

# Stochastic Power Control with Averaging

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## ABSTRACT

Uplink communication in a cellular radio network is considered where the base station in each cell employs a linear or non-linear (decision feedback) multiuser receiver. For any such receiver, the problem of minimizing total transmit power under the constraint that all the users of the network achieve their Quality-of-Service (QoS) objective in terms of signal-to-interference ratio (SIR) has been studied recently. Stochastic approximation based adaptive power control algorithms have been presented and analyzed both in the context of single user receivers and multiuser receivers. The primary focus of this paper is on improving the convergence properties of the standard stochastic approximation based power control strategy by using the more recent results on the stochastic approximation technique with *averaging*. Convergence issues of both the “non-averaged” and “averaged” algorithms are investigated and numerical examples are presented to demonstrate the performance improvement due to averaging.

## I. INTRODUCTION AND SYSTEM MODEL

Recent work on a combined approach to multiuser receivers and power control has considered decentralized, iterative and stochastic power control algorithms for a cellular radio network designed to meet every user’s QoS constraint in terms of SIR, with minimum transmit powers. Power control on the uplink has been discussed for the case where the base stations employ linear single-user receivers [1] or multiuser receivers (with or without decision feedback) [2]. In this paper we introduce the concept of “averaging”, a relatively recent technique in stochastic approximation, that can significantly improve the convergence properties of the stochastic power control algorithms in [1], [2]. The problem of computing the optimum user powers needed to achieve target SIR’s in as few iterations as possible is of singular importance in a non-stationary, cellular system since:

- updated user powers need to be conveyed by the base stations to the in-cell mobiles (using a low-rate feedback channel) and hence, power updates have to be kept to a minimum
- system parameters such as path gains can be assumed to be essentially constant only over a limited period of time (the coherence time in, say, a slow-fading frequency-nonselctive channel) and, therefore, the optimum powers will change every so often and have to be adaptively recomputed.

The issue of convergence rate of general stochastic approximation methods has for a long time been of great interest both from a theoretical and a practical point of view. In a couple of fundamental papers on stochastic approximation, Ruppert [3] and Polyak [4] showed that under certain assumptions and conditions on the step-size sequence used in the adaptation, signif-

icant improvement in convergence can be achieved if the output of the conventional stochastic approximation is averaged. In this paper we show the convergence of the “averaged” stochastic power control algorithm and demonstrate the resulting improvement in performance.

We consider a cellular network [5] in which there are  $B$  base stations and  $K$  active users with  $K_j$  users assigned to base  $j$ . Base  $j$  employs a set of  $N_j$  matched filters matched to orthonormal basis functions of a carefully chosen signal subspace that must include the signals received from the users within its cell. While the transmissions of the mobile users of a particular cell arrive at the bases of other cells symbol-asynchronously, it is assumed, for the sake of simplicity, that they arrive at their own base symbol-synchronously. Base  $j$  has timing information of only in-cell users.

The discrete-time model for the  $N_j$  matched filter outputs at base  $j$  can be expressed as

$$\mathbf{y}_j = \sum_{i=1}^{K_j} \sqrt{w_{ij} g_{ijj}} \mathbf{s}_{ijj} x_{ij} + \sum_{l \neq j}^B \sum_{i=1}^{K_l} \sqrt{w_{il} g_{ilj}} \left( \mathbf{s}_{ilj}^- x_{il}^- + \mathbf{s}_{ilj}^+ x_{il}^+ \right) + \mathbf{n}_j, \quad (1)$$

where  $w_{il}$  and  $x_{il}$  denote the transmit power and the transmitted symbol, respectively, of user  $i$  of base  $l$ .  $g_{ilj}$  is the channel gain of the  $i^{\text{th}}$  user of base  $j$  to base  $l$ . The vectors  $\mathbf{s}_{ijj}$  denote the vector representations (the “signature sequence”) of the signal of user  $i$  of base  $j$  w.r.t. the orthonormal basis functions employed at base  $j$ . The vectors  $\mathbf{s}_{ilj}^-$  and  $\mathbf{s}_{ilj}^+$  denote the segments of the signals associated with the two symbols of user  $i$  of base  $l$  that overlap with the signals of base  $j$ .  $\mathbf{n}_j$  is an  $N_j$ -dimensional zero-mean Gaussian random vector with a covariance matrix equal to  $\sigma_j^2 \mathbf{I}$ . Define the in-cell signal matrix  $\mathbf{S}_{jj} = [\mathbf{s}_{1jj} \mathbf{s}_{2jj} \cdots \mathbf{s}_{K_j jj}]$  whose columns are the signature sequences of the users of cell  $j$ . Also define the signal matrices of out-of-cell users  $\mathbf{S}_{lj}^+ = [\mathbf{s}_{1lj}^+ \mathbf{s}_{2lj}^+ \cdots \mathbf{s}_{K_l lj}^+]$  and  $\mathbf{S}_{lj}^- = [\mathbf{s}_{1lj}^- \mathbf{s}_{2lj}^- \cdots \mathbf{s}_{K_l lj}^-]$ . The received signal can therefore be written as

$$\mathbf{y}_j = \mathbf{S}_{jj} \mathbf{G}_{jj}^{1/2} \mathbf{W}_j^{1/2} \mathbf{x}_j + \sum_{l \neq j}^B \mathbf{S}_{lj}^- \mathbf{G}_{lj}^{1/2} \mathbf{W}_l^{1/2} \mathbf{x}_l^- + \sum_{l \neq j}^B \mathbf{S}_{lj}^+ \mathbf{G}_{lj}^{1/2} \mathbf{W}_l^{1/2} \mathbf{x}_l^+ + \mathbf{n}_j, \quad (2)$$

where  $\mathbf{G}_{lj} = \text{diag}\{g_{1lj}, g_{2lj}, \cdots, g_{K_l lj}\}$ , and  $\mathbf{W}_l = \text{diag}\{w_{1l}, w_{2l}, \cdots, w_{K_l l}\}$ . Moreover,  $\mathbf{x}_j$ ,  $\mathbf{x}_l^+$  and  $\mathbf{x}_l^-$  are vectors whose  $i^{\text{th}}$  elements are  $x_{ij}$ ,  $x_{il}^+$  and  $x_{il}^-$ , respectively.

## II. MULTIUSER RECEIVERS

The presentation here is similar to that in [2] and will be kept brief. Each base station uses either a linear or a decision feedback receiver to decode the information transmitted by users in its own cell. In the decision feedback case, assume for notational simplicity, that the users in each cell are numbered according to the order in which they are decoded. The soft-output of the linear or decision feedback receiver for user  $i$  of base  $j$  can be written in the common form

$$v_{ij} = \mathbf{f}_{ij}^T \left( \mathbf{y}_j - \sum_{k=1}^{i-1} \mathbf{b}_{kij} \sqrt{w_{kj}} g_{kjj} \hat{x}_{kj} \right). \quad (3)$$

$\mathbf{f}_{ij}$  is the feedforward filter. The feedback filters  $\mathbf{b}_{kij} = \mathbf{0}$  for linear receivers (no decision feedback) and for decision feedback receivers, we let  $\mathbf{b}_{kij} = \mathbf{s}_{kjj}$  for  $k = 1, \dots, i-1$ , without loss of generality [7]. It is assumed that  $|\mathbf{f}_{ij}^T \mathbf{s}_{ijj}|^2 > 0$ . The symbol  $T$  denotes transposition.  $\hat{x}_{kj}$  denotes the detected (in uncoded transmission) or decoded (in the coded case) symbols of “past” users  $1, \dots, i-1$  relative to user  $i$  of base  $j$ . Implicit in (3) is the assumption that base  $j$  has knowledge of the codes, signals, transmit powers and channel gains of the users within its own cell. We do not however assume that it has knowledge of these parameters for out-of-cell users. Hence we consider some single-cell receivers, i. e., receivers that require only in-cell information.

First, we note that for the matched-filter (or single-user) receiver, denoted as MF, we have  $\mathbf{f}_{ij} = \mathbf{s}_{ijj}$ . The matched-filter decision feedback (MF-DF) receiver also has the same feedforward filters.

A simple linear multiuser receiver that can be implemented at each base without knowledge of the system parameters corresponding to out-of-cell users is the single-cell decorrelator which for base  $j$  is given as

$$\mathbf{f}_{ij}^T = \text{row}_i(\mathbf{S}_{jj}^T \mathbf{S}_{jj})^{-1} \mathbf{S}_{jj}^T. \quad (4)$$

Note that such a receiver tunes-out the interference for each user that arises due to the other users in the same cell but ignores interference from out-of-cell users.

The single-cell decorrelating decision feedback (D-DF) receiver is specified next: consider a partition of the columns of  $\mathbf{S}_{jj}$  into two matrices, one that contains the first  $i-1$  columns and the second that contains the other  $K_j - i + 1$  columns. Accordingly, let  $\mathbf{S}_{jj} = [\tilde{\mathbf{S}}_{jj}(i) \mathbf{S}_{jj}(i)]$ . The feedforward filter of the single-cell D-DF receiver of user  $i$  is given as

$$\mathbf{f}_{ij}^T = \text{row}_1(\mathbf{S}_{jj}^T(i) \mathbf{S}_{jj}(i))^{-1} \mathbf{S}_{jj}^T(i). \quad (5)$$

This receiver subtracts interference contributed by the already decoded users in cell  $j$  and decorrelates interference from the as yet undecoded users in cell  $j$  [8] without consideration for out-of-cell users.

## III. OPTIMUM POWER CONTROL

Given that each base uses an arbitrary linear or decision feedback receiver (that does not change with the user powers), consider the problem of minimizing total transmitted power subject

to the constraints that the SIR of each user in the network is not less than some target value:

$$\begin{aligned} \min \quad & \sum_{j=1}^B \sum_{i=1}^{K_j} w_{ij} \\ \text{s.t.} \quad & \gamma_{ij} \geq \gamma_{ij}^* \quad i = 1, 2, \dots, K_j, \quad j = 1, 2, \dots, B. \end{aligned} \quad (6)$$

For  $j \in \{1, 2, \dots, B\}$ , define the diagonal matrices  $\mathbf{D}_j$  according to

$$\mathbf{D}_j = \text{diag} \left\{ \gamma_{1j}^* / g_{1jj} |\mathbf{f}_{1j}^T \mathbf{s}_{1jj}|^2, \dots, \gamma_{K_j j}^* / g_{K_j j j} |\mathbf{f}_{K_j j}^T \mathbf{s}_{K_j j j}|^2 \right\}. \quad (7)$$

Let  $\mathbf{t}_j = \sigma_j^2 [\mathbf{f}_{1j}^T \mathbf{f}_{1j}, \dots, \mathbf{f}_{K_j j}^T \mathbf{f}_{K_j j}]^T$ . Define the vector of transmitted powers of users in cell  $j$  as  $\mathbf{w}_j = [w_{1j}, \dots, w_{K_j j}]^T$ . Define matrices  $\{\mathbf{Q}_{jl}\}$  for  $j, l \in \{1, 2, \dots, B\}$ , with  $Q_{jl}(i, k)$  denoting the  $(i, k)^{\text{th}}$  element of  $\mathbf{Q}_{jl}$ , as follows: the matrices  $\mathbf{Q}_{jj}$  are defined for  $j = 1, \dots, B$  as

$$Q_{jj}(i, k) = \begin{cases} |\mathbf{f}_{ij}^T (\mathbf{s}_{kjj} - \mathbf{b}_{kij})|^2 & \text{if } k < i \\ 0 & \text{if } k = i \\ |\mathbf{f}_{ij}^T \mathbf{s}_{kjj}|^2 & \text{if } k > i \end{cases} \quad (8)$$

for  $i, k \in \{1, \dots, K_j\}$  and the matrices  $\mathbf{Q}_{jl}$  for  $j, l \in \{1, 2, \dots, B\}$  with  $j \neq l$  are defined as

$$Q_{jl}(i, k) = |\mathbf{f}_{ij}^T \mathbf{s}_{klj}^-|^2 + |\mathbf{f}_{ij}^T \mathbf{s}_{klj}^+|^2 \quad (9)$$

for  $i \in \{1, \dots, K_j\}$  and  $k \in \{1, \dots, K_l\}$ . With this notation, the SIR constraints in (6) can be written for each  $j$  as

$$\mathbf{w}_j \geq \mathbf{D}_j \left[ \sum_{l=1}^B \mathbf{Q}_{jl} \mathbf{G}_{lj} \mathbf{w}_l + \mathbf{t}_j \right]. \quad (10)$$

Assuming  $\gamma_{ij}^* > 0$ ,  $\sigma_j^2 > 0$  for all  $i \in \{1, \dots, K_j\}$ ,  $j \in \{1, \dots, B\}$  in addition to  $|\mathbf{f}_{ij}^T \mathbf{s}_{ijj}|^2 > 0$ , the target SIR's are said to be *feasible* if there exist positive and finite transmit powers that satisfy the above inequalities. The necessary and sufficient condition for feasibility is stated next.

Define a square, non-negative, block matrix  $\mathcal{X}$  with  $B^2$  matrix elements with the  $(j, l)^{\text{th}}$  element defined as the matrix product

$$\mathbf{X}_{jl} \triangleq \mathbf{D}_j \mathbf{Q}_{jl} \mathbf{G}_{lj} \quad (11)$$

for each  $j, l \in \{1, 2, \dots, B\}$ . Note that this matrix is not necessarily irreducible. To show the feasibility of the target SIR's we require that there be a positive solution to the problem in (10). If we consider the case where (10) holds with equality,

$$\mathbf{w}_j = \mathbf{D}_j \left[ \sum_{l=1}^B \mathbf{Q}_{jl} \mathbf{G}_{lj} \mathbf{w}_l + \mathbf{t}_j \right], \quad (12)$$

i. e., the SIR requirements are met with equality, we can show (using a proof by contradiction) that the positive  $\mathbf{w}_j$  (if they exist) that satisfy equation (12), are the component-wise optimum (minimum) powers  $\{\mathbf{w}_j^*\}$  which solve the power control problem in (6). Our next result establishes conditions under which (12) has a unique positive solution.

**Theorem 1:** A necessary and sufficient condition for (12) to have a unique positive solution, ensuring the feasibility of the target SIR's with minimum powers, is that  $\lambda_0(\mathcal{X}) < 1$ , where  $\lambda_0(\mathcal{X})$  is the maximal eigenvalue of  $\mathcal{X}$  [9].

Defining the matrix  $\mathbf{D} = \text{diag}\{\mathbf{D}_1, \dots, \mathbf{D}_B\}$  and denoting the optimal powers as  $\mathbf{w}^*$ , we have

$$\mathbf{w}^* = (\mathcal{I} - \mathcal{X})^{-1} \mathbf{D} \mathbf{t}. \quad (13)$$

Implicit in this result is the assumption that feedback is perfect in the case of nonlinear receivers. While not valid in general, it is possible, by decoding the users in the decreasing order of their target SIR's, to mitigate the error propagation effects to a large extent [8]. In this regard, equal target SIR's may represent something of a worst-case scenario. In such cases, it may be necessary for better overall performance at the cost of slightly increased power requirement, to set the target SIR's to be slightly different from each other while ensuring that the worst-case target SIR is equal to the required target SIR.

#### IV. DISTRIBUTED STOCHASTIC POWER CONTROL

The solution in (13) can be computed in a centralized way only when all the parameters of the system are known. Based on the point Jacobi method [10] for solving a system of linear equations, one can devise the following convergent distributed algorithm (see [2] for details)

$$w_{ij}(n) = \gamma_{ij}^* \left( \frac{E[|v_{ij}(n)|^2]}{g_{ijj} \mathbf{f}_{ij}^T \mathbf{s}_{ijj}} - w_{ij}(n-1) \right). \quad (14)$$

This can be implemented at the  $(i, j)^{\text{th}}$  mobile or in the  $j^{\text{th}}$  base station provided it has perfect knowledge of the mean-squared value or power of the normalized decision statistic  $v_{ij}(n)/\mathbf{f}_{ij}^T \mathbf{s}_{ijj}$ , in addition to its target SIR value, the path gain to its own base station and the transmit power in the previous interval. In general, the  $(i, j)^{\text{th}}$  mobile or its base station will not have perfect knowledge of the power of the decision statistic  $v_{ij}(n)$ , and we therefore we need to devise a distributed algorithm that requires less information.

If the transmit powers are updated once every several symbols, it is possible for the base station to estimate the mean-squared values of the decision statistics by computing the corresponding sample mean-squared values. A distributed stochastic power control algorithm where the transmit powers are updated (either at the base stations or at the mobiles) iteratively and synchronously once every  $M$  symbols will be presented next, as in [2]. When the power control algorithm is implemented at the base (mobile) stations, the bases communicate the newly updated powers (sample mean-squared values of the decision statistics) to their respective mobiles via low rate feedback channels. The mobiles then transmit the next  $M$  symbols at the newly updated powers.

Form  $M$ -length blocks of the vectors of matched filter outputs at each base. Let  $\mathbf{y}_j(n, m)$  denote the  $m^{\text{th}}$  vector in the  $n^{\text{th}}$  block. The model in (1) can be written including the time indices as  $\mathbf{y}_j(n, m) = \sum_{i=1}^{K_j} \sqrt{w_{ij}(n-1)} g_{ijj} \mathbf{s}_{ijj} x_{ij}(n, m) + \mathbf{n}_j(n, m) + \sum_{l \neq j}^B \sum_{i=1}^{K_l} \sqrt{w_{il}(n-1)} g_{ilj} \left( \mathbf{s}_{ilj}^- x_{il}^-(n, m) + \mathbf{s}_{ilj}^+ x_{il}^+(n, m) \right)$ ,

where  $w_{il}(n-1)$  denotes the transmit power of user  $i$  of base  $l$  during the  $n^{\text{th}}$  block, and  $x(n, m)$  with the appropriate sub- and superscripts denote the corresponding transmitted symbols in the  $m^{\text{th}}$  duration in the  $n^{\text{th}}$  block. The decision statistic for user  $i$  of cell  $j$  is

$$v_{ij}(n, m) = \mathbf{f}_{ij}^T \left( \mathbf{y}_j - \sum_{k=1}^{i-1} \mathbf{b}_{kij} \sqrt{w_{kj}(n-1)} g_{kjj} \hat{x}_{kj}(n, m) \right), \quad (15)$$

where  $\hat{x}_{kj}(n, m)$  is the detected symbol in the uncoded case, and it is the decoded and re-encoded symbol in a coded system with decoders that are based on the soft outputs  $v_{ij}(n, m)$  obtained during the  $n^{\text{th}}$  block. The time-averaged power in the decision statistic is given by

$$z_{ij}(n) = \frac{1}{M-1} \sum_{m=1}^{M-1} |v_{ij}(n, m)|^2. \quad (16)$$

The averaging in (16) is done over  $M-1$  symbol intervals (ignoring the  $M^{\text{th}}$  interval), because the received data in the  $M^{\text{th}}$  interval reflects two different powers for each out-of-cell user. If  $M$  is relatively large, this does not lead to any significant loss of efficiency. Moreover, it allows the convergence analysis to remain fairly simple. Consider, now, the following stochastic power control algorithm, based on the Robbins-Monro stochastic approximation method [11], that iteratively updates the transmit power of mobile  $i$  in cell  $j$ :

$$w_{ij}(n) = w_{ij}(n-1) + a_n \left( \frac{z_{ij}(n) \gamma_{ij}^*}{g_{ijj} \mathbf{f}_{ij}^T \mathbf{s}_{ijj}} - (1 + \gamma_{ij}^*) w_{ij}(n-1) \right), \quad (17)$$

where  $\{a_n\}$  denotes the sequence of step-sizes. The special case of this algorithm for the matched-filter linear receiver reduces to the one proposed in [1]. The mean-square convergence of (17) is stated next. Let  $\mathbf{w}_j(n)$  be the vector of power updates of users in cell  $j$  in the  $n^{\text{th}}$  iteration. Let  $\mathbf{w}(n)$  be the power vector obtained by stacking  $\{\mathbf{w}_j(n)\}_{j=1}^B$ .

**Theorem 2:** If the target SIR's are feasible, and assuming perfect feedback (in the case of decision-feedback receivers), the decentralized stochastic algorithm in (17) converges to the optimal powers in the mean-square sense, i.e.,

$$\lim_{n \rightarrow \infty} E[|\|\mathbf{w}(n) - \mathbf{w}^*|^2] = 0 \quad (18)$$

for a positive-valued step-size sequence  $a_n$  that satisfies the Robbins-Monro conditions [11]  $\sum_{n=1}^{\infty} a_n = \infty$  and  $\sum_{n=1}^{\infty} a_n^2 < \infty$ .

#### V. STOCHASTIC POWER CONTROL WITH AVERAGING

A recent fundamental development in stochastic approximation is the idea of averaging as introduced in [3] and [4]. In the former work, a linear algorithm for the one-dimensional case was considered, and asymptotic normality of the procedure was proved. Multidimensional problems were considered in [4] and under certain assumptions, mean-square convergence for decreasing step-size adaptive algorithms followed by averaging was demonstrated. The improvement in convergence is

essentially a result of averaging a stochastic approximation that uses a step-size sequence that decays more slowly (or, is relatively “larger”) than the  $a/n$  step-size that was used in the original Robbins-Monro formulation. It was further shown in [6] that a.s. convergence is achieved even for a suitable fixed step-size stochastic approximation strategy with averaging, and this was ‘optimal’ in terms of the convergence rate and the asymptotic error covariance matrix. The recursion in (17) can be modified to include an averaging step after the “basic” recursion as follows:

$$w_{ij}(n) = w_{ij}(n-1) + a_n \left( \frac{z_{ij}(n)\gamma_{ij}^*}{g_{ijj} \|\mathbf{f}_{ij}^T \mathbf{s}_{ijj}\|^2} - (1 + \gamma_{ij}^*) w_{ij}(n-1) \right) \quad (19)$$

$$\tilde{w}_{ij}(n) = \frac{1}{n} ((n-1)\tilde{w}_{ij}(n-1) + w_{ij}(n-1)), \quad (20)$$

where  $a_n$  is a suitable decreasing or fixed step size sequence. Note that the system information required and the order of computational complexity for the recursion with averaging remain the same as for the non-averaged algorithm in (17). The “smoothing” effect due to the averaging allows the basic recursion step to use relatively “larger” step-sizes than would be feasible for the non-averaged adaptive rules. This is essentially the basis for the improvement in the rate of convergence that our numerical examples will later demonstrate.

In the non-stationary cellular environment, where the path gains can be assumed to remain essentially constant for a limited period of time, it is critical that the optimum powers be computed as quickly as possible. To ensure tracking capability in the non-stationary environment, one would, typically, use a fixed step size in the basic recursion step and/or either a finite window of past estimates or a “forgetting” factor [12] in the averaging step. For our convergence analysis of the averaging algorithm (19), we shall consider a fixed step size version that averages over all past estimates. Our main convergence result for the averaged algorithm is as follows:

**Theorem 3:** When the target SIR’s are feasible and assuming perfect decision feedback for the nonlinear receivers,

$$0 < \lim_{n \rightarrow \infty} E [n(\tilde{\mathbf{w}}(n) - \mathbf{w}^*)^T (\tilde{\mathbf{w}}(n) - \mathbf{w}^*)] \leq \delta < \infty, \quad (21)$$

where  $\delta$  is a some finite constant and the fixed step-size is chosen sufficiently small.

**Proof:** We will give only an outline here. The details can be worked out along the lines of the proof of Theorem 1, part (b) in [4]. Let  $\Gamma$  be a  $K \times K$  diagonal matrix with the desired SIR’s along its diagonal, i. e.,  $\text{diag}([\gamma_{11}^*, \dots, \gamma_{K_1 1}^*, \dots, \gamma_{1B}^*, \dots, \gamma_{K_B B}^*])$ . Further, define  $\beta(n) = (\Gamma + \mathcal{X}) \mathbf{w}(n-1) + \mathbf{D} \mathbf{t} - \mathbf{D} \mathbf{z}(n)$  where  $\mathbf{z}(n) \triangleq [z_{11}(n), \dots, z_{K_1 1}(n), \dots, z_{1B}, \dots, z_{K_B B}]^T$  and  $\mathbf{t} = [\mathbf{t}_1^T, \dots, \mathbf{t}_B^T]^T$ .  $E[\beta(n) | \mathbf{w}(n-1) = \mathbf{w}] = \mathbf{0}$  and, therefore,  $\beta(n)$  is a zero-mean vector. By the assumption that the power control algorithm with the chosen receivers is feasible, the matrix  $\mathbf{A} \triangleq (\mathbf{I} - \mathcal{X})$  is such that (s.t.)  $\text{Re}(\lambda_i(\mathbf{A})) > 0$ . We now rewrite (19) in terms of the error,  $\epsilon(n) \triangleq \mathbf{w}(n) - \mathbf{w}^*$ , as :

$$\epsilon(n) = (\mathbf{I} - a_n \mathbf{A}) \epsilon(n-1) - a_n \beta(n). \quad (22)$$

With the assumption that user powers are bounded above by a sufficiently large number, i. e.,  $0 < w_{ij}(n) \leq \mathcal{K}_1 < \infty$ , we can show that  $\sup_n E[\|\beta(n)\|^2 | \beta(n-1), \dots, \beta(1)] \leq \mathcal{K}_2 < \infty$  almost surely (a.s.) and further,  $\beta(n)$  is a martingale-difference process [13]. We consider the fixed step size  $a_n = a$  to be s.t.  $0 < a < 2(\min_i \text{Re} \lambda_i(\mathbf{A}))^{-1}$  and denote the error with averaging by  $\bar{\epsilon}(n) = \tilde{\mathbf{w}}(n) - \mathbf{w}^*$ . It can be shown as in [4] that  $\sqrt{n} \bar{\epsilon}(n) = \frac{1}{\sqrt{na}} \mathcal{W}_n \epsilon(0) + \frac{1}{\sqrt{n}} \sum_{j=1}^{n-1} \mathbf{A}^{-1} \beta(j) + \frac{1}{\sqrt{n}} \sum_{j=1}^{n-1} \mathcal{Z}_j^n \beta(j)$ , where  $\mathcal{W}_n, \mathcal{Z}_j^n \in \mathbb{R}^{N \times N}$  are such that  $\|\mathcal{W}_n\| \leq \mathcal{K}_3, \|\mathcal{Z}_j^n\| \leq \mathcal{K}_3$  for some  $\mathcal{K}_3 < \infty$ , and  $\frac{1}{n} \sum_{j=1}^{n-1} \|\mathcal{Z}_j^n\| \rightarrow \mathbf{0}$  as  $n \rightarrow \infty$ . With this result and denoting  $\kappa = \frac{1}{\sqrt{n}} \sum_{j=1}^{n-1} \mathbf{A}^{-1} \beta(j)$  and

$$nE\bar{\epsilon}^T(n)\bar{\epsilon}(n) = E\|\kappa\|^2 + \tau_n, \quad (23)$$

we can further show (using in the process the martingale-difference property of the vector sequence  $\beta(n)$ ) that  $\tau_n \rightarrow 0$  as  $n \rightarrow \infty$ . Therefore,  $\lim_{n \rightarrow \infty} nE\bar{\epsilon}^T(n)\bar{\epsilon}(n) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^{n-1} E\beta(j)^T (\mathbf{A}^{-1})^T \mathbf{A}^{-1} \beta(j) \leq \lim_{n \rightarrow \infty} \frac{1}{n} \|\mathbf{A}^{-1}\|^2 \sum_{j=1}^{n-1} E\|\beta(j)\|^2 \leq \frac{1}{n} \|\mathbf{A}^{-1}\|^2 n \mathcal{K}_2 < \infty$ .

The theorem implies that the MSE error goes to zero in the limit (as opposed to the non-averaged algorithm, where the MSE can only be guaranteed to be bounded for a fixed step-size value). Although a discussion on “optimality” of the rate of convergence due to averaging will not included here (and can be found in [4]), we will see in the following numerical examples section that the rate of convergence does significantly improve with averaging.

## VI. NUMERICAL EXAMPLES

**Example 1:** We consider a 7-cell cellular system as shown in Figure 1 with 20 users in each cell. The gains to every base station for each user is calculated using a  $1/d^4$  path-loss law, where  $d$  is the distance to the base station of interest. The noise power is about 20 dB less than the average received signal power. The processing gain is chosen to be 64, and the signature sequences are chosen to be unit norm random vectors. Once chosen, the signature sequences, remain fixed throughout the adaptation. In this example the sequences of in-cell users are linearly independent.

We consider the single-cell decorrelating receiver (4) and the single-cell decorrelating decision feedback receiver (5) for each user for detection on the uplink. Corresponding to these receivers, the optimum power computation is done at each base station for users in its own cell. The user powers are updated every 20 symbol intervals. We consider the case when the target SIR’s are all equal to 8. The optimum sum of powers (as yielded by a deterministic computation (13)) is 0.89 units for the linear receiver and 0.58 units for the decision feedback receiver - nonlinear detection is 52 % more power-efficient than linear detection. Note that for this example, a common target SIR of 8 is not feasible with matched-filter (single-user) detection with and without decision feedback. Figure 2 shows the sum of the powers (without assuming perfect feedback for the decorrelating-DF receiver) computed by the non-averaged (17) and averaged (19) power control algorithms. The power control

algorithm with averaging uses a suitably larger fixed step size (same for the linear and nonlinear receivers) than that for the non-averaging version (again, same for the linear and nonlinear receivers), and as evident from the figure, achieves a faster rate of convergence. A comparable rate of convergence for the non-averaged algorithm with fixed step-size in either case will come at the price of larger asymptotic MSE. Although with perfect feedback the convergence in the decision-feedback case is found to be closer to the optimum powers, the figure shows that the power adaptation does not suffer in a significant way when feedback is not assumed perfect. In Figure 3 we plot the standard deviation of the SIR's achieved over all the users. Note that the target SIR's are achieved significantly faster with averaging.

## VII. CONCLUSIONS

We have shown in this paper that the stochastic power control algorithms that have been discussed earlier for single-user as well as multiuser receivers to achieve specified SIR performance on the uplink of a cellular system can be considerably improved by using the technique of averaging for stochastic approximation. Since the averaging is achieved without any increase in the order of complexity and enables faster convergence than the corresponding non-averaged algorithms, it makes the practical implementation of the stochastic power control algorithms a more attractive proposition. The convergence of the averaged power control algorithm has been shown analytically and improved rate of convergence demonstrated numerically.

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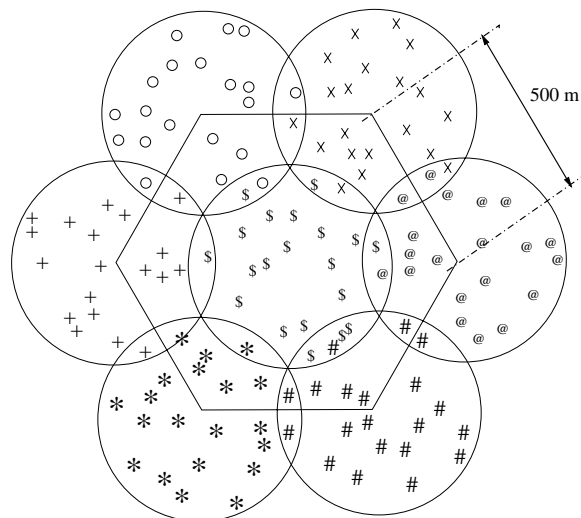


Fig. 1. 7-cell cellular system with 20 users per cell

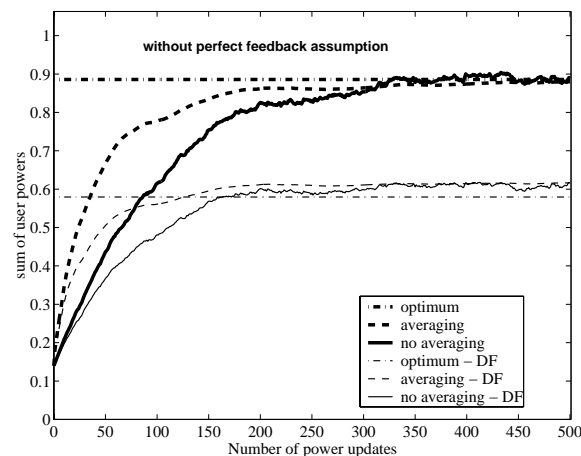


Fig. 2. Sum of powers as a function of iteration index in Example 1 for (linear) single-cell decorrelating and (non-linear) decorrelating decision feedback receivers. Common target SIR is 8. Note slight degradation in adaptation performance due to imperfect feedback.

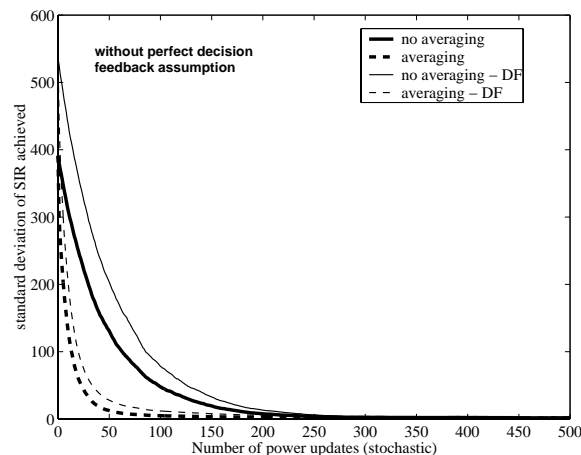


Fig. 3. Standard deviation (over users) of SIR achieved as a function of iteration index for Example 1 when common target SIR is 8.