

Pricing versus Admission Control in Multi-class Loss Networks

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Abstract—In this paper, we study the relative roles of optimal pricing and call admission control (CAC) in network revenue performance optimization from a network provider’s perspective. We consider a multi-class loss network where each traffic class has a differentiated quality of service (QoS), as characterized by a per connection bandwidth, and an associated static charging price which decides the volume of the service demand. We formulate a framework that views pricing and call admission control (CAC) as two stages of a unified call level resource allocation process, and subsequently study the impact of pricing and CAC both analytically and numerically.

We show that both an optimal pricing and an optimal CAC can improve the performance, and the optimal pricing plays a significant role. Moreover, in large networks with many small users, the performance resulting from the greedy CAC and its corresponding optimal pricing approaches the upper bound performance asymptotically, and more complicated joint optimization of pricing and CAC does not provide an additional performance gain.

Keywords— pricing, call admission control, loss network

I. INTRODUCTION

In this paper, we study the relative roles of optimal pricing and call admission control (CAC) in network performance optimization from a network provider’s point of view. We consider a multi-traffic-class capacity-limited network whose performance objective is to maximize the long term revenue. Each traffic class has a differentiated quality of service (QoS), which is characterized by a per connection bandwidth. Each traffic class also has an associated static charging price, which decides the volume of the service demand (i.e., arrival rate), where the service demand decreases with an increasing price. In order to optimize performance, a network provider has the freedom to set a sound pricing policy (i.e., the rule to set prices) and to apply an intelligent CAC policy (i.e., the rule to accept or reject a connection request) for network control.

In a multi-class differentiated services network, connections of different traffic classes have different values to the network in terms of revenue per bandwidth per unit time. Hence, a greedy CAC (i.e., accept all connection requests when the unused bandwidth is available) may not be optimal. Only accepting high value connections may not be optimal either since it may leave unused bandwidth. Meanwhile, the selection of the prices is not trivial. Prices may not be optimal if they are optimized for each traffic class independently, or if they are

designed without the context of an optimal CAC. For example, independent optimizing the price for each traffic class leads to high demand and high blocking, but choosing a higher price could simultaneously increase the value per connection and reduce the demand. Moreover, because a CAC policy can control the relative blocking of different traffic classes, pricing design considering the CAC impact may improve the network revenue performance. Therefore, it is important to use sound pricing and CAC policies in order to achieve an optimal network performance, and this is the subject of our research presented in this paper.

We observe that, fundamentally, both pricing and CAC aim to improve the performance via a two-stage network resource allocation process. In the first stage, a network pricing policy takes effect to influence the traffic received, or seen, by a network. Which of this traffic is accepted into the network is decided in the second stage by a CAC policy. As a result, we formulate a pricing and CAC policy pair as a two-stage call level network resource allocation scheme. To our knowledge, such a unified two-stage formulation of the impact of pricing and CAC on network control and optimization is new. In line with such a characterization, we first present our taxonomy of network resource allocation schemes concerning various pricing and CAC combinations. Subsequently, we examine the interplay of pricing and CAC organized by such a taxonomy. We present our theoretical and numerical performance results of the various resource allocation schemes under different traffic scenarios, and discuss the implications of these results.

The remainder of this paper is organized as follows. In Section II, we review the related research. In Section III, we present our problem formulation and define a taxonomy of the resource allocation schemes and their performances. In Section IV, we develop propositions that identify the relationships of the various performances as defined in Section III. In Section V, we describe our computational methods to these performances. We then give numerical examples in Section VI. Finally, Section VII concludes the paper and addresses the directions of future work.

II. RELATED WORK

Network CAC research was introduced in the late 1980s and its objective is to achieve a performance gain through an

intelligent CAC policy, see [1]–[3]. A trivial baseline CAC policy is called *greedy (aka, complete sharing)* policy, meaning a network always accepts a connection request whenever there is available bandwidth. In contrast, a *dynamic* CAC policy bases its acceptance or rejection of a connection request on both the available bandwidth and the system state at the decision time. In the previous CAC research, the value of a connection, as characterized by the price, is given and fixed. The corresponding CAC solution does not consider whether these underlying prices can be optimized.

Network pricing research was introduced in the early 1990s and its key idea is to use a sound pricing policy as an efficient, viable and fair way to adjust demand on network resources. The pricing research can be broadly classified into two categories which are: *price-based bandwidth allocation* and *price-based volume control*. With a *price-based bandwidth allocation*, given the capacity constraint, a pricing policy is a price vector indicating the price for each traffic class, which in turn decides the obtained transmission quality, characterized by the bandwidth per connection, see [4]–[10]. While some methods eliminate congestion via a proper bandwidth assignment subject to the capacity constraint (e.g., [6], [8]), some others allow congestion and the network will block traffic of the lower-priced class(es) and accept the traffic of the higher-priced class(es) in case of congestion (e.g., [4], [5]). In contrast, with a *price-based volume control*, a price policy is a price vector indicating the price for each class, which in turn determines the obtained transmission quantity, or volume, characterized by the connection arrival rate. Thus, the price is one critical factor in determining the number of users requesting service at a certain time. Meanwhile, the bandwidth per connection is assumed to be given and fixed. In such a context, both *static (or state-independent) pricing* and *dynamic (or state-dependent) pricing* are studied, see [11]–[14]. In case of congestion, a *static pricing* assumes a greedy CAC policy and does not consider the potential congestion control effects of any intelligent CAC. In contrast, a *dynamic pricing* always increases the prices to reduce the amount of traffic arrival to reduce or eliminate congestion.

III. PROBLEM FORMULATION

A. Pricing and CAC models

We adopt the *price-based volume control* with a *static pricing* policy, and we assume a *dynamic CAC* policy. We do not address the issue of price-based bandwidth allocation. Rather, the bandwidth request for each traffic class is fixed which indicates the corresponding QoS class. The number of users requesting service is dynamic, and the *dynamic CAC* policy is used for the accept or reject decision.

Moreover, we model pricing and CAC as two stages of a resource allocation process, where a pricing policy affects the connection requests received by a network, and a CAC policy affects the way that a network handles the received connection requests, as illustrated in Figure 1. In this figure, the arrows represent potential connection requests, and the width of the line represents the required bandwidth and the

number of dollar signs the willingness to pay. Consequently, in our formulation, the optimality of a static pricing policy indicates an *optimal* stationary volume of traffic arrivals for a particular CAC policy, whereas the optimality of a CAC policy is for the underlying prices. Thus, our formulation allows a joint consideration of pricing and CAC for performance analysis.

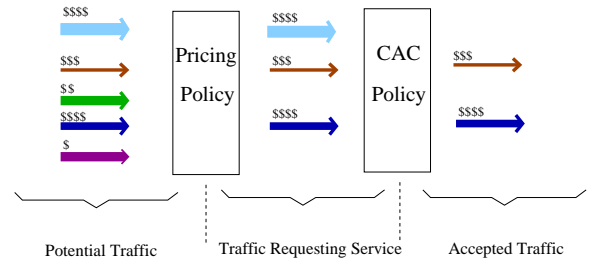


Fig. 1. The effects of pricing and call admission control on network traffic

Furthermore, we assume the CAC decision is only with respect to the bandwidth demand of an incoming connection, and a CAC decision is per connection, rather than per packet. No queueing of incoming connecting requests is assumed. The network collects an immediate revenue that is equal to the price charged to the incoming connection if it is accepted, and collects no revenue if a connection is rejected and the connection leaves the system with no retries.

We note that, a dynamic pricing can subsume a dynamic CAC. At any state in which CAC would block arrivals, a dynamic pricing policy can set the state’s price to infinity leading to a zero arrival rate and subsequently a zero blocking probability, as is done in [11], [12]. Nevertheless, combined with an optimal static pricing, an optimal dynamic CAC in our framework allows congestion and intelligent blocking. In our view, the combination of an optimal static pricing policy and a dynamic CAC policy possesses a practical advantage over a dynamic pricing policy. With a dynamic pricing policy, more interaction of users at a connection setup phase is required in order to relay the current optimal price. As a result, users of a dynamic pricing need to dynamically reassess their willingness to pay. In contrast, the operation of a dynamic CAC policy is internal to a network and it is transparent to users, which can significantly simplify the network services offering.

B. Network and traffic models

We currently consider a single link network with a link capacity C , and a set of different traffic classes for a user to choose from and the traffic class index is in the set $\mathcal{I} = (1, \dots, I)$. We assume that traffic of each class arrives according to a Poisson process and the service times are exponentially distributed. Each traffic class $i \in \mathcal{I}$, is characterized by a fixed price per connection, u_i , a transmission rate (bandwidth), x_i , a mean arrival rate, λ_i , and a mean service rate, μ_i . As in early works, we also call a class- i connection a class- i call, and they are assumed to have constant bit rates (CBR).

The price per call for each traffic class i is determined by a pricing policy. According to the economic theory on consumer demand (see [15]), we assume the class- i connection arrival rate, λ_i , is a function of such a static pricing, i.e., $\lambda_i = \lambda_i(u_i) \geq 0$, where $\lambda_i(\cdot)$ represents a continuous and strictly decreasing known arrival (i.e., demand) function for $0 \leq u_i \leq u_{i,max}$ with $\lambda_i(0) < \infty$ and $\lambda_i(u_{i,max}) = 0$. We also assume no demand substitution effect among different traffic classes in this work.

C. System state definition

The system is dynamic due to the dynamic arrival and departure of connections. We define the system state to be the combination of the system configuration (i.e., the number of calls of each class in the system) and the transition-triggering traffic events. In particular, we let n_i denote the number of ongoing class- i calls in the system. Therefore, the configuration space is defined as follows.

$$\mathcal{N} = \left\{ n := (n_1, \dots, n_I) \mid \sum_{i=1}^I n_i x_i \leq C \right\}$$

We also assume that the number of state transitions within a finite time interval is finite and there is only one traffic event taking place at any instant. Let e_i denote a $(1 \times I)$ unit vector whose i^{th} element equals to 1 and other $I - 1$ elements equal to 0. Subsequently, the event space can be expressed below, where e_i and $-e_i$ denote the arrival and departure of a class- i call, respectively.

$$\Omega = \left\{ \omega \mid \omega \in \{\pm e_i\}, i \in \mathcal{I} \right\}$$

Due to the finite capacity constraint and the finite number of traffic classes, the system is assumed to constitute a finite number of states given by:

$$\mathcal{S} = \left\{ s = (n, \omega) \mid n \in \mathcal{N}, \omega \in \Omega, \text{ and } \omega \neq -e_i \text{ if } n_i = 0 \right\}$$

D. Performance objective

Our performance objective is to maximize the long-term average revenue under a pricing and CAC policy pair. Let L_i denote the long term average loss probability of class- i calls resulting from a dynamic CAC policy, then the long-term average revenue with a static pricing policy can be expressed in a simple form as following:

$$R = \sum_{i=1}^I \lambda_i(u_i) u_i (1 - L_i) \quad (1)$$

E. A taxonomy of resource allocation schemes

We use $\mathcal{U} = \{u \mid u_i \in [0, u_{i,max}], i \in \mathcal{I}\}$ and $\Pi = \{\pi\}$ to denote the set of all pricing policies and the set of all CAC policies, respectively. We also denote the set of average revenue performances by $\mathcal{R} = \{R(u, \pi) \mid u \in \mathcal{U}, \pi \in \Pi\}$.

We let u^a denote an *arbitrary* pricing, and let π^g denote the trivial greedy CAC policy. Additionally, we let $u_{\pi^g}^*$ denote the optimal price vector with respect to the greedy CAC policy π^g . We express it in (2) and called it the *greedy optimal*

pricing. We also let π^{*a} denote the optimal CAC policy given an underlying arbitrary pricing, u^a , as expressed in (3).

$$u_{\pi^g}^* = \arg \max_u R(u, \pi^g) \quad (2)$$

$$\pi^{*a} = \arg \max_{\pi} R(u^a, \pi) \quad (3)$$

Furthermore, we define a pair of joint optimal pricing and CAC based upon Proposition 1 and 2 developed below¹.

Proposition 1: There exists a finite upper bound, R^* , such that $R(u, \pi) \leq R^*, \forall u \in \mathcal{U}, \forall \pi \in \Pi$. In addition, there exists a (not necessarily unique) pair of pricing, $u_{\pi^{*j}}^*$, and CAC, π^{*j} , such that,

$$(u_{\pi^{*j}}^*, \pi^{*j}) = \arg \max_{u, \pi} R(u, \pi) = R^* \quad (4)$$

Proposition 2: The pricing and CAC of a policy pair $(u_{\pi^{*j}}^*, \pi^{*j})$, as defined in (4), are optimal with respect to each other, that is:

$$u_{\pi^{*j}}^* = \arg \max_u R(u, \pi^{*j}) \quad (5)$$

$$\pi^{*j} = \arg \max_{\pi} R(u_{\pi^{*j}}^*, \pi) \quad (6)$$

Consequently, we call $u_{\pi^{*j}}^*$ and π^{*j} the *joint optimal* pricing and CAC.

Since computing R^* may be difficult, we seek a simple bound on the performance, denoted as R_{ub} . Specifically, we define R_{ub} to be the maximal revenue performance assuming all arriving traffic is admitted into the system without blocking, and the system is only constrained by the average load being less than the capacity. In other words, the loss probabilities $L_i = 0$ for $i \in \mathcal{I}$. Thus, R_{ub} is only a function of the prices but independent of CAC. Let \bar{n}_i denote the expected number of class- i calls in such a system when the arrival rate is λ_i , for $i \in \mathcal{I}$. According to Little's Theorem in [19], $\bar{n}_i = \lambda_i / \mu_i$. Consequently,

$$R_{ub} = \max_{u, \pi} R(u, \pi) |_{L_i=0, i \in \mathcal{I}} = \max_u \sum_{i=1}^I \lambda_i(u_i) u_i \quad (7)$$

subject to

$$\sum_{i=1}^I \bar{n}_i x_i \leq C \quad \text{where} \quad \bar{n}_i = \lambda_i / \mu_i \quad (8)$$

Let

$$R_{ub}(u) = \begin{cases} \sum_{i=1}^I \lambda_i(u_i) u_i, & \text{if (8) is satisfied,} \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Subsequently, we denote the optimal prices to obtain R_{ub} by u_{ub}^* and express it in (10). We also call u_{ub}^* the *optimal upper bound pricing*.

$$u_{ub}^* = \arg \max_u R_{ub}(u) \quad (10)$$

¹Proofs for Proposition 1 and 2 are omitted for space reasons.

Using the defined pricing and CAC policies, we now define a set of performances concerning various pricing and CAC combinations as follows:

$$\begin{aligned} R_0 &= R(u^a, \pi^g) & R_0^* &= R(u^a, \pi^{*a}) \\ R_1 &= R(u_{\pi^g}^*, \pi^g) & R_1^* &= R(u_{\pi^g}^*, \pi^{*g}) \\ R_2 &= R(u_{\pi^{*j}}^*, \pi^g) & R_2^* &= R(u_{\pi^{*j}}^*, \pi^{*j}) \\ R_{ub} &= R_{ub}(u_{ub}^*) \end{aligned}$$

where R_0 is the long-term average revenue with the arbitrary pricing and the greedy CAC policy; R_0^* represents the average revenue with the arbitrary pricing but the corresponding optimal CAC policy; and R_1, R_1^*, R_2, R_2^* and R_{ub} are similarly defined².

IV. ANALYTICAL RESULTS OF PERFORMANCE PROPERTIES

In this section, we address the relationships of various performances that are defined in Section III-E analytically. First, we establish the performance relationship in a general network.

Proposition 3: Given the performance values R_0, R_1, R_1^*, R_2^* and R_{ub} , as defined in Section III-E, we have

$$R_0 \leq R_1 \leq R_1^* \leq R_2^* \leq R_{ub} \quad (11)$$

Proof

- 1) Let L_i^g denote the loss probability of class- i calls with the greedy CAC. Per our definition, $R_0 = R(u^a, \pi^g)$ and $R_1 = R(u_{\pi^g}^*, \pi^g)$, where u^a is any arbitrarily selected price vector, and

$$\begin{aligned} u_{\pi^g}^* &= \arg \max_u R(u, \pi^g) \\ &= \arg \max_u \sum_{i=1}^I \lambda_i(u_i) u_i (1 - L_i^g) \end{aligned}$$

If $R_0 > R_1$, then $u_{\pi^g}^*$ can not be the optimal price vector associated with the greedy CAC, which is contradictory to the assumption. Therefore $R_0 \leq R_1$.

- 2) Per our definition, $R_1 = R(u_{\pi^g}^*, \pi^g)$ and $R_1^* = R(u_{\pi^g}^*, \pi^{*g})$. If $R_1 > R_1^*$, then π^{*g} can not be the optimal CAC associated with the greedy optimal pricing, $u_{\pi^g}^*$, since it achieves a worse performance than the greedy admission control policy π^g , given that $u_{\pi^g}^*$ is applied to both cases. This is contradictory to the assumption. Therefore $R_1 \leq R_1^*$.
- 3) Per our definition, $R_2^* = R^*$, and per Proposition 1, $R(u, \pi) < R^*, \forall u, \forall \pi$. Therefore $R_1^* \leq R^* = R_2^*$.
- 4) When the joint optimal price vector, $u_{\pi^{*j}}^*$, and the corresponding optimal CAC, π^{*j} , are applied, we denote class- i calls' arrival rate as $\lambda_i^* = \lambda_i(u_{\pi^{*j}}^*)$ and the corresponding successful transmitted traffic rate as $\lambda_i^{I*} =$

² $R_2^* = R^*$ is renamed for a clear presentation of the structure of our taxonomy. R_1 and R_{ub} are the same as the performance value of the *optimal static pricing policy* and the *upper bound performance*, respectively, as defined in [11].

$\lambda_i^*(1 - L_i)$ is the successful transmit rate, for $i \in \mathcal{I}$. The number of class- i calls in such a system is a random variable n_i^* , and we denote its mean as \bar{n}_i^* . Since at any time, $\sum_i n_i^* x_i \leq C$, we have $\sum_i \bar{n}_i^* x_i \leq C$. According to Little's Theorem in [19], $\lambda_i^{I*} = \mu_i \bar{n}_i^*$. Therefore \bar{n}_i^* and λ_i^{I*} form a feasible solution of the upper bound problem (7). Consequently,

$$R_2^* = \sum_{i=1}^I \lambda_i^* (1 - L_i) u_{\pi^{*j}}^* = \sum_{i=1}^I \lambda_i^{I*} u_{\pi^{*j}}^* \leq R_{ub}$$

- 5) Based upon the above results in step [1-4], the result follows. *Q.E.D.*

Next, we address the performance relationship in large networks with many small users.

Definition 1: Consider a base system whose capacity and arrival rates are C , and λ_i , for $i \in \mathcal{I}$, respectively. We define a *scaled system* to be a network whose capacity is $C_\beta = \beta C$ and arrival rates are $\lambda_{i,\beta} = \beta \lambda_i$, for $i \in \mathcal{I}$. We also define the *limiting regime* to be such a scaled system when $\beta \rightarrow \infty$.

Such a limit regime represents a network where there are many small users and the system capacity is large compared to the bandwidth requirement of each user. Accordingly, we use $R_{1,\beta}, R_{1,\beta}^*, R_{2,\beta}, R_{ub,\beta}$ to represent the corresponding values for the scaled system, and establish the following relationship.

Proposition 4: In the limiting regime of a large system with many small users, we have:

$$\lim_{\beta \rightarrow \infty} \frac{1}{\beta} R_{1,\beta} = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} R_{1,\beta}^* = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} R_{2,\beta}^* = \lim_{\beta \rightarrow \infty} \frac{1}{\beta} R_{ub,\beta}$$

Proof

As we noted in Section III, our performance upper bound R_{ub} and R_1 are the same as the upper bound performance and optimal static pricing performance, respectively, as defined in [11]. According to Theorem 7 in [11],

$$R_{ub,\beta} = \beta R_{ub}$$

$$\lim_{\beta \rightarrow \infty} \frac{1}{\beta} R_{1,\beta} \geq R_{ub}$$

Additionally, according to our Proposition 3, we have

$$R_{1,\beta} \leq R_{1,\beta}^* \leq R_{2,\beta}^* \leq R_{ub,\beta}$$

Therefore, the result follows. *Q.E.D.*

V. COMPUTATIONAL METHODS

In this section, we address the methods to compute the various revenue performances defined in Section III-E.

A. Greedy optimal pricing

With our static pricing model, if a trivial greedy CAC policy is assumed, we can compute the corresponding call blocking probabilities analytically using the method in [16]. Then we can apply a multi-dimensional optimization technique to maximize the performance objective expressed by equation (1) with respect to the price vector u . Thus, the resulting optimal price vector is the greedy optimal pricing, $u_{\pi^g}^*$.

B. Optimal dynamic CAC

Per our system state definition in Section III-C, let $s(t) := (n_1(t), \dots, n_I(t), \omega(t))$ be the state of the system at time t . The state transitions and control selection take place at discrete times, but the time from one transition to the next is random. A CAC policy can therefore be modeled as the solution to a semi-Markov decision process (SMDP) problem, see [1]–[3]. We assume a deterministic CAC policy. Therefore, an admission control policy can be defined by a mapping $\mathbf{f} = (f_1, \dots, f_I)$, where $f_i \rightarrow \{0, 1\}$, $i = 1, \dots, I$, and $f_i(s)$ takes the value 0 or 1 when an arriving class- i connection request is rejected or accepted, respectively. We model such a policy as the solution to a continuous-time average reward SMDP problem, and solve it using the dynamic programming (DP) methods, see [17], [18]. The details of this approach are omitted for space reasons.

C. Joint optimal pricing and CAC

It is challenging to find the joint optimal pricing and CAC (i.e. $u_{\pi^*j}^*$ and π^*j) analytically. This is because it is generally difficult to express an optimal CAC policy and the corresponding optimal price vector in closed forms. For the cases where the number of traffic classes I is small, e.g. $I \leq 2$, we can solve such a problem through a brute-force discrete approximation. Specifically, we can first generate a price grid that covers the entire price range $[0, u_{i,max}]$, $i \in \mathcal{I}$, with a fine granularity. Next, we can simulate the optimal CAC policy for each price vector via the DP method. Subsequently, the price vector and the CAC policy corresponding to the resulting maximal performance point are the joint optimal pricing and CAC, namely, $u_{\pi^*j}^*$ and π^*j .

D. A performance upper bound

For the performance upper bound formulated in Section III-E, the values of R_{ub} and u_{ub}^* can in general be solved numerically via a multi-dimensional constrained optimization technique. When the arrival function is differentiable (a condition satisfied by the linear arrival function expressed by (14) in Section VI), this is also a standard optimization problem that can be solved using Lagrange multipliers. Writing the Lagrangian function as

$$Q = \sum_{i=1}^I \lambda_i(u_i) u_i - p \left(\sum_{i=1}^I \frac{\lambda_i(u_i) x_i}{\mu_i} - C \right) \quad (12)$$

where p is the Lagrange multiplier.

Set $\frac{\partial Q}{\partial u_i} = 0$, then we have:

$$u_{i,ub}^* = \frac{p x_i}{\mu_i} - \frac{\lambda_i(u_i)}{\lambda_i'(u_i)} \quad (13)$$

For any given arrival function, $\lambda_i(u_i)$ and $\lambda_i'(u_i)$ are known, so that the Lagrange multiplier p can be solved through inserting (13) into the constraint function (8). Subsequently, $u_{i,ub}^*$ and R_{ub} can be readily computed via (13) and (7), respectively.

VI. NUMERICAL EXAMPLES

In this section, we present the results of our numerical studies obtained using the methods described in Section V. We first define a base system, which resembles a small network, and we then scale the base system to evaluate the performance of larger networks as a function of the scaling factor.

Specifically, for the base system, we assume the total number of traffic classes $I = 2$, and the link capacity is $C = 10$. We also assume the bandwidth requirements of class-1 and class-2 traffic are $x_1 = 6$ and $x_2 = 1$, respectively, and their service rates are $\mu_1 = 0.167$ and $\mu_2 = 0.0167$. Additionally, we assume that the connection arrival rate λ_i , $i = 1, 2$, is a linear function of the price u_i charged for class- i calls, as expressed in (14). This model satisfies our *continuous* and *strictly decreasing* assumptions for traffic arrival functions as stated in Section III-B, and we set $k_{1,1} = 2$, $k_{1,2} = 1.6$, $k_{2,1} = 2$ and $k_{2,2} = 1.6$.

$$\lambda_i = \lambda_i(u_i) = k_{i,1} - k_{i,2} u_i, \quad u_i \in \left[0, \frac{k_{i,1}}{k_{i,2}} \right], \quad i \in \mathcal{I} \quad (14)$$

We then scale the network capacity and the traffic load simultaneously using the method specified in Definition 1 with a scaling factor β , for $\beta = 1, 2, \dots, 10$, and evaluate the respective performance of $R_{1,\beta}$ and $R_{1,\beta}^*$. To obtain $R_{1,\beta}$ and $R_{1,\beta}^*$, we first identify the corresponding price vectors, $u_{\pi^*j}^*$, using the corresponding methods in Section V and the optimal dynamic CAC policy. Subsequently, we simulate over 3×10^6 test traffic events (i.e., call arrivals and departures) to generate the result of $R_{1,\beta}$ and $R_{1,\beta}^*$ of one test. We repeat each test ten times and use the mean values as the simulated performance results. The standard deviations for the simulated revenue performance results are less than 7.3×10^{-4} . We then plot the performance gap, defined by $\frac{R_{1,\beta}^* - R_{1,\beta}}{R_{1,\beta}^*} \times 100\%$, as a function of β , with the solid curve in Figure 2.

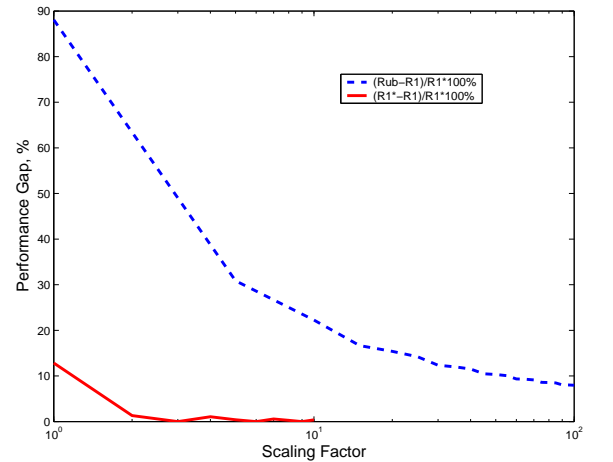


Fig. 2. Performance Gaps between $R_{1,\beta}^*$ and $R_{1,\beta}$, and between $R_{ub,\beta}$ and $R_{1,\beta}$

Next, we examine the asymptotic performances of the scaled

large networks. We compute $R_{1,\beta}$ and $R_{ub,\beta}$ analytically for the base network (i.e., $\beta = 1$) and the scaled networks using β in the range of 5 to 100 with an increment of 5. We then plot the performance gap, defined by $\frac{R_{ub,\beta} - R_{1,\beta}}{R_{1,\beta}} \times 100\%$, as a function of β , with the dashed curve in Figure 2.

A. Discussions

Despite the use of the greedy optimal pricing, an additional optimal dynamic CAC policy can further improve the revenue performance when the network is small. As shown by the solid curve in Figure 2, such a performance gain is about 13 percent for the case of $\beta = 1$. However, such a gain due to the optimal dynamic CAC dissolves when the network grows larger, as shown in the cases of $\beta \geq 2$ in the solid curve in Figure 2. This is also supported by the facts: (a) the performance upper bound, R_{ub} , asymptotically approaches the revenue performance resulting from the greedy optimal pricing and the greedy CAC, R_1 , as shown by the dashed curve in Figure 2 and Proposition 4, and (b) $R_1 \leq R_1^* \leq R_{ub}$. Additionally, the combination of the facts that $R_1 \leq R_2^* \leq R_{ub}$ and R_{ub} asymptotically approaches to R_1 show that the joint optimal pricing and CAC (i.e., the pricing and policy pair for R_2^*) do not bring significant performance benefits in the limiting regime relative to the use of the greedy optimal pricing and the greedy CAC. Consequently, the above results indicate that the greedy optimal pricing plays a significant role in the revenue performance optimization in networks of both small and large sizes.

VII. CONCLUSIONS AND FUTURE WORKS

We have formulated a framework to view pricing and call admission control as two stages of a call level network resource allocation process. Within such a context, we have studied the relative roles of an optimal pricing policy and an optimal admission control policy in the revenue performance of multi-class loss networks. We have shown that both an optimal pricing and an optimal CAC can improve the performances, and the greedy optimal pricing plays a significant role. Moreover, in large networks with many small users, the performance resulting from the greedy optimal pricing and the greedy CAC policy approaches the upper bound performance asymptotically, and the pricing resulting from the joint pricing and CAC optimization does not provide an additional performance gain. A potential direction for the future research is to further investigate the relative performance benefits of various pricing and CAC in networks away from the limiting regime. Extensions of the analysis to the long-range dependent traffic (see, [20]), multi-link networks and demand functions allowing price substitution would also be valuable.

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