Design of the Series Resonant Converter for Minimum Component Stress

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For a given output voltage and power, the peak resonant capacitor voltage and peak inductor and switch currents of the series resonant converter depend strongly on the choice of transformer turns ratio and of tank inductance and capacitance. In this paper the particular component values which result in the smallest component stresses are determined, and a simple design strategy is developed. The procedure is illustrated for an off-line 200 W, 5 V application, and it is shown that an incorrect choice of component values can result in significantly higher component stresses than are necessary.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>C</td>
<td>Resonant (tank) capacitance.</td>
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<tr>
<td>f₀</td>
<td>Resonant frequency of the tank circuit.</td>
</tr>
<tr>
<td>fₛ</td>
<td>Switching frequency.</td>
</tr>
<tr>
<td>I</td>
<td>Output current of the converter.</td>
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<tr>
<td>Iₘₚ</td>
<td>Peak value of inductor current.</td>
</tr>
<tr>
<td>J</td>
<td>Normalized output current.</td>
</tr>
<tr>
<td>Jₘₚ</td>
<td>Normalized peak inductor current.</td>
</tr>
<tr>
<td>J₀</td>
<td>Intercept used in linear approximations (19)–(22).</td>
</tr>
<tr>
<td>k</td>
<td>Mode index.</td>
</tr>
<tr>
<td>L</td>
<td>Resonant (tank) inductance.</td>
</tr>
<tr>
<td>M</td>
<td>Normalized output voltage.</td>
</tr>
<tr>
<td>Mₛₚ</td>
<td>Normalized peak voltage on the tank capacitor.</td>
</tr>
<tr>
<td>n</td>
<td>Transformer turns ratio.</td>
</tr>
<tr>
<td>Pₒ</td>
<td>Output power.</td>
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<tr>
<td>Rₖₗₒ</td>
<td>Load resistance.</td>
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<tr>
<td>R₀</td>
<td>Characteristic impedance of the tank circuit.</td>
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<tr>
<td>S₁</td>
<td>Stress function for tank components.</td>
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<td>S₂</td>
<td>Stress function for switching transistors.</td>
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<tr>
<td>V</td>
<td>Output voltage of the converter.</td>
</tr>
<tr>
<td>Vₛₚ</td>
<td>Peak voltage on the tank capacitor.</td>
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<tr>
<td>Vᵣ</td>
<td>Voltage on the dc blocking capacitors, C1 and C2.</td>
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<tr>
<td>Vᵯᵣ</td>
<td>DC input voltage.</td>
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<tr>
<td>γ</td>
<td>Normalized switching frequency.</td>
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<tr>
<td>ω₀</td>
<td>Angular resonant frequency (in radians per second).</td>
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1. INTRODUCTION

In the design of the series resonant converter, Fig. 1, it is necessary to select values of resonant inductance, capacitance, and transformer turns ratio to obtain a given output voltage and power. Experience has shown that peak component stresses can vary by an order of magnitude or more depending on the choice of these component values. It is not clear at first which values of characteristic tank impedance \( R₀ = \sqrt{L/C} \) and transformer turns ratio \( n \) result in the lowest overall stresses. Therefore it is of interest to seek a systematic method of choosing the characteristic impedance and turns ratio to achieve the goal of an efficient design.

A solution to this design problem is based on the exact transcendental equations \([1, 2]\) which relate the peak capacitor voltage and tank current to the output voltage and current, the transformer turns ratio, and the characteristic tank impedance. Approximation of these exact expressions provides simple explicit equations for peak voltage and current in terms of the other quantities. Next, as a measure of component size, two stress functions are defined in terms of the peak voltage and current, one for stress on the resonant capacitor and inductor, and one for stress on the switching transistors. The minima of these stress functions are then found; these minima represent the value of tank impedance and transformer turns ratio that result in the smallest components and the most cost-effective design. However, in some cases the approximate expressions are not...
accurate enough, and hence iterative computer routines are employed to refine the answers given by the approximate method. Finally, to illustrate the results of the analysis, a design example is given in which the same set of specifications is satisfied by a number of different designs, and it is shown that stresses are far lower at the recommended operating points.

II. THE NORMALIZED OUTPUT PLANE

Clearly the specification of output voltage and current is equivalent to the choice of an operating point \((V, I)\) in the unnormalized output plane. However, it is possible to obtain the same \(V\) and \(I\) in many different ways by variation of characteristic impedance, turns ratio, and switching frequency. In order to generalize the problem of finding the best component values for a given \(V\) and \(I\), we can normalize the output voltage and load impedance as follows:

\[
M = \frac{V}{nV_g} \tag{1}
\]

\[
R = \frac{R_{\text{load}}}{n^2 R_0} \tag{2}
\]

The output voltage is normalized with respect to the turns ratio, and the load impedance normalized with respect to the tank impedance. Division of normalized output voltage by the normalized load yields the normalized output current:

\[
J = \frac{nR_0I}{V_g} \tag{3}
\]

We can now speak of finding the normalized operating point \((M, J)\) that yields the lowest peak stresses for a given amount of output power. When \(M\) and \(J\) are denormalized for a specific output voltage and current, they yield the values of characteristic impedance and transformer turns ratio which minimize the peak stresses for the given application. Furthermore, since the output characteristics of the series resonant converter are known for constant switching frequency [1], we may relate the operating point \((M, J)\) to its corresponding switching frequency through (4)

\[
J = \frac{2}{\gamma} \left[ (-1)^{k+1} + \left( k + \frac{(-1)^k + 1}{2} \right) \sqrt{1 + \left( k + \frac{(-1)^k + 1}{2} \right)^2 \tan^2 \frac{\gamma}{2}} \right] \tag{4}
\]

where \(\gamma = \pi f_0 f_s\) is the normalized switching frequency and \(k = 0, 1, 2, \ldots\), is the mode index. This equation is valid for any continuous conduction mode \(k\) as long as the switching frequency is in the range

\[
\frac{f_0}{(1 + k)} \leq f_s \leq \frac{f_0}{k}. \tag{5}
\]

For constant values of switching frequency, (4) describes a family of ellipses on the output plane. We are now in a position to relate the generalized operating point \((M, J)\) to the peak capacitor voltage \(V_{CP}\) and the peak inductor current \(I_{LP}\).

III. EXACT AND APPROXIMATE EXPRESSIONS FOR PEAK VOLTAGE AND CURRENT

In this section the exact expressions relating peak resonant voltage and current to output voltage and current are reviewed, and linear explicit equations for the peak stresses are developed by approximation of the exact equations.

As shown in [1], the combination of (4) with circuit equations containing peak voltage and current enables one to obtain a set of equations relating the normalized output current to either \(V_{CP}\) and \(M\), or \(I_{LP}\) and \(M\). To be consistent with the normalized output variables, we normalize peak capacitor voltage and inductor current in the same manner as the output quantities.

\[
M_{CP} = \frac{V_{CP}}{V_g} \tag{6}
\]

\[
J_{LP} = \frac{R_0 I_{LP}}{V_g} \tag{7}
\]

The derivation of the equations is given in [1], and the results are summarized here for convenience.

Above resonance \((k = 0)\):

\[
J = \frac{M_{CP}}{\tan^{-1} \sqrt{\frac{(M_{CP} + 1)^2 - 1}{1 - M^2}}} \tag{8}
\]

\[
J = \frac{J_{LP} - 1 + M}{\tan^{-1} \sqrt{\frac{(J_{LP} + M)^2 - 1}{1 - M^2}}}, \tag{9}
\]

for \(1 - M^2 / M < J_{LP}\)
When the exact equations are plotted in this manner, it is apparent that the peak stresses are nearly independent of M, and increase almost linearly with normalized output current. This is to be expected: since the output current is simply the rectified tank current, the two should be directly related. In addition, because the tank capacitor voltage is a function of the charge transferred by the resonant current, the peak capacitor voltage also varies directly with output current. This explanation is true to the extent that tank current waveshape is independent of output voltage, a good first-order approximation. The curves in Fig. 2 do exhibit a second-order dependence on output voltage. These observations suggest that, in order to obtain an expression that may be solved explicitly for peak voltage or current, we allow M to be constant in (8)–(12), and write the output current as a linear function of peak voltage or current.

As an example of the approximation of an exact equation, consider the case of peak tank capacitor voltage above resonance. Equation (8) is plotted for M = 0 in Fig. 3. We seek an approximation of the form

\[ J = gM_{CP} + J_0. \]

In order to find the slope g, recall that the series expansion of \( \tan^{-1} u \) for \( u > 1 \) is

\[ \tan^{-1} u = \frac{\pi}{2} - \frac{1}{u} + \frac{1}{3u^2} - \frac{1}{6u^3} \ldots \]

Letting \( u \) be the appropriate quantity in (8), we can see that

\[ J = \frac{M_{CP}}{\tan^{-1} u} = \frac{M_{CP}}{\pi/2 - 1/u + 1/3u^2 - 1/6u^3 \ldots} \]

It is clear that as \( M_{CP} \) goes to infinity, \( u \) also goes to infinity, hence

\[ \lim_{u \to \infty} \frac{M_{CP}}{\tan^{-1} u} = \frac{2}{\pi} M_{CP} \]

so the slope is \( g = 2/\pi \). Similarly, the intercept \( J_0 \) can be found from the expression

\[ J_0 = \lim_{u \to \infty} \frac{M_{CP}}{\tan^{-1} u} - gM_{CP} = \frac{4}{\pi^2} \sqrt{1 - M^2}. \]
Substitution of the slope and intercept into (13) yields an explicit equation for $M_{CP}$:

$$M_{CP} = \frac{\pi}{2} J - \frac{2}{\pi} \sqrt{1 - M^2}.$$  \hfill (18)

When this process is repeated for the other peak stresses above and below resonance, the following results are obtained.

**Above resonance** ($k = 0$):

$$M_{CP} = \frac{\pi}{2} J - \frac{2}{\pi} \sqrt{1 - M^2}$$

$$J_{LP} = \frac{\pi}{2} J - \left( M - 1 + \frac{2}{\pi} \sqrt{1 - M^2} \right).$$ \hfill (20)

**Below resonance** ($k = 1$):

$$M_{CP} = \frac{\pi}{2} J + \frac{2}{\pi} \sqrt{1 - M^2}$$

$$J_{LP} = \frac{\pi}{2} J + M - 1 + \frac{2}{\pi} \sqrt{1 - M^2}.$$ \hfill (22)

These approximations agree well with our intuitive explanation of the converter behavior. In fact, when $M = 1$ the intercept reduces to zero in each case. The resulting expression for peak voltage or current is $\pi/2$ times the average output current. This corresponds to the peak-to-average ratio of a rectified sine wave, and correctly models the converter behavior at resonance. Similar approximations have previously been made for the parallel resonant converter [3]. In summary, (19)--(22) are the explicit approximate equations needed to solve for the operating points with minimum stress.

**IV. TANK COMPONENT STRESS**

In order to quantify the effect that the characteristic tank impedance $R_0$ and transformer turns ratio $n$ have on the tank inductor and capacitor, a function is defined in this section that relates $R_0$ and $n$ to the peak stresses on these components. The minimum of this function yields the desired operating point with lowest component stresses and the best values of $R_0$ and $n$.

One measure of the size of the resonant inductor and capacitor is the magnitude of the peak energy stored in each element during each switching cycle. Hence an elementary stress function could be the sum of the peak energy in the two components:

$$S = \frac{1}{2} L_{LP}^2 + \frac{1}{2} C V_{CP}^2.$$ \hfill (23)

However, if we simply vary the peak voltage and current, then the output voltage and current will vary as well. Therefore it is necessary to normalize (23) with respect to the output power. The stress function then becomes the ratio of the peak energy stored in the inductor and capacitor to the average energy transferred to the output.

By recalling the definition of $M$ and $J$ from Section II, one can see that the output power $P_0 = V I$ is

$$P_0 = \frac{M J V_k^2}{R_0}.$$ \hfill (24)

Combination of (23) and (24) yields the stress function

$$S_1 = \frac{1}{2} L_{LP}^2 + \frac{1}{2} C V_{CP}^2 \quad \frac{M J V_k^2}{R_0}.$$ \hfill (25)

Consider first the below-resonance case. Recalling the fact that peak stresses are nearly independent of normalized output voltage $M$, we choose $M = 1$ as the simplest form of (21) and (22) and substitute the resulting expressions into (25):

$$S_1 = \frac{L/V_k/R_0^2((\pi/2)J)^2 + C V_{CP}^2((\pi/2)J)^2}{2MJV_k^2/R_0}.$$ \hfill (26)

The definition of the characteristic tank impedance $R_0 = \omega_0 L = 1/\omega_0 C$ can be applied to eliminate $R_0$ from (26):

$$S_1 = \frac{\pi^2 J^2}{4 \omega_0 M}.$$ \hfill (27)

Evidently the tank stresses are minimized for maximum $M$ and minimum $J$. Below resonance, this corresponds to $M = 1$ and $J = 2/\pi$, the mode boundary between $k = 1$ continuous mode and $k = 2$ discontinuous mode. A similar analysis of stress in the $k = 2$ discontinuous mode (see Appendix A) shows the peak stresses form vertical lines in the output plane (Fig. 4), and that the tank component stress increases with decreasing $J$, so $J = 2/\pi$ is indeed the minimum stress operating point below resonance.

The same line of reasoning may be applied to finding the minimum tank stresses above resonance. In fact, (27) is valid in this case also: the maximum $M$ is $M = 1$, and the minimum $J$ is $J = 0$. However, this raises a further question because $J = 0$ is precisely the region in which the exact equations deviate from the linear behavior predicted by the approximations (19)--(22) (see Fig. 3).

![Fig. 4: Peak inductor current in normalized output plane, for $k = 1$ continuous mode and $k = 2$ discontinuous mode. In continuous mode stress depends primarily on output current, while in discontinuous mode stresses are entirely dependent on output voltage.](image)
To verify the above conclusions, an iterative computer routine was employed to find the exact peak capacitor voltage and inductor current for a given operating point, enabling us to find the exact tank stresses as well. The result is shown in Fig. 5. This figure indicates that the approximate analysis is substantially correct, although the minimum with respect to \( M \) occurs at approximately \( M = 0.95 \) instead of \( M = 1 \) as predicted. We may conclude that the operating point for minimal tank component stresses is at \( M = 0.95 \) and \( J \) as close to zero as other considerations will allow.

\[ S = \frac{2V_g I_{LP}}{M J V_o^2 / R_0}. \]
\[ (28) \]

When either (20) or (22) is evaluated at \( M = 1 \) and substituted into (28) the stress function becomes

\[ S = \frac{\pi}{M}. \]
\[ (29) \]

At first glance it would appear that transistor stress is independent of \( J \) and that the function is minimal anywhere along a vertical line in the output plane at \( M = 1 \). However, it is necessary to verify this conclusion near \( J = 0 \) where the approximations (20) and (22) are not accurate.

A numerical computer routine was employed to find the exact value of the stress function. To eliminate \( I_{LP} \) from the calculation, we first rewrite (28) as

\[ \frac{R_0 I_{LP}}{V_g} = \frac{M J S_2}{2}. \]
\[ (30) \]

Substitution of (30) into (12) yields an exact expression strictly in terms of the variables \( M, J \), and \( S_2 \):

\[ J + \frac{(M - 1 - M J S_2/2)}{\pi - \tan^{-1}[(M J S_2/2 - M J^2 - 1)/(1 - M^2)]} = 0. \]
\[ (31) \]

This equation may be solved iteratively to obtain exact values of \( S_2 \) for differing operating points. The results are shown in Figs. 6 and 7, below and above resonance.
respectively. Fig. 6 shows that, below resonance, the switch stress is approximately constant for constant $M$, but that near the $M$ axis, $J < 1.4$ and the switch stress goes to infinity. Fig. 7 reveals that the switch stress above resonance does not go to infinity for small $J$, but does increase 30 to 40 percent for values of $J$ less than about $J = 0.2$. A series expansion of the exact equation for switch stress above resonance reinforces the conclusion that $S_2$ does not go to infinity as $J$ goes to zero, but instead goes to a constant, $4/M$. To summarize, the minimum transistor stress below resonance ($k = 1$) occurs at operating points $M = 1$, $J > 1.4$, and the smallest stress above resonance ($k = 0$) occurs at operating points $M = 1$, $J > 0.2$.

VI. DISCUSSION OF COMBINED RESULTS

In the ideal case, the most efficient and cost-effective converter would have both minimum tank stresses and minimum switch stresses, yet we can see that these two requirements conflict. Tank stress is minimized for decreasing $J$, and transistor stress is minimized for increasing $J$. Clearly a design tradeoff must occur. Below resonance, transistor stress is not appreciably reduced for $J > 1.4$; hence a good compromise point is $M = 1$, $J = 1.4$. Above resonance the transistor stress is minimized for $J > 0.2$, hence a good operating point is $M = 0.95$, $J = 0.2$. One further conclusion is immediately apparent. Because the operating point above resonance has a lower normalized output current than the operating point below resonance, any series resonant converter can be designed above resonance with lower stresses than below resonance. Hence, it is better to operate above resonance than below in applications not requiring the commutation of thyristor switches.

VII. APPLICATION OF MINIMAL STRESS ANALYSIS TO A SPECIFIC DESIGN

In this section an off-line 200 W dc-to-dc converter is designed to meet the following specifications: input voltage of 160 V ($V_g = 80$ V), output voltage and current of 5 V and 40 A, and switching frequency of 50 kHz. The design for the recommended below-resonance operating point is developed; then a number of different designs that meet the same specifications are discussed. Finally, a topology is presented that eliminates the half-bridge blocking capacitors, further reducing the size and component count of the supply.

Consider the operating point $M = 0.9$, $J = 1.4$, below resonance. This point should have low transistor stress and small component size, as stated in Section VI. The first design step is to obtain the normalized switching frequency $\gamma$, in this case $\gamma = 3.79$ rad, either by solving (4) iteratively or by estimating $\gamma$ graphically from Fig. 2. The resonant frequency may then be found from the definition of $\gamma$:

$$f_0 = \frac{\gamma f_s}{\pi} = 60.2 \, \text{kHz}. \quad (32)$$

The transformer turns ratio is determined by the normalized output voltage:

$$n = \frac{V}{MV_g} = 0.0694 \quad (33)$$

which enables us to solve directly for the characteristic impedance of the tank:

$$R_0 = \frac{V_g}{nI} = 40.3 \, \Omega. \quad (34)$$

The resonant frequency and characteristic impedance provide two equations in terms of the two unknowns, resonant inductance and capacitance.

$$L = \frac{R_0}{2\pi f_0} = 106.5 \, \mu\text{H} \quad (35)$$

$$C = \frac{1}{2\pi f_0 R_0} = 65.5 \, \text{nF}. \quad (36)$$

All that remains is to find peak inductor current and capacitor voltage, which may be obtained approximately from (21) and (22), graphically from Fig. 2, or exactly from iterative solutions of (11) and (12). In this case the exact peak current is 5.06 A, and the exact peak capacitor voltage is 211.7 V. The switching transistors must sustain the peak resonant current and the dc input voltage; hence transistors must be chosen that will reliably operate with 5.06 A peak and 160 V dc.

The same design procedure may be followed for any specific operating point; a summary is given in Table I for above resonance, and Table II for below resonance. Some instructive conclusions may now be drawn. First, the peak stresses in the converter are highly dependent on the operating point. Compare the peak capacitor voltage in case B above resonance to the peak capacitor voltage in case C: 15.8 V compared with 607.3 V for the same input and output power. In addition, peak current in case B is less than half the peak current in case D: 4.13 A

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>Summary of Designs with Different Operating Points Above Resonance</th>
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<tbody>
<tr>
<td>$M, J$</td>
<td>$f_0$ (kHz)</td>
</tr>
<tr>
<td>A 0.9, 0.05</td>
<td>14.6</td>
</tr>
<tr>
<td>B 0.9, 0.2</td>
<td>31.37</td>
</tr>
<tr>
<td>C 0.9, 5</td>
<td>48.4</td>
</tr>
<tr>
<td>D 0.5, 0.2</td>
<td>15.15</td>
</tr>
</tbody>
</table>

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compared with 9.46 A. It is clear that the reputation of
the series resonant converter for high peak stresses is
undeserved; it simply must be designed to operate at the
correct point. Secondly, it is distinctly better to operate
above resonance than below. Compare case B in Table I,
the minimal stress point above resonance, with case B in
Table II, the minimal stress point below resonance. The
peak capacitor voltage above resonance is only 15.8 V,
or 7.5 percent of the peak voltage below resonance and
the peak resonant current is 4.13 A, or 82 percent of the
peak current at the minimal stress point below resonance.
Although the capacitance in this case is larger above
resonance than below, it is still small (0.88 μF) and is
more than compensated for in cost by a smaller
inductance (29.2 μH compared with 106.5 μH). Finally,
an inspection of cases A and B in both tables confirms
the fact that transistor stress increases both above and
below resonance when normalized output current is
decreased below the minimum stress point.
Before concluding this section on design, it is
appropriate to note that the component count of the half-
bridge series resonant converter can be reduced. Referring
to Fig. 8(a) it can be seen that the entire function of the
two half-bridge capacitors is to bear the dc component of
the voltage created by the switching between the input
source and ground. This dc voltage is dependent entirely
on the duty ratio of the switches, which is always
D = 0.5 for a frequency-controlled resonant converter.
Hence we may eliminate the half-bridge capacitors by
simply letting the resonant capacitor bear both the ac and
the dc component of the voltage induced by the switched
input source, as shown in Fig. 8(b). In this case the peak
capacitor voltage is \( V_c \) plus the peak ac component,
\( V_e + V_{CP} \). The cost-effectiveness of this approach will
depend on the tradeoffs involved in a particular design
situation.

VIII. CONCLUSIONS

If the series resonant converter is to be designed
effectively, there must be a systematic procedure for
selecting the characteristic impedance of the tank, \( R_0 = \sqrt{L/C} \), and the transformer turns ratio \( n \). These two
quantities must be chosen to yield the lowest possible
peak voltages and currents in the converter for a given
output power. In this paper the solution to this problem is
found by normalizing the output voltage and current with
respect to \( n \) and \( R_0 \), then seeking the normalized value of
output voltage and current \((M, J)\) that yields the lowest
peak stresses in the general case.

To accomplish this task, two stress functions are
defined to allow measurement of the required size of the
components, one for the size of the tank inductor and
capacitor, and one for the transistor switches. Each
function is then analyzed to find the normalized output
operating point that yields the smallest stresses. It is
found that in all cases the component stresses are
minimized for large normalized output voltage \( M \) near
unity. However, the tank component stress decreases with
decreasing \( J \), and the transistor stress increases for
decreasing \( J \); hence a tradeoff must be made. The best
points for combined low stresses appear to be \( M = 1, J = 1.4 \) below resonance, and \( M = 0.95, J = 0.2 \) above
resonance.

Application of these results to a design example
reveals that for the same output power, the peak capacitor
voltage can vary by an order of magnitude depending on
the normalized operating point. The peak transistor and
inductor currents can be twice as large as necessary if the
converter is not properly designed. Furthermore, it is
shown that significantly smaller stresses can be obtained
for the same converter by operating above resonance
instead of below. Consequently, in applications not
requiring the commutation of thyristors, it is best to
operate in the above resonance mode.

APPENDIX A. STRESSES IN THE EVEN
DISCONTINUOUS MODE

As shown in [2], the normalized output voltage in the
even discontinuous mode is
and the circuit equations for peak voltage and current are

\[ V_{CP} = \frac{2kR_{load}}{R_0\gamma} \text{ and } k = 2, 4, 6, \ldots \]  

(A1)

\[ V_{CP} = V_g \left[ 2 - \frac{4kR_{load}}{R_0\gamma} + \frac{2k^2R_{load}}{R_0\gamma} \right] \]  

(A2)

\[ I_{LP} = \frac{V_g}{R_0} [1 - M + kM]. \]  

(A3)

Equation (A3) is already in the desired form, and we may express (A2) in terms of \( M \) by substituting (A1) for the appropriate quantities:

\[ V_{CP} = V_g [2 + (k - 2)M]. \]  

(A4)

Hence, in the discontinuous mode, the peak stresses are independent of normalized output current and linearly dependent on \( M \), corresponding to vertical lines in the output plane. In the discontinuous mode it is the output voltage alone that determines when the output bridge is reverse biased: hence determining both the duration and the charge transferred to the output (magnitude) of the resonant current.

We may now find the stress functions for the discontinuous mode. In order to find the tank component stress, substitute (A3) and (A4) into (25):

\[ S_2 = \frac{(1 + (k-1)M)^2 + (2 - (2-k)M)^2}{2\omega_0MJ}. \]  

(A5)

for the \( k = 2 \) discontinuous mode, this reduces to

\[ S_2 = \frac{5 + 2M + M^2}{2\omega_0MJ}. \]  

(A6)

Partial differentiation of (A6) demonstrates that the tank component stress in this mode decreases with increasing \( M \) and increases with decreasing \( J \), so the operating point with minimal tank stress in this mode is at the boundary \( M = 1, J = 2/\pi \). In the same manner the transistor stress function may be found by substitution of (A3) into (28):

\[ S_2 = \frac{2(1 + M(k - 1))}{MJ}. \]  

(A7)

In this case, the minimum transistor stress occurs at the same point as the minimum tank component stress, \( M = 1, J = 2/\pi \).

REFERENCES


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