SYNTHESIS OF SWITCHED-MODE CONVERTERS

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ABSTRACT

An analytical synthesis method is described which is capable of generating the complete set of switching converters which possess given attributes. This leads to a number of useful new converter topologies, as well as to a more fundamental understanding of switched-mode conversion.

Two examples are considered. It is found that the class of single-inductor duo-topology converters contains seven basic members. In addition, a class of converters with nonpulsating input and output currents also contains seven members. Zero input or output current ripple may be obtained in three of these converters.

1. INTRODUCTION

The number of applications of switched-mode converters has increased rapidly over the past decade. In addition to their widespread use as dc regulators, these converters have found employment in a wide variety of applications involving ac signals, including dc-to-ac inverters for solar arrays [15], ac-to-dc unity power factor controlled rectifiers for battery charger applications [16], ringing tone generators in telephone systems [17], variable speed drive of induction motors [18], servo amplifiers [19], and audio amplifiers [4].

These new applications have generated a need for converter topologies with new properties, such as the capability for two or four quadrant operation or the ability to produce input or output waveforms which are low in harmonics. This has prompted the search for new converter topologies, and a variety of techniques have been developed for their derivation.

All of these techniques begin with a basic switching converter such as the buck, boost, buck-boost, or Cuk converters shown in Fig. 1. Various circuit manipulations are then performed in such a way that a converter which implements the desired functions is obtained. For example, a new converter may be created by cascade connection of two existing converters as in Fig. 2; this is the method in which the Cuk converter was originally derived [1, 2]. Inversion of line \( V_l \) and load \( V \) connections [3,14], as in Fig. 3, results in a new converter whose dc conversion ratio \( V_0/V \) is the reciprocal of that of the original converter. Duality transformations [3] can be used to generate related converters with dual properties. Finally, the differential connection of the load across the outputs of two converters as in Fig. 4 can be used to obtain a two or four quadrant output [4].

The above listed methods have yielded insight into the basic relationships between converters, and have resulted in a number of new configurations. In some instances, however, converters have been
Fig. 2. The cascade connection of basic converters to generate a new converter with improved properties.

Fig. 3. Inversion of source voltage and load transforms the buck converter into the boost converter.

Fig. 4. The differential connection of the load across the outputs of two de-to-de converters produces a de-to-ac inverter or amplifier system.

Fig. 5. Two basic converters which are not derived from the buck or boost converters via cascade or differential connections or inversion and duality transformations: (a) Watkins-Johnson converter; (b) SEPIC.

In light of this, one might ask whether other useful topologies exist. More fundamentally, it is desired to determine, in a systematic way, the complete set of converters which possess given desired properties. In Section 2, an analytical synthesis technique is presented from which every possible converter with given characteristics may be derived. With this technique, entire classes of converters may be found which possess desirable properties such as nonpulsating input and output current ripple, low power-stage parts count, and a given dc conversion ratio.

Two simple classes of converters are explored as examples in Section 3. The first is the basic class of converters with a single inductor and two switching intervals. It is found that this class contains exactly seven members; in addition to the basic buck, boost, and buck-boost converters, two converters appear which appear suitable for dc-to-ac applications, and two others appear fit for ac-to-ac applications. The second example deals with a class of converters with nonpulsating input and output currents. It is found that the inductors of three converters in this class, including the Cuk converter, may be coupled and adjusted for zero current ripple.

The complete procedure is summarized in Section 4.
2. SYNTHESIS

Power converters are periodically-switched linear networks. A number of reactive elements are connected between the input line voltage $V_L$ and the output load voltage $V_O$, first in one topology during the first switching interval, then in different topologies during the second and any succeeding switching intervals. Semiconductor switches are used to cyclically connect the circuit elements in these various configurations.

The choice of linear circuit topologies determines the functions performed by the converter, as well as its input and output properties. The synthesis problem therefore involves the selection of these topologies to obtain a converter with desired properties. In this section, a general form of the system state equations is described which is suitable for synthesis. It is found that the basic transformerless converters without parasitics are characterized by a small number of parameters, each of which is either +1, 0 or -1. Those parameters describe the interconnections between the reactive elements, line, and output during each switching interval. The choice of these parameters therefore determines the complete converter topology.

The second subsection relates these parameters to the overall properties of the converter. The specific attributes considered here are: (1) the dc conversion ratio $N/V_L$, (2) the absence of pulsating input or output currents, (3) the number of reactive elements employed, and (4) the number of switching intervals used.

Next, degenerate and redundant cases are eliminated. There are a number of different ways of writing the state equations of a given switching converter. For example, the reversal of the defined polarity of the output voltage changes the system state equations but does not change the function of the converter. Those redundancies are eliminated in the third subsection.

The complete procedure described here involves the formulation of the general form of the state equations for the class of converters with desired properties. Degenerate and redundant cases are eliminated, leaving a number of distinct useful cases. For a given case, the linear circuits for each switching interval are then restructured from these state equations. Finally, switches are added to obtain the complete converter.

2.1 General Form of the State Equations

The first step is the determination of the general form of the converter state equations. It is shown here that the state equations which describe any basic ideal switching converter during one switching interval may be written in a very specific way. The derivation closely follows the RLC network state variable formulation derivation given in [7].

Since the objective is the synthesis of basic ideal switching converters, the effects of parasitics and of transformers are neglected here. Hence, the linear networks considered contain only inductors, capacitors, independent voltage sources and output load resistors bypassed by capacitors. Under those assumptions, and if the network contains no capacitor loops or inductor cutsets, then as shown in [7], the loop and node equations of the linear network can be written in the form

$$\begin{align*}
L_i \frac{di}{dt} + F_{gr} i + F_{gl} i = 0 \\
R_c \frac{dr}{dt} + F_{cr} r + F_{cl} r = 0 \\
V_g - F_{gr} i - F_{rl} r = 0 \\
V_l - F_{gl} i - F_{cl} r = 0
\end{align*}$$

where $V_g$, $i_g$ = voltages and currents of independent voltage sources

$\gamma_c$, $i_c$ = voltages and currents of network capacitors

$\gamma_r$, $r_r$ = voltages and currents of output loads

$\gamma_l$, $i_l$ = voltages and currents of network inductors

$F_{cl}$, $F_{cr}$, $F_{gl}$, and $F_{gr}$ are matrices which describe the connections between the network components.

Eqs. (1) and (2) are the Kirchhoff current laws and Eqs. (3) and (4) are the Kirchhoff voltage laws. If no transformers are present, then all elements of $F_{cl}$, $F_{cr}$, $F_{gl}$, and $F_{gr}$ are either +1, 0 or -1. In addition, if all output loads are directly bypassed by a capacitor, then these loads are not directly connected to an independent voltage source, and hence $F_{gr} = 0$.

The relations between the branch voltages and currents are given by the impedance relations

$$\begin{align*}
\gamma_g &= F_{gr} r \\
\gamma_l &= L_c \frac{di}{dt} \\
\gamma_c &= \frac{dv_c}{dt}
\end{align*}$$

where $F_c$ and $C$ are diagonal positive definite matrices containing system resistance and capacitive values, respectively, $L$ is a symmetrical positive definite matrix containing system inductance values. It may not be diagonal if coupled inductors are present.

Combination of Eqs. (1) through (7) yields

$$\begin{align*}
\frac{di}{dt} &= F_{cl} v_c + F_{cl} v_L \\
\frac{dv_c}{dt} &= -F_{cl} i - F_{cr} r - L_c \frac{di}{dt} - F_{cr} F_{gl} v_L \\
\gamma_L &= \frac{di}{dt}
\end{align*}$$

Upon placing these equations in matrix form and accounting for the fact that $F_{gr} = 0$, one obtains
\[ \begin{bmatrix} L_0 \ \ 0 \\ 0 \ \ C \end{bmatrix} \begin{bmatrix} I_t \\ V_c \end{bmatrix} = \begin{bmatrix} 0 & F_{11}^T \\ F_{11} & F_{11} \end{bmatrix} \begin{bmatrix} I_t \\ V_g \end{bmatrix} + \begin{bmatrix} I_{g1} \\ 0 \end{bmatrix} V_g \]  
with \( F_{11} = D_1 F_{11} + D_2 F_{12} \)

\( F_{12} = \text{steady-state duty ratio} \)

\( D_1 = 1 - D_2 \)

This assumes that the converter always operates in the continuous conduction mode [11]. The dc converter model may now be solved to find the average dc values of the relevant converter voltages and currents. In particular, the line-to-output conversion ratio \( M(D) \), and its dependence on the choice of \( F_{g1} - F_{g2} \) and \( F_{g1} + F_{g2} \), can be found. The proper selection of these matrices such that a given \( M(D) \) is obtained then becomes apparent.

Thus, the state equations of the network can be written in a very specific form, and the topology of any basic switching converter is completely described by a number of parameters whose values may be \(+1, 0, \text{or} -1\).

2.2 Some Properties of Switching Converters

It is now desired to relate the topological parameters of the previous section to the basic physical properties of the converter. This will allow the selection of these parameters such that a given basic property is obtained. The specific properties considered here are: (1) the number of reactive elements used, (2) the number of switching intervals employed, (3) the dc conversion ratio \( M(D) \), and (4) the absence of pulsating input or output currents.

The number of reactive elements used determines the complexity and size of the converter, and often is a quantity to be minimized. The number of reactive elements also determines the dimensions of the state equations and the relevant matrices. If a given converter contains \( n \) inductors, \( m \) capacitors, and \( p \) independent voltage sources, then matrices \( F_{g1} \) and \( F_{g2} \) are of dimension \( n \times m \), and matrices \( F_{g1} + F_{g2} \) and \( F_{g1} - F_{g2} \) are of dimension \( p \times n \). Hence, there are a local of \( m(n+p) \) parameters to choose for every switching interval, and each parameter may be \(+1, 0, \text{or} -1\). It will be seen in Section 2.3, however, that most of these combinations are redundant, degenerate or not allowed.

Thus, the actual number of distinct converters is much less than the total possible combinations of \( F_{g1} \) and \( F_{g2} \).

The synthesis procedure is not restricted to converters with only two switching intervals. Any number of switching intervals may occur, each of which is described by a different choice of linear circuit topology, and hence by a different value of \( F_{g1} \) and \( F_{g2} \). In this case, synthesis involves the selection of a distinct circuit topology for each of the three or more intervals. A wide variety of dc conversion ratios may be obtained.

Once the number of reactive elements and switched intervals has been chosen, it remains to select the individual topologies such that desired properties are obtained. In the next subsection, the choice of topologies for the realization of a given dc conversion ratio \( M(D) \) is examined.
2.2.1 Synthesis of Converter With a Given DC Conversion Ratio

The dc line-to-output voltage conversion ratio \( M(D) \) is perhaps the most important attribute of a switched-mode converter. Consequently, it is a natural property to study. In particular, it is desired to determine the range of possible conversion ratios, the general form of the conversion ratio for a class of converters with a given number of reactive elements and switching intervals, and the correct choice of topology (i.e., the correct choice of \( F_{bi} \) and \( F_Si \)) to obtain a desired conversion ratio \( M(D) \).

This is accomplished by solution of the general dc converter equations (13). It is found that the dc conversion ratio can always be expressed as the ratio of two polynomial functions of the duty ratio. In addition, if the number of inductive states \( n \) is equal to the number of capacitive states \( m \), then \( M(D) \) is independent of load for ideal converters. If \( n \) differs from \( m \), then the output voltage in the ideal case may vary with load. Finally, it is seen that the elements of the \( F_{bi} \) and \( F_Si \) matrices may be chosen in a straightforward manner to obtain a desired feasible \( M(D) \). Once the elements of these matrices are known, the complete converter topology is easily constructed.

The steady-state average values of the inductive currents and capacitor voltages are given by Eq. (13), repeated below:

\[
\begin{align*}
0 &= T_{S} V_0 + F_{bi} V_S \\
0 &= -F_S I_0 - G_0 V_0
\end{align*}
\]

Eq. (14)

These equations may be easily solved to find \( I_0 \) and \( V_0 \). As described above, there are two cases of interest, depending on whether the number of inductive states \( n \) is equal to the number of capacitive states \( m \).

Case 1: \( n = m \)

Since the dimensions of \( F_{bi} \) are \( n \times m \), \( F_{bi} \) is a square matrix in this case. The solution to Eq. (14) is straightforward, and is given by

\[
\begin{align*}
I_0 &= F_{bi}^{-1} G_0 + T_{S} V_S \\
V_0 &= -F_{bi}^{-1} F_S V_S
\end{align*}
\]

Eq. (15)

where \( F_{bi}^{-1} \) denotes the inverse of the transpose of \( F_{bi} \)

valid if \( F_{bi} \) is nonsingular. Note that \( V_0 \) is independent of \( G_0 \), and hence the system voltages are not a function of the load resistance. Furthermore, since the elements of \( F_{bi} \) and \( F_Si \) are constant or linear functions of duty ratio, as given by Eq. (13), then the elements of \( F_{bi}^{-1} \) are the ratios of polynomial functions of duty ratio.

Case 1 is the most common circumstance. For example, the basic buck, boost, and buck-boost converters each contain a single inductor and single capacitor; hence, \( m = 1 \). The dc conversion ratios of these converters, when operated in the continuous conduction mode, are \( M(D) = D/(1-D) \), and \(-D/(1-D) \), respectively. In each case, the conversion ratios are the ratio of polynomial functions of \( D \), and are independent of load resistance.

Case 2: \( n \neq m \)

\( F_{bi} \) is not a square matrix in this case. The solution to Eq. (14) is given by:

\[
\begin{align*}
I_0 &= \left(F_{bi}^{-1} F_S\right)^{-1} F_{bi} V_S \\
V_0 &= -G_0 F_{bi}^{-1} F_S^{-1} \left(F_{bi}^{-1} F_S\right)^{-1} F_{bi} V_S
\end{align*}
\]

Eq. (16)

valid provided that both \( G_0 \) and \( (F_{bi}^{-1} F_S) \) are nonsingular. Since \( G_0 \) is a function of the load resistances, it is apparent that in this case the output voltage is no longer independent of load. For this reason, case two should be avoided whenever low output impedance is required.

In either case, synthesis of a converter with a given dc conversion ratio is straightforward. Eq. (15) or (16) is evaluated to find the dc conversion ratio as a function of the elements of \( F_{bi} \) and \( F_Si \). The elements of these matrices may then be chosen to obtain the desired \( M(D) \), and the complete converter circuit constructed. An example of this procedure is given in Section 3.

2.2.2. Synthesis of Converters With Nonpulsating Input or Output Currents

Another desirable attribute is the absence of pulsating currents at the converter input or output terminals. Converters with this feature are also easily synthesized; expressions are found for the average line or output current during each switching interval which must be equal if the current is nonpulsating. Hence, equating these expressions yields the correct condition.

The Kirchhoff node equation which describes the line current during one switching interval is of the form given by one row of Eq. (1), as shown below:

\[
l_{im} = -f_{gi} l_{im} - f_{gi} l_{im}
\]

Eq. (17)

where \( l_{im} \) is the line current, and is one element of the vector \( L_0 \) of Eq. (1). \( f_{gi} \) and \( f_{gi} \) are each one row of the matrices \( f_{gi} \) and \( f_{gi} \).

As noted previously, \( f_{gi} \) is usually zero in ideal converters, and hence \( f_{gi} = 0 \). Hence, the line current is given by:

\[
l_{im} = -f_{gi} l_{im} \quad \text{during interval } D_1 T_S \]

Eq. (18)

\[
l_{im} = -f_{gi} l_{im} \quad \text{during interval } D_2 T_S \]

Eq. (19)

Similar expressions occur for any additional switching intervals.

If the line current is nonpulsating, then Eqs. (18) and (19) must be equal, and hence the following
condition must hold:

\[ (E_{g1}^T - E_{g2}^T) \mathbf{i}_1 = 0 \]  

(20)

It is desired to obtain nonpulsating line current independently of circuit conditions and hence for all allowed values of \( I_1 \). Therefore, the condition becomes:

\[ E_{g1}^T - E_{g2}^T \]  

(21)

This is the basic condition for nonpulsating line current. It states that the row of \( F_{g1} \) (and hence the column of \( F_{g1} \) in Eq. (10)) which corresponds to the line current must remain the same for each switching interval. As demonstrated in Section 3, converters are easily synthesized using this criterion.

A similar argument applies for nonpulsating output currents. If the output current is nonpulsating, then the current in the capacitor which bypasses the load must be nonpulsating. During a given switching interval, this current is of the form described by one row of Eq. (2), as shown below:

\[ i_{out} = \begin{pmatrix} -T_f & 1_r & -T_f & 1_l \end{pmatrix} \mathbf{i}_1 \]  

(22)

where \( i_{out} \) is the output capacitor current, and is one element of the vector \( \mathbf{i}_c \) of Eq. (2). \( T_f^T \) and \( T_f^T \) are each one row of the matrices \( F_{g1} \) and \( F_{g2} \), respectively. Eq. (22) is the Kirchhoff node equation for the current in the output capacitor. Hence, this current is given by:

\[ i_{out} = \begin{pmatrix} -T_f & 1_r & -T_f & 1_l \end{pmatrix} \mathbf{i}_1 \]

(23)

\[ i_{out} = \begin{pmatrix} -T_f & 1_r & -T_f & 1_l \end{pmatrix} \mathbf{i}_1 \]

(24)

If the output current is nonpulsating, then Eqs. (23) and (24) must be equal, and therefore the following condition must hold:

\[ (T_f^T - T_f^T) \mathbf{i}_1 + (T_f^T - T_f^T) \mathbf{i}_1 = 0 \]  

(25)

Since it is assumed that output load is always bypassed by the output capacitor, then \( T_f^T = T_f^T \) and the first term of Eq. (25) is zero. Also, it is desired to obtain nonpulsating output current independently of circuit conditions, and hence for all allowed values of \( I_1 \). Therefore, the condition becomes:

\[ E_{f1}^T = E_{f2}^T \]  

(26)

This is the basic condition for nonpulsating output current. It states that the row of \( F_{f1} \) (and hence the row of \( F_{f1} \) in Eq. (10)) which corresponds to the output capacitor current must remain the same for each switching interval. Again, it is demonstrated in Section 3 that converters are easily synthesized using this criterion.

Thus, the synthesis of switching converters involves the selection of the elements of the matrices \( F_{g1} \) and \( F_{g2} \) for each switching interval. These elements may each be either +1, 0, or -1, and they describe the way in which the converter reactive elements, line, and load are connected together during each switching interval. Relations have been found in this section which reveal the correspondences between these matrices and the important properties of dc conversion ratio \( \mu \), nonpulsating line current, and nonpulsating output current. Thus, it is now apparent how one may choose the elements of \( F_{g1} \) and \( F_{g2} \) such that these properties are obtained.

**2.3 Elimination of Degenerate, Redundant, and Nonrealizable Cases**

There are a number of ways of writing the state equations for a given switching converter: as a result, redundancies arise in the synthesis procedure. Also, degenerate cases may occur in which the magnitude of at least one capacitor voltage or inductor current in the converter is independent of duty ratio. Finally, not all combinations of \( F_{g1} \) and \( F_{g2} \) in Eq. (10) can be realized using only two-terminal inductors, capacitors, resistors, and independent voltage sources. As a result of these redundant, degenerate, and nonrealizable cases, a large percentage of the possible combinations of \( F_{g1} \) and \( F_{g2} \) do not yield interesting converters, and need not be investigated.

Redundant cases occur when there is more than one way of writing the state equations for what is normally considered the same converter. In practice, there are four, eight, or more ways of doing this. First, reversal of the defined polarity of any capacitor voltage, inductor current, or independent source voltage changes the state equations but does not change the topology of connections between circuit elements. The number of redundant cases which this causes depends on the number of reactive elements and voltage sources in the class of converters being considered. Second, interchange of the defined order of the switching intervals (for example, redefinition of which interval is called \( I_{1}F_{g1}^T \) and which one is called \( I_{2}F_{g2}^T \)) does not change the functions performed by the converter. For a two-topology converter, this corresponds to interchanging the choice of \( F_{g1} \) with \( F_{g2} \) and \( F_{g1} \) with \( F_{g2} \); hence, this doubles the number of redundant cases.

Degenerate cases may arise when the magnitude of at least one inductor current or capacitor voltage of the converter is independent of duty ratio under both dc and ac conditions (i.e., when a state is uncontrollable). The most obvious degenerate cases occur when the output voltage is independent of duty ratio; the dc conversion ratio is usually zero, one, or infinity in this instance. Degenerate cases may also occur when any other state of the converter is independent of duty ratio, or when the output voltage is independent of a converter state; this usually means that the corresponding reactive element has no effect on the operation of the converter.

In higher-order systems, it is possible to select the elements of \( F_{g1} \) and \( F_{g2} \) in such a way that Eq. (10) cannot be realized using only two-terminal inductors, capacitors, resistors, and in-
dependent voltage sources. An example is given by

\[
F_{g1} = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix}, \quad F_{g2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

(27)

For this choice of \( F_{g1} \) and \( F_{g2} \), the two Kirchhoff loop equations (4) are not consistent, and hence Eq. (27) represents a nonrealizable case.

Thus, the analytical synthesis process begins with the determination of the general form of the converter state equations. The circuit topologies are specified by the elements of the matrices \( F_{g1} \) and \( F_{g2} \). Next, the range of possible choices of \( F_{g1} \) and \( F_{g2} \) is narrowed by elimination of those combinations which lead to unsuitable converter properties; the two properties considered here are dc conversion ratio and nonpulsating input and output currents. Third, degenerate, redundant, and nonrealizable cases are excluded; the remaining choices of \( F_{g1} \) and \( F_{g2} \) are distinct, nondegenerate converters. For a given choice, the individual circuit topologies for each switching interval are then constructed. Finally, switches are added, yielding the final converter.

3. Examples

Two examples are considered here to illustrate the synthesis procedure. First, the simple class of two-topology single-inductor converters is explored, and it is found that this class contains exactly seven distinct members. Three of these converters are well-suited for dc-to-ac applications, two appear useful in ac-to-ac applications, and the final two appear appropriate for ac-to-dc applications.

The second example is a class of converters with nonpulsating input and output currents. This class also contains seven basic members, each of which possesses the same dc conversion ratio as one of the converters of the first example. Finally, it is shown that the inductors of three members of this class may be coupled. By proper magnetic design, zero input or output current ripple can be obtained.

3.1 The Class of Two-Topology Single-Inductor Converters

In a two-topology single inductor converter the line, inductor, and load are connected together in one topology during the first switching interval, and then in a second topology during the second switching interval, as illustrated in Fig. 6. The choice of the two topologies determines the functions performed by the converter.

The analysis of this class is very simple. The converters contain only one inductor, capacitor and independent voltage source; hence, \( n = m + p - 1 \). As a result, the matrices \( F_{g1} \) and \( F_{g2} \) defined in Eq. (9) become scalars. \( F_{g1} \) and \( F_{g2} \) describe the converter topology during the first switching interval, and \( F_{g2} \) and \( F_{g2} \) describe the topology during the second switching interval. Since each of these parameters may be either \(+1\), \(0\), or \(-1\), there are a total of 9 different topologies possible for each interval, as illustrated in Fig. 7.

Evaluation of Eq. (10) for this class yields the following general form of the converter state equations during one switching interval:

\[
L_{d1}\frac{dv(t)}{dt} = F_{g1} v(t) + F_{g2} \frac{v(t)}{R}
\]

(27a)

\[
C_{d1}\frac{dv(t)}{dt} = -F_{g1} i(t) - \frac{v(t)}{R}
\]

(27b)

Eq. (27a) is the Kirchhoff loop equation which describes the inductor voltage, and Eq. (27b) is the Kirchhoff node equation which describes the capacitor current.

Next, the general form of the dc conversion ratio \( M(D) \) for any converter in this class is found. Evaluation of Eq. (15) yields

\[
V_0 = \frac{D_1 F_{g1} + D_2 F_{g2}}{D_1 F_{g1} + D_2 F_{g2}} V_L
\]

(28a)

\[
I_0 = -\frac{1}{D_1 F_{g1} + D_2 F_{g2}} V_L
\]

(28b)
The nine possible ways of connecting the inductor between the line voltage $V_g$ and the output load.

Again, $P_{g1}$, $P_{g2}$, $P_{g3}$, and $P_{g4}$ determine the converter topologies during the two switching intervals. The dc conversion ratio is given by

$$M(D) = \frac{V_0}{V_g} = \frac{D_1 P_{g1} + D_2 P_{g2}}{D_3 P_{g3} + D_4 P_{g4}}$$

(29)

Thus, the dc conversion ratio for every converter in this class is independent of load and is a bilinear function of $D_1$.

The next step is the investigation of all possible choices of $P_{g1}$, $P_{g2}$, $P_{g3}$, and $P_{g4}$ to determine all converters in this class. This task at first appears formidable since there are a total of 81 combinations; however, as noted in Section 2.3, most of these cases are either degenerate or redundant. In particular, there are nine degenerate cases in which the same topology is chosen for both switching intervals, and twenty-four cases in which different topologies are used but the conversion ratio is nonetheless independent of duty ratio. The forty-eight remaining cases are mostly redundant; upon elimination of these redundancies, the seven distinct converters of Fig. 8 remain. Since all possible combinations have been exhausted, this is the complete set of distinct, nondegenerate converters in this class.

The first converter is the well-known buck converter. It corresponds to the choice $P_{g1} = 0$, $P_{g2} = 1$, $P_{g3} = 1$, $P_{g4} = 0$, and $M(D) = D_1$. Hence, from Eqs. (11) and (12), the network equations are:

\[
\begin{align*}
L \frac{di(t)}{dt} &= -v(t) + V_g \\
C \frac{dv(t)}{dt} &= i(t) - v(t)/R
\end{align*}
\]

(30)

The linear networks corresponding to these equations may be easily constructed. Upon the addition of switches, the circuit of Fig. 8a is obtained.

The second and third converters are the well-known boost and buck-boost converters. The boost converter corresponds to the choice $P_{g1} = 0$, $P_{g2} = 1$, $P_{g3} = 1$, $P_{g4} = 1$, and the buck-boost converter corresponds to the choice $P_{g1} = 0$, $P_{g2} = 1$, $P_{g3} = 1$, $P_{g4} = 0$. As with the buck converter, the converter topology may be constructed by use of these parameters.

The fourth converter is the well-known bridge converter of Fig. 8d, which corresponds to the choice $P_{g1} = 1$, $P_{g2} = 1$, $P_{g3} = 1$, $P_{g4} = -1$. Unlike the first three converters, which produce a dc output of one polarity, the bridge converter is capable of producing an output voltage of either polarity since its dc conversion ratio is $M(D) = 2D_1 - 1$. This converter requires a floating load, and is useful for dc-to-ac applications.
A similar current-fed converter, Fig. 8a, is described by the choice \( F_B = 1, F_D = 1, F_L = 1, F_p = 1 \). This converter, which also requires a floating load, appears useful for ac-to-dc applications (controlled rectification with any desired power factor) since its dc conversion ratio is

\[
\frac{V_o}{V_i} = \frac{1}{D_1} - \frac{1}{D_2} = 2D_1 - 1
\]

(32)

Thus, if \( V_i \) is sinusoidal, \( V_o = A \sin \omega t \), and if the duty ratio is varied sinusoidally, \( D = 0.5 + b \sin \omega t \), then for a sufficiently small the output becomes dc:

\[
V_o = \frac{A}{2b}
\]

(33)

The sixth converter is the "Watkins-Johnson" converter [51], as depicted in Fig. 8f. This converter is also capable of producing output voltages

![Diagrams representing various converter configurations and their duty ratio characteristics.](image)

**Fig. 8.** The seven distinct members of the class of single-inductor, two-topology converters: (a) buck; (b) boost; (c) buck-boost; (d) bridge; (e) current-fed bridge; (f) Watkins-Johnson; (g) inversion of Watkins-Johnson converter.
of either positive or negative polarity, and it therefore appears suitable for dc-to-1c applications. The magnetic assembly functions as a single inductor; the use of two windings allows the reduction of the number of switches necessary. This converter is described by the choice Fig. 17 = 1, Fig. 18 = 1, Fig. 20 = 0, Fig. 21 = 1.

The advantages of this converter over the bridge converter are its ability to drive a ground-referred, rather than floating, load, and the reduced number of switches required. The transistor stresses are higher in the Watkins-Johnson converter, however. Disadvantages include the pulsating nature of the input and output currents, and the nonlinear nature of its dc conversion ratio: \( M(D) = (D_1 - D_2)/D_1 \).

The final converter, Fig. 8g, can be derived from the Watkins-Johnson converter by inversion of source and load [3, 14]. It is described by the choice Fig. 1 = 1, Fig. 2 = 1, Fig. 3 = 1, Fig. 4 = 0, and its dc conversion ratio is \( M(D) = D_2/(D_1 - D_2) \).

This converter may find use in ac-to-dc applications. It shares many of the attributes of the Watkins-Johnson converter: at the expense of pulsating input and output currents and of a nonlinear dc conversion ratio, this converter can supply dc to a ground-referred load using fewer switches than required in the converter of Fig. 8d.

Thus, each complete converter topology in the basic class of single-inductor two-topology converters is described by the four parameters Fig. 1, Fig. 2, and Fig. 3. This class contains exactly seven distinct nondegenerate members. Three of these, the well-known buck, boost, and buck-boost converters, are suitable for dc-to-dc applications. Two additional converters occur which are suitable for dc-to-ac applications, and the final two converters appear fit for ac-to-dc applications.

3.2 A Class of Converters with Nonpulsating Input and Output Currents

The second general class of converters considered here are those in which the dc line voltage \( V_L \) two inductors, a capacitor, and a load bypassed by a second capacitor are connected in one topology during the first switching interval and in another topology during the second switching interval. It is desired to choose these two topologies such that nonpulsating current waveforms are obtained at both the input and output. This is done by use of the conditions derived in Section 2.2.2.

Since the converters of this class contain two inductors, two capacitors, and a single independent voltage source, then \( n = 2 \), \( m = 2 \), and \( p = 1 \). As a result, matrices \( F_{gl} \) and \( F_{g2} \) are of dimension \( 2 \times 2 \), and matrices \( F_{gl} \) and \( F_{g2} \) are of dimension \( 1 \times 2 \). It is apparent that there are a total of six parameters to select for each of the two switching intervals.

Define

\[
F_{gl} = \begin{bmatrix}
  a_1 & a_2 \\
  a_3 & a_4 \\
  b_1 \\
  b_2 \\
  v_1(t) \\
  v_2(t)
\end{bmatrix}
\]

\[
F_{g2} = \begin{bmatrix}
  a_1 & a_2 \\
  a_3 & a_4 \\
  b_1 \\
  b_2 \\
  v_1(t) \\
  v_2(t)
\end{bmatrix}
\]

where \( v_1(t) \) is the voltage of the first capacitor \( v_2(t) \) is the voltage of the output capacitor.

The condition for nonpulsating input current, Eq. (21), then becomes

\[
F_{gl} = F_{g2}
\]

Hence,

\[
b_1 = b_2 \quad b_1
\]

\[
b_2 = b_2 \quad b_2
\]

This says that the input voltage source \( V_L \) is connected to the inductors in the same way during the two switching intervals.

The condition for nonpulsating output current, Eq. (26), becomes

\[
a_1 = a_2 \quad a_3
\]

\[
a_4 = a_4 \quad a_4
\]

Likewise, Eq. (37) states that the output capacitor is connected to the inductors in the same way during both switching intervals.

The general form of the dc conversion ratio for this class of converters may be found by use of Eq. (15). Substitution of Eq. (30) into Eq. (15) and solution for \( V_2/V_L = M(D) \) yields an expression which is the ratio of two parabolic functions of \( D_1 \) and \( D_2 \) as shown below:

\[
M(D) = -\frac{C_1D_1^2 + C_2D_1D_2 + C_3D_2^2}{C_4D_1 + C_5D_1D_2 + C_6D_2^2}
\]

where \( C_1 \) are constants which depend on the elements of \( F_{gl} \) and \( F_{g2} \). Hence, the dc conversion ratio for converters with two inductors may vary as \( D_1 \).

However, when the conditions for nonpulsating input and output current, Eqs. (26) and (37), are substituted into Eq. (38), the general form of the dc conversion ratio becomes
Thus, the requirement for nonpulsating input and output currents reduces the dependence of the dc conversion ratio on duty ratio to a bilinear form.

The next step is the investigation of all allowed values of the matrices $F_{q1}$, $F_{q2}$, and $F_{q3}$ to determine all converters in this class. After elimination of redundant, degenerate, and unrealizable cases, it is found that this class contains the seven distinct converters of Fig. 9, each of which corresponds to a converter of the previous section.

The first two converters are merely a buck converter with input filter (Fig. 9a), and a boost converter with output filter (Fig. 9b). The fourth and fifth converters are a bridge converter with input filter (Fig. 9d), and a current-fed bridge with output filter (Fig. 9e). Thus, four of the seven members of this class consist merely of the addition of an L-C filter to one of the single-inductor converters of the previous section.

The other three converters are not as simple. The Cuk converter [1, 2], Fig. 9c, is the member of this class which possesses the dc conversion ratio $M(D) = -D_1/D_2$. It is described by the choice of parameters $a_{11}=0$, $a_{21}=-1$, $a_{12}=1$, $a_{22}=0$, $a_3=0$, $a_4=-1$, $b_1=1$, $b_2=0$.

The converter of Fig. 9f possesses the same dc conversion ratio as the Watkins-Johnson converter; hence, it may find use in dc-to-ac applications. This converter requires the same number of components as the bridge converter with input filter of Fig. 9d. At the expense of a nonlinear dc conversion ratio, this converter is capable of driving a ground-referred load. As shown in the next section, an additional advantage of this converter is the fact that its inductors may be coupled on the same core; this yields a number of benefits including reduced magnetics parts count and the ability to null the inductor current ripple at either the input or output of the converter. The choice of topological parameters which describe this converter are $a_{11}=-1$, $a_{21}=0$, $a_{12}=1$, $a_{22}=-1$, $a_3=0$, $a_4=-1$, $b_1=1$, $b_2=0$.

Fig. 9. The seven distinct members of the class of two-inductor, two-topology, nonpulsating input and output current converters: (a) buck with input filter; (b) boost with output filter; (c) Cuk; (d) bridge with input filter; (e) current-fed bridge with output filter; (f) analog of Watkins-Johnson converter; (g) analog of inversion of Watkins-Johnson converter. The ideal dc conversion ratio of each of these converters is identical to the ideal conversion ratio of the corresponding converter of Fig. 8.
The final converter in this class, Fig. 9g, possesses the same dc conversion ratio as the converter of Fig. 8g; hence, it may find employment in ac-to-dc applications. It shares many of the attributes of the converter of Fig. 9f; at the expense of a nonlinear ac conversion ratio \( \frac{V_2}{V_1} = \frac{1}{(2\pi - 1)} \), this converter can supply dc to a ground-referred load using the same number of components as is required by the current-fed bridge with input filter. The inductors of this converter may also be coupled, thereby reducing the magnetizing parts count and allowing zero inductor current ripple to be obtained at either the input or output of the converter. The topology of this converter is described by \( a_1 = a_2 = 1, a_2 = 1, a_2 = 1, a_2 = 1, b_1 = b_2 = 0 \).

Thus, the class of two-inductor, two-topology converters with nonsaturating input and output currents contains seven distinct, nondegenerate members. Four of these converters consist merely of the addition of an input filter to the single-inductor converters of the previous section. Of the remaining three converters, one is the well-known Cuk converter while the other two are previously unknown converters. Hence, the synthesis method is capable of generating new converters, as well as determining the complete set of converters which possess a given set of attributes.

As described in [11, 12, 13], the magnitudes of the inductor current ripples in the coupled-inductor Cuk converter depend on two important properties of the coupled-inductor structure: the coefficient of coupling \( k \) and the effective turns ratio \( n \), defined as follows:

\[
k = \frac{L_1}{\sqrt{L_1 L_2}} \quad n = \frac{L_1}{\sqrt{L_2}}
\]

where \( L_1 \) is the primary self-inductance, \( L_2 \) is the secondary self-inductance, and \( L_{m} \) is the mutual inductance. The condition for zero output current ripple is given as

\[
n = \frac{1}{k}
\]

alternatively, zero current ripple occurs at the converter input when

\[
n = \frac{1}{k}
\]

Thus, zero ripple in the Cuk converter is determined by the properties of the coupled-inductor assembly alone, and is independent of converter component values, switching frequency, duty ratio, etc.

The converters of Figs. 9f and 9g also possess the property of proportional inductor voltage waveforms. The relevant waveforms for the converter of Fig. 9f are shown in Fig. 11. It is seen that the voltage across the output inductor \( V_{L2} \) is always twice as large as the voltage across the input inductor \( V_{L1} \). This occurs independently of duty ratio, component values, etc.; hence, the inductors of this converter may be coupled as in Fig. 12a, and the condition for zero ripple will not depend on circuit values or operating conditions.

Fig. 10. Since the inductor voltage waveforms of the Cuk converter are identical, the inductors may be coupled on the same leg. By proper design of the magnetic assembly, zero current ripple may be obtained at either the input or output.

3.3 Coupled-inductor Extensions

It is well-known that the two inductors of the Cuk converter may be wound on a common core [11, 12, 13], as shown in Fig. 10. This is possible because the voltage waveforms impressed on the two inductors by the converter are identical; therefore, with proper choice of turns ratio and coupling coefficient, the inductors may be coupled. Besides the obvious economic advantage of reducing the number of magnetic assemblies, this allows the reduction of the magnitude of the inductor current ripple at the converter input and output. In fact, it has been shown [11, 12, 13] that by proper magnetic design, the inductive current ripple at either the converter input or output may be reduced to essentially zero magnitude.

Fig. 11. The inductor voltage waveforms for the converter of Fig. 9f. Note that \( V_{L1} = 2V_{L2} \), independent of duty ratio, component values, and circuit conditions; hence, the inductors may be coupled.
Likewise, zero ripple occurs at the input when

\[ n = \frac{2}{k} \]  \hspace{1cm} (46)

Thus, the inductors of three of the seven converters of Section 3.2 may be coupled. By proper choice of effective turns ratio \( n \) and coupling coefficient \( k \), the inductive current ripple at either the converter input or output can be reduced to essentially zero magnitude.

4. CONCLUSIONS

Studies of cascade and differential connections, and of duality and inversion transformations in switched-mode converters [1, 2, 3, 4, 14] have generated a number of useful new converter topologies and have allowed insight to be gained into the basic conversion processes. However, these operations and transformations have not yielded the complete set of basic converter topologies, and consequently it is natural to ask whether other as yet unknown useful converters can be found.

In light of this, it is desired to study switched-mode converter topologies in a more systematic way, such that the unaided set of converters which possess properties well-suited for a given application can be determined. An analytical synthesis method is described here which can be used to identify entire classes of converters which possess desirable attributes such as nonpulsating input and output currents, a low number of reactive components, or a given dc conversion ratio.

The basis of the approach taken is the recognition that reactive elements are switched between different circuit topologies in a switched-mode converter, and the choice of these topologies alone determines the overall properties of the system. Subject to the assumptions noted in Section 2, these topologies may be described by a number of parameters, each of which is either \(-1\), \(0\), or \(1\), and is an entry in the system state equations. The synthesis process therefore involves the selection of these parameters, thereby selecting the various circuit topologies, in such a way that the desired properties are obtained. The synthesized circuits are then constructed, and switches are added to obtain the final converter.

The examples of Section 3 demonstrate the significance of the analytical synthesis approach. First, it is found that the class of basic single-inductor converters with two switching intervals contains exactly seven distinct, nondissipative members. In addition to the buck, boost, and buck-boost converters, four converters occur which can produce output voltages of either positive or negative polarity. Second, a class of converters with nonpulsating input and output currents is found to also contain seven distinct members. Zero input or output current ripple [11, 12, 13] may be obtained in three of these converters by coupling of inductors. Thus, the analytical synthesis technique can be used to establish not only the existence of new converters, but also to determine the complete set of converters in a given class. The different
types of converters can be correlated, and now converters which possess desirable properties such as four-quadrant operation or nonpulsating input and output currents can be synthesized.

REFERENCES


