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Dynamic Channel Assignment in Shotgun Cellular Systems

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Abstract

This paper analyzes dynamic channel assignment in cellular systems where the base stations are placed at random. Using a simple dynamic channel assignment algorithm, random systems are compared to an ideal hexagonal systems. Under log-normal shadow fading the random system has capacity within 1dB of the hexagonal system. This paper looks at both the uplink and downlink and finds that performance is similar to previous downlink analysis. Thus, at a modest cost in performance, deployment effort is greatly reduced.

1. Introduction

Current cellular and PCS systems, with their small cell sizes, depart significantly from the ideal hexagonal layout due to terrain variations, difficulties in site acquisition, and space variations in mobile station density. In quickly deployed ad hoc military or emergency communication systems, little or no planning may be possible. Distributed campus wireless LANs may add communication elements in a distributed fashion with little coordination between departments. Indoor environments have high variability in signal levels and base stations are placed under architectural constraints. As an extreme we can consider the base station placement in these systems as “random”. Such random systems we denote as *shotgun* cellular systems.

Previous work developed and analyzed the shotgun concept [2,3] and showed that the shotgun system with random fixed channel assignment (FCA) differs from an ideal hexagonal system by 4 dB in signal levels and 2dB in spectrum efficiency. While the shotgun system minimizes planning, non-random channel assignment has the potential to improve system quality and capacity. Unfortunately, finding an optimal FCA in a shotgun system is not straightforward and posed as an optimization task is computationally complex. In this paper we show that dynamic channel assignment (DCA) [6,8] is ideal for the shotgun system and improves the shotgun performance to nearly that of an ideal hexagonal system. Furthermore, the analysis provides insights into the gains possible due to DCA in realistic, non-ideal systems.

The paper first develops the system model, discusses DCA, and describes simulation experiments using DCA and FCA for shotgun and standard hexagonal systems.

2. Model:

In the shotgun system, base stations are distributed as a 2-D Poisson process where the average density of stations is λ and the probability of a base station in a small area dA is λdA. All base stations have identical transmit power, antenna gains, etc., and the path loss is an inverse power law with path loss exponent ε. All antennas are omni-directional. Background and thermal noise are assumed zero. Shadow fading (aka slow fading) is modeled as independent log-normally distributed multiplicative noise, Ψ, on the signal strength received from each base station. It is well modeled by a log-normal density [4]:

$$p(\psi) = \frac{1}{\psi\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\ln\psi}{\sigma}\right)^2}, \tag{1}$$

so that the fading factor has mean 1 and one standard deviation includes from $1/\sigma$ to σ.

In this initial study, users are uniformly distributed, do not move while communicating, and remain on the same channel throughout the call. N orthogonal channels are available and each channel carries one user. Being orthogonal, adjacent channel interference is ignored and only co-channel interference is considered. Calls are generated via a Poisson process with exponential call holding time. Call set-up is instantaneous with no distinction between user originating or user terminating calls. Channel quality is measured as the uplink and downlink carrier to interference ratio (C/I):

$$C/I = d_{\mathrm{S}}^{-\varepsilon} \Big/ \left(\sum_i d_{\mathrm{i}}^{-\varepsilon}\right), \tag{2}$$

where d_{S} is the distance to the signal base station and $\{d_i\}$ is the set of distances to co-channel interfering base stations (on the downlink) or mobiles (on the uplink).

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3. Dynamic Channel Assignment:

Many DCA schemes are known [6] and the goal of this paper is not to find the best of these schemes. The goal instead is to show representative performance with DCA as compared to FCA algorithms.

The *uniform* FCA scheme chooses a reuse number *R*, divides the channels into *R* disjoint equal size sets, and permanently assigns each base station one channel set. A base station blocks a call whenever *N*/*R* calls are already active. The number of channel groups, *R*, depends on the radio environment and base station layout. In the regular hexagonal layout, optimal assignments are well known and easily computed [7]. For a uniform distribution of users and a regular hexagonal grid of base stations, the uniform FCA is optimal (among FCA). For non-uniform distribution of either users or base stations the traffic load varies from cell to cell and a non-uniform FCA is optimal.

In the random shotgun layout, the best FCA is, in general, non-uniform because the cells cover different areas and have different loads. Finding an assignment that maximize system capacity requires solving a complex optimization [3,5,6]. This is true even if each cell's traffic load is the same. To avoid this optimization, and to maximize the total capacity of the system we look at DCA.

We consider the most general form of DCA. At each call arrival, the base station can choose to assign any channel it is not using in its cell to the new user subject to some criteria. The first criteria is that the uplink and downlink for the chosen channel must meet the required C/I for the radio. The second criteria is that the addition of this new call cannot cause any existing radio link to drop below the required C/I. Within this criteria, several channels may be available which could be ranked by the carrier or interference signal strength or the C/I. A number of algorithms are available (e.g. [1]).

We choose a simple algorithm. The channels are numbered from 1 to *N*. The smallest numbered channel that meets the criterion is assigned. Note that only the base station with the strongest signal, call it *b*, will ever be used by a mobile. The C/I on a channel is maximized by using the strongest available base station for the signal. If this channel is not occupied at *b* then *b*, by definition, is that base station. If this channel is occupied at *b*, than using any other (weaker) base station yields a downlink C/I < 1. Note also, that under shadow fading the strongest base station is not necessarily the closest.

Calls are only blocked if no channel that meets the required C/I is found. To be clear, this means that base stations have sufficient radios to carry all acceptable calls. In FCA the required number of radios is fixed and known. A base station will block calls whenever *N*/*R* channels are occupied. In DCA, calls are blocked when co-channel interference is unacceptable on every possible channel and not when the base station runs out of assigned channels.

To compare different channel assignments and different base station layouts we look at a measure of spectrum efficiency which we call the *effective reuse number.* In a uniform FCA, the effective reuse number is simply *R*. In DCA the effective reuse number is the total number of system channels, *N*, divided by the effective number of channels per cell. Given *E* Erlangs per cell, the effective number of channels per cell is the number of channels per cell that uniform FCA would require in order to have the same blocking as DCA. A smaller reuse number is better with a minimum of 1 when every channel is used in every cell.

4. Simulation:

To evaluate the dynamic channel assignment for different channel and base station layouts, we use a simulated cellular system. This section describes the details of this simulation. The reader may wish to skip to the next section on results and refer back to this section if needed. The simulation proceeds as follows:

1. A set of base stations is placed on a two dimensional plane.
2. The Erlangs per cell is chosen and a sequence of discrete call arrival and departure events is generated.
3. For each call arrival: the new user is placed on the two dimensional plane; every channel is evaluated; if a channel meets the criteria, then it is assigned to the call, otherwise the call is blocked.
4. For each call departure, the call is simply removed.

Each of these steps is described in detail. A number of base stations, *B*, is chosen. For the hexagonal layout, base stations are placed in a hexagonal grid and the *B* base stations closest the to the origin are chosen. For the shotgun layout, the base stations are placed according to a two-dimensional Poisson point process. In this process, if *K* is the random variable for the number of base stations in area *a*:

$$\mathrm{Prob}\{K = k|a\} = \frac{(\lambda a)^k}{k!}e^{-\lambda a}. \tag{3}$$

Let d_k be the distance from an arbitrary point to the *k*th nearest base station to that point.

$$p(d_k = r_k | d_{k-1} = r_{k-1}) = \lambda\pi 2 e^{-\lambda\pi(r_k^2 - r_{k-1}^2)} r_k. \tag{4}$$

Letting $d_0 = 0$ (but placing no base station at the origin), we can iteratively generate the *B* nearest base stations to the origin.

The Poisson call process has *E* Erlangs per cell.

The total traffic in the system is $E_T = EB$. In this simulation time is not important, and we only are concerned with the sequence of call and departure events. We note:

$$E_T = \lambda\mu , \tag{5}$$

where λ is the arrival rate and μ is the call holding time. The rate call arrival rate is fixed at λ and the call departure rate depends on the number of calls in progress, n_u. Given n_u, the rate of call departures is n_u/μ. The probability that the next event is a call arrival is given by:

$$p^{n_u}_{\text{arrival}} = \frac{\lambda}{\lambda + n_u/\mu} = \frac{E_T}{E_T + n_u}. \tag{6}$$

In this sequence, we ignore statistics from the first $5EB$ call arrivals to allow the system reach steady state. After this initial period blocking statistics on the next A arrivals is recorded.

For call arrivals, the user is first placed. For the hexagonal system the user is randomly and uniformly placed within one of the cellular hexes. For the random system, we define a circle centered on the origin with radius equal to the distance from the origin to the furthest base station. The user is placed with a uniform distribution within this circle.

The signal power from the user to each base station is defined via:

$$p_{ij} = \Psi_{ij} K d_{ij}^{-\varepsilon} \tag{7}$$

where p_{ij} is the power of the signal from user i to base station j, K is a constant containing the transmit power, antenna gains, etc., d_{ij} is the distance from user i to base station j, and Ψ_{ij} is a the log normal shadow random variable distributed as in (1). Since we are only concerned with power ratios we let $K = 1$ and use p_{ij} equally for both up and down link powers. If c_i is the channel of user i, n_u is the number of users, and b_i is the base station with the strongest signal to user i, then the carrier to interference ratio for channel c at user ν follows from (2):

$$\left(\frac{C}{I}\right)^{c,\nu}_{\text{downlink}} = \frac{p_{\nu b_\nu}}{\sum\limits_{\{i \mid c_i = c,\, i \neq \nu\}} p_{\nu b_i}} \tag{8}$$

$$\left(\frac{C}{I}\right)^{c,\nu}_{\text{uplink}} = \frac{p_{\nu b_\nu}}{\sum\limits_{\{i \mid c_i = c,\, i \neq \nu\}} p_{i b_\nu}} \tag{9}$$

Channel c meets all C/I criteria if (8) and (9) are above a required C/I threshold, T, for every ν (i.e. for the new call and every existing call). If no channels meet the criterion, the call is blocked, otherwise the smallest numbered c that meets the criterion is assigned

The blocking probability, p_B, is simply the ratio of the number of blocked arrivals over the total number of arrivals.

Table 1: Simulation Parameters

Parameter	Symbol	Default Value
Number of Channels	N	200
Number of base stations	B	100
Erlangs per base station	E	(dynamic)
Pathloss exponent	ε	4
Shadow fading std. dev.	σ	10dB
Required C/I	T	10dB
Arrivals simulated	A	100,000

Given E and p_B, the effective number of channels n_e is found by numerically solving for n in the Erlang B formula:

$$p_B = \frac{E^n/n!}{\sum\limits_{i=1}^{n} \frac{E^i}{i!}} \tag{10}$$

Since n is an integer n_e is an interpolation between the integers above and below the solution to (10). The effective number of channels depends (weakly) on E so an E near a typical operating point is chosen by simulating at different E until p_B is between 1% and 3%. The scenario is repeated with different random number seeds and the effective number of channels computed.

Table 1 gives a parameter summary. The number of channels and base stations is chosen as large as could be reasonably simulated. The channel parameters are typical values from [7]. The arrivals simulated balances getting useful statistics in one run against the initial arrivals ignored while reaching steady state (e.g. at 100 Erlangs per cell, the initial period is $5EB$ = 50,000 arrivals).

5. Results:

This paper introduces two new features to shotgun cellular systems. The first is combined uplink and downlink analysis, and the second is dynamic channel assignment. We look at each of these separately for the four combinations of hexagonal vs. shotgun layout and shadow fading vs. no shadow fading ($\sigma = 0$).

Previous papers on shotgun cellular systems were able to analytically derive performance in the downlink direction [2,3]. In other words, only (8) was used as a criterion. One question we seek to answer is whether the downlink performance differs significantly from applying (9) or from applying (8) and (9) jointly. To see the difference we plot the effective number of channels for the four cases. The results are shown in Figure 1.

These results indicate that the uplink constraint is 5% more optimistic than with the downlink constraint. While the up and down link constraints separately produce similar results, the combined constraint is 5-12% more pessimistic than with the downlink constraint. This indicates the blocking due to downlink and uplink

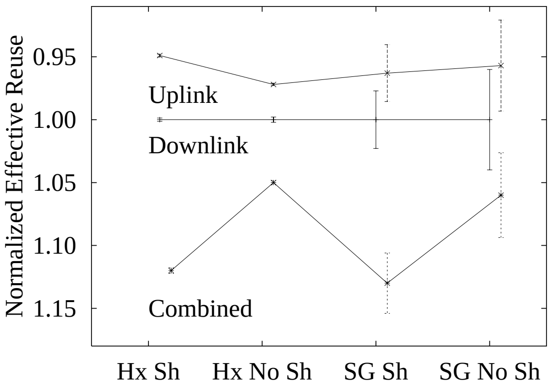

Figure 1: The effective reuse number for hexagonal (Hx) and shotgun (SG) base station layouts with shadow fading (Sh) and with no shadow fading (No Sh). Average and s.d. of 10 runs. The lines are for comparison purpose. Inverted *y* scale.

constraints do not always correspond. In any case, the differences are small so that the downlink analysis does provide a bound on performance with small error.

The relative ordering between uplink, downlink and combined constraint results was always the same. The error bars indicate the performance variance due to different instances of random cellular systems. As indicated by the hex layout results, repeated experiments for a particular base station layout produced little variation.

We next turn to the performance of FCA and DCA for hexagonal and shotgun layouts with and without shadowing. Only results with the combined uplink and downlink constraints are plotted. For FCA the effective reuse number was found by applying a load of $E = 9.83$ Erlangs (2% blocking for a trunk group of 16 channels) and 16 channels per cell and finding the smallest reuse number, R, that yielded less than 4% blocking. The results are similar for the downlink only analysis of [2,3].

Results are shown in Figure 2. We make three observations. First, as is well documented, DCA provides significant improvements in channel utilization; 2dB in the hexagonal layout, and 4dB in the shotgun layout. Second, shadow fading improves the performance of the shotgun layout (+1.5dB) while degrading the performance of the hexagonal layout (–1.5dB). This corresponds with earlier studies where it was noted all layouts converged to the same performance with increasing variance in the log-normal shadow fading. Third, while the hexagonal layout always has higher capacity than the shotgun layout, under dynamic channel assignment with shadow fading the difference is less than 1dB.

6. Conclusion

This paper showed two results. The first is that downlink analysis that ignores the uplink provides a good guide to performance. This is useful since the downlink often yields to analysis. The main result indicates that a shotgun base station placement augmented by the self organizing DCA can lead to near ideal performance with little or no planning. The effective reuse number is a very efficient $R = 3$. Further work is building on these initial results including user mobility, and handoff analysis. Exploring alternative DCA algorithms may yield further improvements.

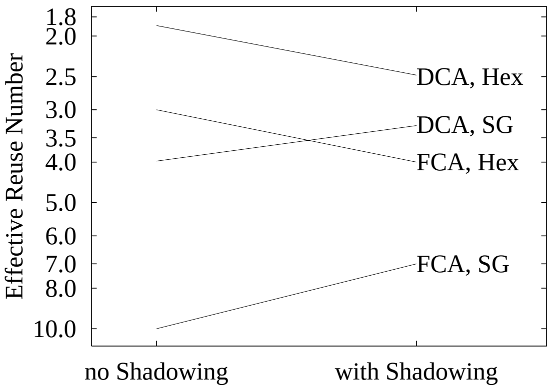

Figure 2: Effective reuse number with and without log normal shadow fading, for dynamic channel assignment (DCA) and fixed channel assignment (FCA) for shotgun (SG) and hexagonal (Hex) base station layouts.

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