Spectrally Efficient Multiuser Space–Time Modulation

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Abstract — **To achieve full transmit diversity for a single user with N_T antennas the coherence time of the channel has to be at least N_T time-slots long. The same minimum number of slots (or dimensions) guarantees full transmit diversity for every user in a "narrowband" multiuser system in which all users communicate simultaneously and occupy the same bandwidth. However, with an increase in the number of users, an increasing signal-to-noise ratio (SNR) is required of each user to achieve the same error probability as in the single-user case. We showed earlier that this SNR penalty can be alleviated, and even eliminated for sufficiently high SNR, with only a marginal increase of the bandwidth and a judicious design of "spreading matrices" that spread a single-user space–time constellation or (block-) code. In other words, such signaling exploits multiuser diversity and ensures that each user enjoys single-user like performance for sufficiently high SNR. For instance, for N_T-transmit antennas per user and a single-user space–time constellation that requires only the minimum time-slots N_T, no more than $N_T + 1$ slots are required for the common signal space of any number of users. We present here simplified design criteria to obtain the spreading sequences/matrices. We also show that the optimum multiuser receiver can be implemented by the (generalized) sphere decoder of Damen *et al.***

I. Introduction

The modulation method we proposed in [1] leverages existing single-user space–time constellations and uses spreading matrices to alleviate–and to even eliminate asymptotically (for high SNR)–the SNR penalty of the narrowband multiuser channel. At the same time only a small increase in signal space dimensions (time-slots) is required, leading to spectrally efficient multiuser communications.

In particular, the single-user $D_{SU} \times N_T$ space–time symbol matrices $\{\mathbf{B}_m\}_{m=1}^{M}$ of each of the K users are multiplied by a $D \times D_{SU}$ low-dimensional spreading matrix $\mathbf{F}_k$ with orthonormal columns. Thus in place of transmitting $\mathbf{B}_m$ in D_{SU} dimensions (time-slots) the k^{th} user transmits $\mathbf{F}_k\mathbf{B}_m$ in D dimensions. The spreading matrix's row dimension D is just greater than D_{SU}, the (time)-dimension of the single-user space–time constellation, so as to achieve high aggregate spectral efficiencies while markedly improving performance relative to the narrowband channel. In our examples, D typically equals $D_{SU} + 1$, independently of the number of users. For one transmit antenna, the "spreading matrices" become vectors (sequences) and the system model resembles that of conventional CDMA. However, in the typical CDMA scheme $D \gg K$ whereas we propose $D = 2$ (for $N_T = 1$) and judiciously design signals to achieve asymptotic single-user like performance.[1] Thus for $N_T = 1$ the new signaling scheme may be thought of as *enlightened* or low-rank CDMA signaling in that the spread factor is just two.

II. Signal Design Criterion

We obtain the spreading matrices $\{\mathbf{F}_k\}_{k=1}^{K}$ from numerically minimizing

$$c = \max_{1\le k\le K} \sum_{\substack{l=1\\ l\neq k}}^{K} \left|\mathbf{F}_k\mathbf{F}_k^{\dagger} + \mathbf{F}_l\mathbf{F}_l^{\dagger}\right|^{-N_R}, \tag{1}$$

under the constraint that each matrix $\mathbf{F}_k$ has orthonormal columns. This criterion is inspired by a rigorous error probability analysis, which was then simplified to obtain a criterion that is independent of the specific single-user space–time constellation. It ensures in a "fair" way that all matrices $[\mathbf{F}_l, \mathbf{F}_k]$ $(l \neq k)$ have full rank. This in turn ensures that any error event involving two or more users has a diversity order of at least $DN_R > N_T N_R$ and thus for sufficiently high SNR these multiuser error events do not contribute to the k^{th} user's error probability. In other words, the scheme exploits multiuser diversity and ensures asymptotic single-user like performance.

III. Sphere Decoding

The optimum multiuser receiver has a complexity of M^K, i.e., it is exponential in the number of users. However, for space–time codes for which

$$\mathbf{B}_m\mathbf{h}_{kn} = \widetilde{\mathbf{H}}_{kn}\mathbf{u}_m, \tag{2}$$

where $\widetilde{\mathbf{H}}_{kn}$ is an equivalent channel/code matrix and $\mathbf{u}$ is a vector of independent PAM/QAM or PSK symbols, the sphere decoder of [2] can be applied to obtain a solution in polynomial time. Space–time codes that fulfill (2) are, for example, the algebraic space–time codes of [3] and the orthogonal designs [4], if PAM or QAM symbols are employed. For instance, the equivalent channel/code matrix for the Alamouti scheme corresponding to the k^{th} user's n^{th} receive antenna is

$$\widetilde{\mathbf{H}}_{kn} = \begin{bmatrix} h_{1kn} & jh_{1kn} & h_{2kn} & jh_{2kn} \\ h_{2kn} & -jh_{2kn} & -h_{1kn} & jh_{1kn} \end{bmatrix}, \tag{3}$$

where h_{mkn} is the k^{th} user's fading coefficient from transmit antenna m to receive antenna n. The length-4 vector $\mathbf{u}_m$ contains the real and imaginary parts of the two complex QAM data symbols.

References

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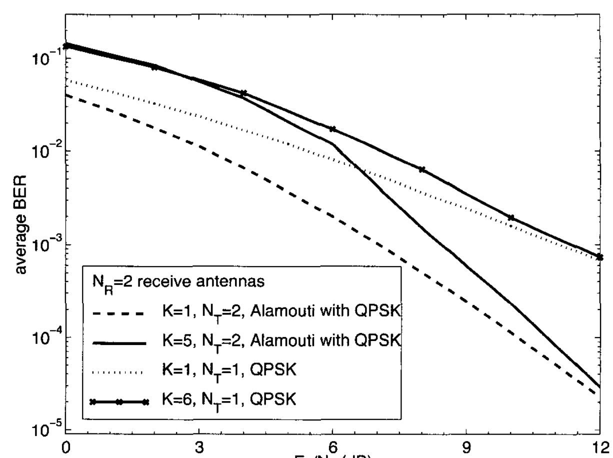

Fig. 1. The figure depicts the performance of $K = 5$ users, each employing two transmit antennas and the Alamouti scheme with QPSK symbols (SE $= 6\frac{2}{3}$ bits/dimension). For comparison, the performance of $K = 6$ users with one transmit antenna each is given (QPSK, SE $= 6$ bits/dimensions). Both multiuser schemes use the minimum $D = N_T + 1$ and approach the performance of the corresponding single-user, as predicted.

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[1] By an "asymptotic single-user like" performance we mean that for high SNR the upper bound on the multiuser bit error rate (BER) converges to the upper bound on the single-user BER.

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