

A Stochastic Algorithm for the Joint Optimization of Power Control and Multiuser Receivers *

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Abstract

Uplink communication in a cellular radio network is considered where the base station in each cell employs a linear or decision feedback (DF) multiuser receiver. The problem of minimizing the total transmit power over the receiver filter coefficients is considered under the constraint that all the users of the network achieve their signal-to-interference ratio (SIR) objectives. A distributed, stochastic algorithm is obtained that converges to the jointly optimal pair of power allocations and maximum SIR decision feedback receivers.

Index Terms: Power control, cellular networks, stochastic approximation, blind receivers.

1 Introduction and System Model

The problem of interest in this paper is that of minimizing the total transmit power under the constraint that all the users of the network achieve their Quality-of-Service (QoS) objective in terms of signal-to-interference ratio (SIR) on the uplink, i. e., at a base station in a cellular system. The minimization of total power required to achieve the SIR objectives is done over all linear and decision feedback (DF) multiuser receivers (both feedforward and feedback filter coefficients in the DF case) to yield the most power efficient solution when the receivers are constrained to be of the linear or decision feedback type. This problem was considered in [1]. A similar problem restricted to the case of linear multiuser receivers was studied in [2]. In this paper, we follow closely the model and problem formulation given in [1].

It is assumed that the decision feedback at each base station involves only decisions made for in-cell users. With complete knowledge of system parameters, the joint multiuser receiver-power optimization essentially consists of alternating between an optimum power computation step and a maximum SIR (MSIR) linear or decision feedback (non-linear) receiver computation step. Although decentralized and stochastic versions of this iterative optimization algorithm have been proposed in [1] [2], no convergence analysis was presented. In this paper, a different iterative algorithm is introduced for directly

*This work was supported in part by NSF grants CCR-9814996 and ANI-9725778.

computing the jointly optimum powers that is decoupled from the computation of the optimum (MSIR or MSIR-DF) receivers. In particular, a distributed, stochastic version of our power control algorithm is shown to converge to the (jointly) optimum powers.

We consider a cellular network in which there are B base stations with K_j users assigned to base j . Base j employs a set of N_j matched filters matched to orthonormal basis functions of a carefully chosen signal subspace that must include the signals received from the users within its cell. While the transmissions of the in-cell mobiles are assumed to be received at the base station symbol-synchronously, those of out-of-cell users are not. Base j is assumed to have knowledge of the common timing of only in-cell users.

The complex baseband discrete-time model for the N_j matched filter outputs at base j can be expressed as

$$\mathbf{y}_j = \sum_{i=1}^{K_j} \sqrt{w_{ij}g_{ijj}} \mathbf{s}_{ijj} x_{ij} + \sum_{l \neq j}^B \sum_{i=1}^{K_l} \sqrt{w_{il}g_{ilj}} (\mathbf{s}_{ilj}^- x_{il}^- + \mathbf{s}_{ilj}^+ x_{il}^+) + \mathbf{n}_j, \quad (1)$$

where w_{il} and x_{il} denote the transmit power and the transmitted symbol, respectively, of user i of base l . The channel gain of the i^{th} user of base j to base l is denoted by g_{ilj} . The (possibly complex-valued) vector \mathbf{s}_{ijj} denotes the vector representation (the ‘‘signature sequence’’) of the signal of user i of base j with respect to the orthonormal basis functions employed at base j . The vectors \mathbf{s}_{ilj}^- and \mathbf{s}_{ilj}^+ denote the vector representations of the segments of the signals associated with the two symbols of user i of base l that overlap with the signals of base j . \mathbf{n}_j is an N_j -dimensional zero-mean, circularly symmetric, Gaussian random vector with a covariance matrix equal to $\sigma_j^2 \mathbf{I}$. Define the signal matrix of in-cell users as $\mathbf{S}_{jj} = [\mathbf{s}_{1jj} \mathbf{s}_{2jj} \cdots \mathbf{s}_{K_j jj}]$. Also define the signal matrices of out-of-cell users $\mathbf{S}_{lj}^+ = [\mathbf{s}_{1lj}^+ \mathbf{s}_{2lj}^+ \cdots \mathbf{s}_{K_l lj}^+]$ and $\mathbf{S}_{lj}^- = [\mathbf{s}_{1lj}^- \mathbf{s}_{2lj}^- \cdots \mathbf{s}_{K_l lj}^-]$, the gain matrices $\mathbf{G}_{lj} = \text{diag} \{g_{1lj}, g_{2lj}, \cdots, g_{K_l lj}\}$, and power matrices $\mathbf{W}_l = \text{diag} \{w_{1l}, w_{2l}, \cdots, w_{K_l l}\}$.

2 Multiuser Receivers

Each base station uses either a linear or a decision feedback receiver to decode the information transmitted by users in its own cell. In the decision feedback case we assume that the users in each cell are numbered according to the order in which they are decoded. The soft-output of the linear or decision feedback receiver for user i of base j can be written as

$$v_{ij} = \mathbf{f}_{ij}^\dagger \left(\mathbf{y}_j - \sum_{k=1}^{i-1} \mathbf{b}_{kij} \sqrt{w_{kj}g_{kjj}} \hat{x}_{kj} \right). \quad (2)$$

‘‘ \dagger ’’ denotes conjugate-transpose and \mathbf{f}_{ij} is the feedforward filter. The feedback filters $\mathbf{b}_{kij} = \mathbf{0}$ for linear receivers (no decision feedback) and for decision feedback receivers, we let $\mathbf{b}_{kij} = \mathbf{s}_{kjj}$ for $k = 1, \cdots, i-1$, without loss of generality [3]. \hat{x}_{kj} denotes the detected (in uncoded transmission) or decoded (in the coded case) symbols of ‘‘past’’ users $1, \cdots, i-1$ relative to user i of base j . Implicit in (2) is the assumption that base j has knowledge of the codes, signals, transmit powers and channel gains of the users within its own cell. We do not however assume that it has knowledge of these parameters for out-of-cell users. Single-cell receivers, i. e., receivers that require only in-cell information, were studied in [1] as were the MSIR linear and decision feedback receivers.

In this paper we focus on receivers designed for the SIR criterion. First define matrices $\{\mathbf{Q}_{lj}\}$ for $l, j \in \{1, 2, \cdots, B\}$, with $Q_{lj}(i, k)$ denoting the $(i, k)^{\text{th}}$ element of \mathbf{Q}_{lj} , as follows:

the matrices \mathbf{Q}_{jj} are defined for $j = 1, \dots, B$ as

$$Q_{jj}(i, k) = \begin{cases} |\mathbf{f}_{ij}^\dagger(\mathbf{s}_{kjj} - \mathbf{b}_{kij})|^2 & \text{if } k < i \\ |\mathbf{f}_{ij}^\dagger \mathbf{s}_{kjj}|^2 & \text{if } k \geq i \end{cases} \quad (3)$$

for $i, k \in \{1, \dots, K_j\}$ and the matrices \mathbf{Q}_{lj} for $j, l \in \{1, 2, \dots, B\}$ with $j \neq l$ are defined as $Q_{lj}(i, k) = |\mathbf{f}_{ij}^\dagger \mathbf{s}_{klj}^-|^2 + |\mathbf{f}_{ij}^\dagger \mathbf{s}_{klj}^+|^2$ for $i \in \{1, \dots, K_j\}$ and $k \in \{1, \dots, K_l\}$.

Assuming perfect feedback in the case of decision feedback receivers, the SIR in the soft output v_{ij} of user i in base j can be expressed as

$$\gamma_{ij} = \frac{w_{ij} g_{ijj} |\mathbf{f}_{ij}^\dagger \mathbf{s}_{ijj}|^2}{\sum_{l=1}^B I_{ijl} - w_{ij} g_{ijj} |\mathbf{f}_{ij}^\dagger \mathbf{s}_{ijj}|^2 + \sigma_j^2 \mathbf{f}_{ij}^\dagger \mathbf{f}_{ij}}, \quad (4)$$

where $I_{ijl} = \sum_{k=1}^{K_l} Q_{lj}(i, k) w_{kl} g_{klj}$.

The MSIR receivers, with and without decision feedback (that are also known, with suitable scaling, to minimize mean-squared error) are given by the vectors \mathbf{f}_{ij}^{msir} that satisfy the equations (cf. [4], [3])

$$\mathbf{A}_{ij} \mathbf{f}_{ij}^{msir} = \mathbf{s}_{ijj} \quad i = 1, \dots, K_j, \quad j = 1, \dots, B, \quad (5)$$

with

$$\begin{aligned} \mathbf{A}_{ij} = & \left(\chi \sum_{k=1}^{i-1} g_{kjj} w_{kj} \mathbf{s}_{kjj} \mathbf{s}_{kjj}^\dagger + \sum_{k=i}^{K_j} g_{kjj} w_{kj} \mathbf{s}_{kjj} \mathbf{s}_{kjj}^\dagger \right. \\ & \left. + \sum_{l \neq j}^B \mathbf{S}_{lj}^- \mathbf{G}_{lj} \mathbf{W}_l (\mathbf{S}_{lj}^-)^\dagger + \sum_{l \neq j}^B \mathbf{S}_{lj}^+ \mathbf{G}_{lj} \mathbf{W}_l (\mathbf{S}_{lj}^+)^\dagger + \sigma_j^2 \mathbf{I} \right), \end{aligned} \quad (6)$$

where the indicator $\chi = 1$ for linear MSIR and $\chi = 0$ for MSIR-DF. The MSIR-DF receiver can have a markedly improved performance relative to the MSIR linear receiver in both the single- and multi-cell channels [3]. Implicit in this result is the assumption that feedback is perfect in the case of the DF receiver. As has been pointed out in [1], while decision feedback is not perfect in general, it is possible, by decoding the users in the decreasing order of their target SIR's, to mitigate the error propagation effects to a large extent [5]. In the case of equal target SIR's, it may be possible to change the problem slightly and set the target SIR's to be slightly different from each other while ensuring that the worst-case target SIR is equal to the required target SIR. The slight increase in the powers required to achieve these higher set of target SIR's may be a small price to offset the effects of error propagation and achieve better overall performance.

Note that in contrast to the single-cell receivers, the MSIR linear and DF receivers depend on the system parameters corresponding to all users including those outside the cell of interest. Since such parameters are not available at each base station, it is of interest to consider blind adaptive multiuser receivers for the cellular channel (cf. [1]).

3 Joint Receiver-Power Control Optimization

We now consider our central optimization problem - minimizing the minimum total power required to achieve the SIR requirements over all possible linear or decision feedback multiuser receivers.

3.1 Algorithms for Joint Optimization

The joint optimization of multiuser receivers and power control can be formulated (equivalently to the optimization in [1]) as:

$$\min_{\substack{\mathbf{f}_{ij} \in C^N \\ i=1, \dots, K_j \\ j=1, \dots, B}} \min_{\substack{\mathbf{w} > \mathbf{0} \\ \gamma_{ij}/\gamma_{ij+1} \geq \gamma_{ij}^*/\gamma_{ij+1}^*+1}} \sum_{j=1}^B \sum_{i=1}^{K_j} w_{ij}, \quad (7)$$

where $\mathbf{w} = [w_{11}, \dots, w_{K_1 1}, \dots, w_{1B}, \dots, w_{K_B B}]^T$ (note that the constraints $\gamma_{ij} \geq \gamma_{ij}^*$ are equivalent to $\frac{\gamma_{ij}}{\gamma_{ij+1}} \geq \frac{\gamma_{ij}^*}{\gamma_{ij+1}^*}$). The above problem can be solved in a synchronous (each base station updates powers and receivers for all its users at the same time), distributed manner by the following deterministic iteration (cf. [1]):

$$w_{ij}(n+1) = \frac{\gamma_{ij}^* \mathbf{f}_{ij}^\dagger(n) \mathbf{A}_{ij}(n) \mathbf{f}_{ij}(n)}{(\gamma_{ij}^* + 1) \mathbf{f}_{ij}^\dagger(n) \mathbf{s}_{ijj} \mathbf{s}_{ijj}^\dagger \mathbf{f}_{ij}(n) g_{ijj}}, \quad (8)$$

$$\mathbf{f}_{ij}(n+1) = (\mathbf{A}_{ij})^{-1}(n) \mathbf{s}_{ijj}, \quad (9)$$

Assuming the target SIRs are feasible (i. e., there exists some feasible power vector for which the SIR constraints can be met), it can be shown that the two-step distributed, synchronous, iterative algorithm (8, 9) converges to the *unique* optimum pair of receivers and power allocations for all users in the network. The proof of convergence for the linear receiver case was shown in [2], and that for the decision feedback receiver case is similar. We will consider here a different algorithm to compute the optimum powers and receivers that will be shown in the following section to be more suitable for a stochastic implementation.

We begin by noting that, given there exists some feasible power vector, the unique solution to the deterministic iteration in (8, 9) is a function only of the user codes, path gains and noise power. In particular, since for user powers \mathbf{w} , the SIR achieved with the MSIR receiver is given as $\mathbf{s}_{ijj}^\dagger (\mathbf{A}_{ij})^{-1} \mathbf{s}_{ijj} g_{ijj} w_{ij}$ for user i in cell j , the optimum powers \mathbf{w}^* , are such that each w_{ij}^* is given by the solution of the nonlinear equation

$$\mathbf{s}_{ijj}^\dagger (\mathbf{A}_{ij})^{-1} \mathbf{s}_{ijj} g_{ijj} w_{ij} = \frac{\gamma_{ij}^*}{1 + \gamma_{ij}^*}, \quad \text{and} \quad (10)$$

and the corresponding SIR maximizing MSIR receiver for the $(i, j)^{th}$ user is given in terms of the optimum powers by

$$\mathbf{f}_{ij}^* = (\mathbf{A}_{ij}^*)^{-1} \mathbf{s}_{ijj}. \quad (11)$$

With complete knowledge of system parameters, we can also devise the following deterministic power-receiver recursions for the $(i, j)^{th}$ user

$$w_{ij}(n+1) = w_{ij}(n) - a_n \left[\mathbf{s}_{ijj}^\dagger (\mathbf{A}_{ij})^{-1}(n) \mathbf{s}_{ijj} w_{ij}(n) g_{ijj} - \frac{\gamma_{ij}^*}{1 + \gamma_{ij}^*} \right], \quad (12)$$

$$\mathbf{f}_{ij}(n) = (\mathbf{A}_{ij})^{-1}(n) \mathbf{s}_{ijj}. \quad (13)$$

The above two-step joint optimization iterative procedure will be used in the next section to derive the corresponding distributed and stochastic algorithms.

4 Stochastic Joint Optimization

It was mentioned in [2] in the context of linear receivers that one could try to achieve the jointly optimum receiver-power solution by alternately running a blind MSIR receiver adaptation (with the powers fixed) until convergence is achieved, and a stochastic power adaptation (with the receivers fixed) until convergence is achieved. However, switching back and forth between a receiver adaptation and a power adaptation entails waiting at each step for convergence. This may not be feasible within a time frame where the path gains (and hence the optimum joint solution) remain the same. In a practical implementation, where convergence cannot be truly realized at any stage, analytical results that give appropriate stopping criteria are needed so as to guarantee the convergence of this procedure. Unfortunately, no such results are available at this time.

As another attempt at a blind, distributed stochastic joint optimization, the work in [1] suggests a two step stochastic approximation based strategy, one for the power computation and one for the filter computation, in each iteration, to mimic the deterministic iterative solution of (8, 9). Numerical simulations suggest that the combination of the two such coupled adaptations does not necessarily converge to the jointly optimal solution of (8, 9). Therefore, there is a need to analyse the pair of iterations in a framework that provides conditions, say, on the step sizes, under which there is guaranteed convergence in some sense. There does not appear to be an easy way to accomplish this from what is known in the stochastic approximation literature.

We therefore seek a stochastic algorithm that is based on (12, 13) which allows us to compute the jointly optimum powers of the joint receiver-power control optimization solution directly in a manner that is decoupled from the receiver computation. Such an algorithm must be distributed, in the sense that it must be implementable at each base station with a knowledge of only the in-cell parameters available at that base station. It must also be blind in that there should be no need for training sequences.

We form M -length blocks of the received vectors at each base. Let $\mathbf{y}_j(n, m)$ denote the m^{th} vector in the n^{th} block at base j given as

$$\begin{aligned} \mathbf{y}_j(n, m) &= \sum_{i=1}^{K_j} \sqrt{w_{ij}(n)} g_{ijj} \mathbf{s}_{ijj} x_{ij}(n, m) + \mathbf{n}_j(n, m) \\ &+ \sum_{l \neq j}^B \sum_{i=1}^{K_l} \sqrt{w_{il}(n)} g_{ilj} \left(\mathbf{s}_{ilj}^- x_{il}^-(n, m) + \mathbf{s}_{ilj}^+ x_{il}^+(n, m) \right), \end{aligned} \quad (14)$$

where $w_{il}(n)$ denotes the transmit power of user i of base l during the n^{th} block, and $x(n, m)$ with the appropriate sub- and superscripts denote the corresponding transmitted symbols in the m^{th} duration in the n^{th} block. From the M -length block of data, base j computes an unbiased estimate of the terms $\{\mathbf{s}_{ijj}^\dagger (\mathbf{A}_{ij})^{-1} \mathbf{s}_{ijj}\}$, $i \in \{1, \dots, K_j\}$. Let us denote these estimates as $\{\mathbf{s}_{ijj}^\dagger (\tilde{\mathbf{A}}_{ij})^{-1}(n) \mathbf{s}_{ijj}\}$, respectively. We now use these estimates in the following power recursion:

$$w_{ij}(n+1) = w_{ij}(n) - a_n \left[\mathbf{s}_{ijj}^\dagger (\tilde{\mathbf{A}}_{ij})^{-1}(n) \mathbf{s}_{ijj} w_{ij}(n) g_{ijj} - \frac{\gamma_{ij}^*}{1 + \gamma_{ij}^*} \right]. \quad (15)$$

The step size sequences a_n can be chosen to be fixed or decreasing. The estimates $\{\mathbf{s}_{ijj}^\dagger (\tilde{\mathbf{A}}_{ij})^{-1}(n) \mathbf{s}_{ijj}\}$ can be formed in the following way: First we estimate $(\mathbf{A}_{ij})^{-1}$ using

the recursion (over $M - 1$ iterations within the M -length block):

$$(\hat{\mathbf{A}}_{ij})^{-1}(m+1, n) = (\hat{\mathbf{A}}_{ij})^{-1}(m, n) - \frac{(\hat{\mathbf{A}}_{ij})^{-1}(m, n) \tilde{\mathbf{y}}_{ij}(n, m) \tilde{\mathbf{y}}_{ij}(n, m)^\dagger (\hat{\mathbf{A}}_{ij})^{-1}(m, n)}{1 + \tilde{\mathbf{y}}_{ij}(n, m)^\dagger (\hat{\mathbf{A}}_{ij})^{-1}(m, n) \tilde{\mathbf{y}}_{ij}(n, m)}, \quad (16)$$

where $\tilde{\mathbf{y}}_{ij}(n, m) = \mathbf{y}_j(n, m) - \sum_{k=1}^{i-1} \mathbf{b}_{kij} \sqrt{w_{kj}(n)} g_{kjj} \hat{x}_{kj}(n, m)$. Note that the averaging is done over $M - 1$ symbol intervals (ignoring the M^{th} interval), because the received data in the M^{th} interval reflects, due to asynchronicity of out-of-cell users, two different powers for each out-of-cell user. If M is large, leaving out one symbol interval does not cause any significant loss of efficiency. Moreover, it makes the convergence analysis somewhat simpler. With the final output of the $(M - 1)$ -length recursion in (16) $(\hat{\mathbf{A}}_{ij})^{-1}(M - 1, n)$, we then calculate $\mathbf{s}_{ijj}^\dagger (\hat{\mathbf{A}}_{ij})^{-1}(n) \mathbf{s}_{ijj} = \mathbf{s}_{ijj}^\dagger (\hat{\mathbf{A}}_{ij})^{-1}(M - 1, n) \mathbf{s}_{ijj} \mu_{bias}(n)$, where $\mu_{bias}(n)$ is chosen to make the estimate unbiased. With the assumption that the received signal is zero-mean Gaussian (as is usually done for the analysis of the Recursive Least Squares (RLS) algorithm) $\mu_{bias}(n)$ is the constant term $\frac{(M-1)-N-1}{M}$.

The MSIR filters corresponding to the optimum powers can be adaptively obtained either concurrently with the power updation, or only when the powers have sufficiently converged. In the latter case, with the powers fixed at $\mathbf{w}(n_N)$, (where $n_N > 0$ is large enough to ensure that the user powers are sufficiently close to the optimum powers), the MSIR filters can be obtained blindly using well known adaptive algorithms (cf. [1]). For blindly estimating the optimum MSIR receivers \mathbf{f}_{ij}^* concurrently with the power updation, one can use the following stochastic approximation based recursion indexed by (n, m) , where m is the updation number within the n^{th} $(M - 1)$ -length block of received vectors (recall that the powers are updated only after every $(M - 1)$ -length interval and, therefore, remain constant for each block):

$$\mathbf{f}_{ij}(n, m+1) = \mathbf{f}_{ij}(n, m) - b_n \left[\tilde{\mathbf{y}}_{ij}(n, m) \tilde{\mathbf{y}}_{ij}(n, m)^\dagger \mathbf{f}_{ij}(n, m) - \mathbf{s}_{ijj} \right]. \quad (17)$$

The step size b_n is chosen to be a sufficiently small fixed number, such that the filter recursion in (17) can track the changing powers from (15) and estimate the optimum filters within an acceptable asymptotic mean squared error.

5 Convergence Analysis

We shall first show that the deterministic iteration in (12) converges to \mathbf{w}^* , the optimum powers of the joint receiver-power control optimization. Let the vector valued function

$$\begin{aligned} \mathbf{p}(\mathbf{w}(n)) &\triangleq \text{diag} \left[\mathbf{s}_{111}^\dagger (\mathbf{A}_{11})^{-1}(n) \mathbf{s}_{111} g_{111}, \dots, \mathbf{s}_{K_B B B}^\dagger (\mathbf{A}_{K_B B})^{-1}(n) \mathbf{s}_{K_B B B} g_{K_B B B} \right] \mathbf{w}(n), \\ &\triangleq \Theta(n) \mathbf{w}(n), \end{aligned} \quad (18)$$

and $\Gamma \triangleq \left[\frac{\gamma_{11}^*}{1 + \gamma_{11}^*}, \dots, \frac{\gamma_{K_B B}^*}{1 + \gamma_{K_B B}^*} \right]^T$. Clearly, $\Theta(n)$ is a function of $\mathbf{w}(n)$. If we denote its value for $\mathbf{w}(n) = \mathbf{w}^*$ as Θ^* , then $\Theta^* \mathbf{w}^* = \Gamma$. For $\mathbf{w} > 0$, we note that the function $\mathbf{p}(\mathbf{w})$ is continuously differentiable and denote its derivative by $\mathbf{p}'(\mathbf{w})$. Let us consider the case when the step-size $a_n = a > 0$.

Theorem 1 *When the joint receiver-power control optimization has a solution, there exists $\bar{a} > 0$ such that when $0 < a < \bar{a}$, the algorithm in (12) is locally convergent to \mathbf{w}^* .*

Proof: Let us define $\epsilon(n) = \mathbf{w}(n) - \mathbf{w}^*$ and let $\|\epsilon(n)\|$ be its norm. We can locally expand $\mathbf{p}(\mathbf{w}(n))$ as

$$\mathbf{p}(\mathbf{w}(n)) = \mathbf{p}(\mathbf{w}^*) + \mathbf{p}'(\mathbf{w}^*)\epsilon(n) + o(\|\epsilon(n)\|), \quad (19)$$

and rewrite the recursion in (12) as

$$\epsilon(n+1) = \mathbf{D}\epsilon(n) + o(\|\epsilon(n)\|), \quad (20)$$

where $\mathbf{D} \triangleq \mathbf{I} - a\mathbf{p}'(\mathbf{w}^*)$. We can show that $\mathbf{p}'(\mathbf{w}^*)$ is an M-matrix [8] and specifically, it is *positive stable*, i. e., the real part of each eigenvalue of $\mathbf{p}'(\mathbf{w}^*)$, $\text{Re } \lambda_k(\mathbf{p}'(\mathbf{w}^*))$, is positive. Now consider $\bar{a} = 2 \min [\text{Re } \lambda(\mathbf{p}'(\mathbf{w}^*)) / |\lambda(\mathbf{p}'(\mathbf{w}^*))|^2]$, which, by the positive-stableness of $\mathbf{p}'(\mathbf{w}^*)$, is positive too. Let $\mu_k(\mathbf{D})$ be the k^{th} eigenvalue of \mathbf{D} , so that $\mu_k(\mathbf{D}) = 1 - a\lambda_k(\mathbf{p}'(\mathbf{w}^*))$. Therefore, when $0 < a < \bar{a}$, and denoting $\mu_k(\mathbf{D})$ simply as μ_k and $\lambda_k(\mathbf{p}'(\mathbf{w}^*))$ as λ_k , we have

$$|\mu_k|^2 = |1 - a\lambda_k|^2 = 1 + a^2|\lambda_k|^2 - 2a\text{Re}\lambda_k = 1 - a|\lambda_k|^2 \left(\frac{2\text{Re } \lambda_k}{|\lambda_k|^2} - a \right) < 1. \quad (21)$$

But, if all the eigenvalues of \mathbf{D} have modulus less than 1, we have $\|\epsilon(n)\| \rightarrow 0$ at the rate of a geometric progression when $\|\epsilon(0)\|$ is small. \diamond

We shall now address the important problem of convergence of the stochastic power control algorithm in (15). We first make the assumption that our estimates $\{\mathbf{s}_{ijj}^\dagger(\tilde{\mathbf{A}}_{ij})^{-1}(n)\mathbf{s}_{ijj}\}$ are unbiased. Similar to the definition of $\Theta(n)$ in (18), we define the matrix $\tilde{\Theta}(n) = \text{diag} [\mathbf{s}_{111}^\dagger(\tilde{\mathbf{A}}_{11})^{-1}(n)\mathbf{s}_{111}g_{111}, \dots, \mathbf{s}_{K_B B B}^\dagger(\tilde{\mathbf{A}}_{K_B B B})^{-1}(n)\mathbf{s}_{K_B B B}g_{K_B B B}]$ and the vector $\xi(n) \triangleq \tilde{\Theta}(n)\mathbf{w}(n) - \Theta(n)\mathbf{w}(n)$. Let the error vector be defined as $\epsilon(n) = \mathbf{w}(n) - \mathbf{w}^*$ and the vector $\zeta(n) \triangleq \Theta(n)\mathbf{w}(n) - \Gamma$. The recursion in (15) can be rewritten in terms of the error, as

$$\epsilon(n+1) = \epsilon(n) - a_n [\zeta(n) + \xi(n)]. \quad (22)$$

We consider the proof technique in [7] to show *local* convergence for our algorithm with the following standard conditions on the step-size sequence: $\sum_n a_n = \infty$, $\sum_n a_n^2 < \infty$ (note that the technique in [9] does not seem applicable). We next define the set $\Omega = \{\epsilon(n) : \|\epsilon(n)\|^2 \leq v\}$ for all n , where $v > 0$ is some number such that a local expansion of the vector function $\Theta(n)\mathbf{w}(n)$ in (19) is valid. We then use Lyapunov's theorem for stable matrices to get suitable bounds on quadratic terms involving $\epsilon(n)$ in order to satisfy the conditions in [7] that make the following result applicable in proving local convergence:

Theorem 2 *When the joint receiver-power control optimization has a solution, then the following is true of the algorithm in (15):*

$$P(\epsilon(n) \in \Omega \forall n) \geq 1 - \delta, \quad \delta = v^{-1} \left(\|\epsilon(0)\|^2 + K \sum_{i=0}^{\infty} a_i^2 \right), \quad (23)$$

$$(24)$$

where $0 < K < \infty$.

From the local convergence result in Theorem 2, we can deduce that δ is small, when the domain in which the conditions of the theorem are fulfilled, is large, i. e., v is large, when the initial approximation is good, i. e., $\|\epsilon(0)\|^2$ is small, and when the step size sequence is small, i. e., $\sum a_n^2$ is small.

6 Numerical Examples

Example 1 We consider the 7-cell cellular system shown in Figure 1 with 20 users in each cell. The noise power is about 20 dB less than the average received signal power. The processing gain is 64, and the signature sequences are unit norm random vectors. Once chosen, the signature sequences, remain fixed throughout the adaptation. The target SIR for each user is 8. For this example, the (deterministic) jointly optimum sum of powers for the linear MSIR based power control is a little more than 1.2 times the power required by the MSIR-DF based power control (a saving of 20% with DF).

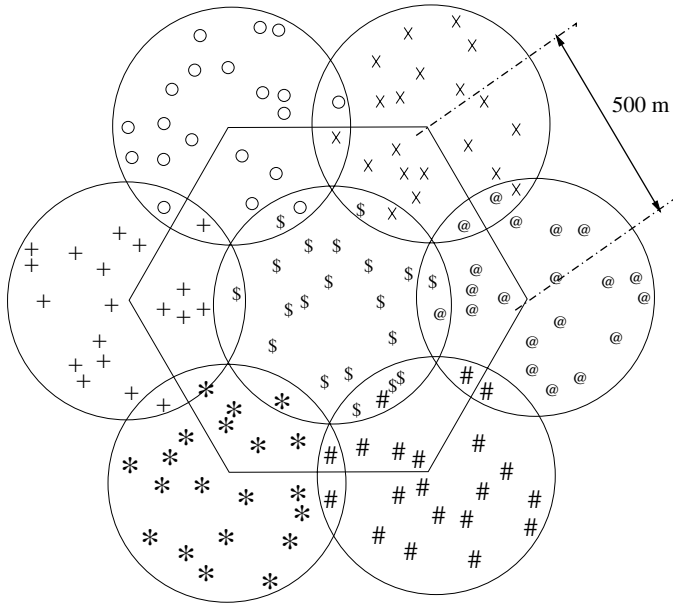


Figure 1: 7-cell cellular system with 20 users per cell

The powers are updated every 100-length block of symbol intervals for both the MSIR-DF-PC (with and without perfect feedback) and MSIR-PC algorithms. The filters are estimated concurrently with the powers as described in (17).

Figure 2 shows the estimated sum of user powers for both the linear MSIR and MSIR-DF based power control algorithms relative to their corresponding optimum (deterministic) values plotted against the iteration index.

Figure 3 plots the average (over users) SIR performance of the estimated MSIR and MSIR-DF receivers (relative to the desired common SIR of 8). Both the power control and the receiver algorithms were run with suitably chosen fixed step sizes, the step sizes being the same for the linear MSIR and MSIR-DF cases.

It is clear from the figure that the estimated sum of powers as well as the average SIR converge to their respective desired values. Note that although the MSIR-DF approach with perfect feedback assumption is more efficient than the linear MSIR approach (both in terms of sum of powers required and convergence rate of the stochastic algorithms), the effects of error propagation in the DF case does have a slightly adverse effect. This example with equal SIR's is, however, the worst case scenario in terms of error propagation. It was also observed over different numerical examples that when the powers are updated less frequently and estimates of $(\tilde{\mathbf{A}}_{ij}^{DF})^{-1}(n)$ are computed over more symbol intervals (yielding, therefore, better estimates), the convergence (for a given number of power updates) improves. Since it is desirable to get acceptably close to the optimum powers over a limited number of symbol intervals there is a trade-off (in determining the

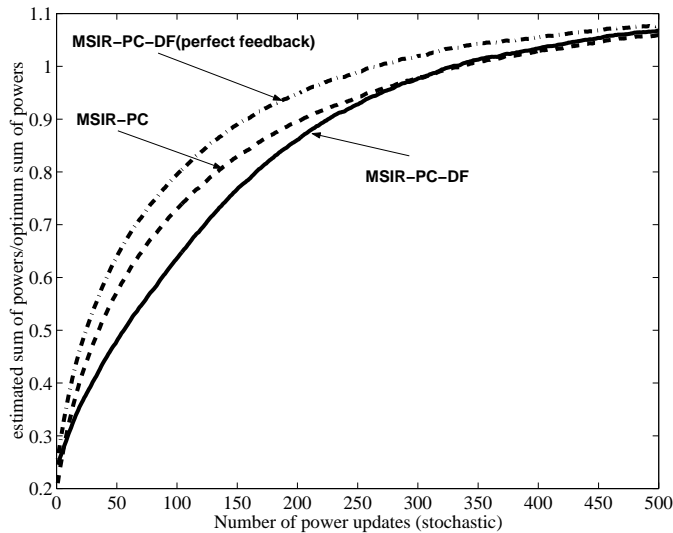


Figure 2: Sum of user powers required to achieve the desired SIR's with the joint stochastic optimization MSIR-PC and MSIR-PC-DF (with and without perfect feedback), plotted *relative* to their respective optimum values.

overall performance) between the number of symbol intervals allocated to the estimation of $(\tilde{\mathbf{A}}_{ij}^{DF})^{-1}(n)$ between power updations, i. e., the frequency of power updates, and the total number of power updates possible within the allowed limit.

7 Conclusions

The joint multiuser receiver-power control problem has been solved on the uplink using a new approach. The earlier known algorithms for this problem required alternating between an optimum power computation step (given the current receivers) and a SIR maximizing MSIR receiver computation step (given the current powers). In this paper the jointly optimum powers are computed independently of the MSIR receivers. Convergence has been shown analytically for not only the deterministic algorithm but, unlike in the earlier approaches, also for the blind stochastic algorithm.

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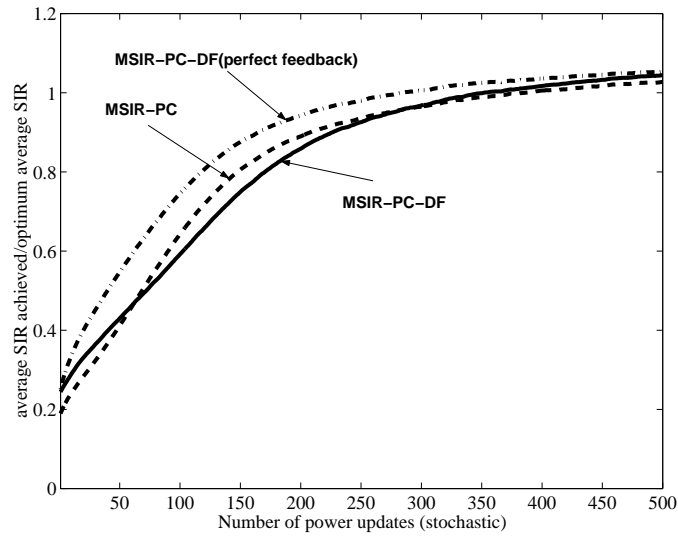


Figure 3: Average SIR achieved with the joint stochastic optimization MSIR-PC and MSIR-PC-DF (with and without perfect feedback), plotted *relative* to the common target SIR of 8.

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