple targets. Future work includes extending this approach for more general distributed fusion architectures as well as simulation evaluation and comparison of this approach with other distributed fusion algorithms.

References

[1] Y. Bar-Shalom. "On the Track-to-Track Correlation Problem," *IEEE Trans. Aut. Ctrl.*, 26(2): 571–572, 1981.

[2] Y. Bar-Shalom. "Multitarget-Multisensor Tracking: Principles and Techniques," UCLA Extension Short Course Notes, Univ. California, Los Angeles, 1993.

[3] Y. Bar-Shalom and T. Fortmann. *Tracking and Data Association*, Academic Press, San Diego, CA, 1988.

[4] C. B. Chang and L. C. Youens. "Measurement Correlation for Multiple Sensor Tracking in a Dense Target Environment," *IEEE Trans. Aut. Ctrl.*, 27(6): 1250–1252, 1982.

[5] C. Y. Chong, K. C. Chang, and S. Mori. "Distributed Tracking in Distributed Sensor Networks," *Proc. American Ctrl. Conf.*, pp. 1863–1868, 1986.

[6] O. E. Drummond. "Feedback in Track Fusion Without Process Noise," *SPIE Signal and Data Processing of Small Targets*, Vol. 2561, pp. 369–383, 1995.

[7] G. Frenkel. "Multisensor Tracking of Ballistic Targets," *SPIE Signal and Data Processing of Small Targets*, Vol. 2561, pp. 337–346, 1995.

[8] L. Y. Pao. "Centralized Multisensor Fusion Algorithms for Tracking Applications," *Control Engineering Practice*, 2(5): 875–887, Oct. 1994.

[9] L. Y. Pao. "Measurement Reconstruction Approach for Distributed Multisensor Fusion," *J. Guid., Ctrl., & Dyn.*, 19(4): 842–847, 1996.

[10] L. Y. Pao and M. K. Kalandros. "Algorithms for a Class of Distributed Architecture Tracking," *Proc. American Ctrl. Conf.*, pp. 1434–1438, 1997.

[11] D. J. Salmond. "Mixture Reduction Algorithms for Target Tracking in Clutter," *SPIE Signal and Data Processing of Small Targets*, Vol. 1305, pp. 434–445, 1990.

[12] L. L. Scharf. *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*, Addison-Wesley, Reading, MA, 1991.