Diversity Order Analysis of Training Codes for MIMO Block Fading Channels

Pranav Dayal, Shivratna Giri Srinivasan, and Mahesh K. Varanasi
e-mail: {dayalp, srinivsg, varanasi}@dsp.colorado.edu
Electrical & Computer Engineering Department
University of Colorado, Boulder, CO 80309

Abstract — **A general class of training-based space-time codes is considered for the noncoherent *block correlated* Rayleigh fading channel. Such codes can be efficiently decoded by first forming the MMSE estimate of the channel and then decoding the underlying coherent space-time code using the channel estimate as if it were perfect. The diversity order of this (in general, sub-optimal) estimator-decoder is obtained, and it is shown that under certain conditions on the channel correlation matrix, training codes inherit the diversity order of the underlying coherent code.**

I. Introduction

Training based schemes for the MIMO Rayleigh fading channel were recently studied in [1], [2], [3]. In [1], a lower bound on the capacity for training based schemes is derived. This bound is maximized by suitable choices of training matrix, training power, and training period. In [2] and [3], the authors analyze and optimize, via the pair-wise error probability (PEP), general finite-length training codes that leverage coherent space-time codes, under the assumptions of block fading and i.i.d. path gains. In this paper, we introduce training codes for the more general correlated Rayleigh fading channel, the capacity considerations for which are addressed in [4].

Consider an $N_{\rm T}$-transmit, $N_{\rm R}$-receive antenna noncoherent fading channel. Let $\mathcal{C}$ be a space-time code consisting of codewords of the form $\mathbf{S} = [\mathbf{S}_1^T \mathbf{S}_2^T \ldots \mathbf{S}_K^T]^T \in \mathbb{C}^{D \times N_T}$, where the k-th sub-block $\mathbf{S}_k = \frac{1}{\sqrt{K}}[\sqrt{\tau}\mathbf{T}^{\rm T}\sqrt{1-\tau}\mathbf{B}_k^{\rm T}]^{\rm T} \in \mathbb{C}^{L \times N_T}$. The sub-block $\mathbf{S}_k$ undergoes fading characterized by a channel matrix $\mathbf{H}_k$ so that the received statistics are $\mathbf{Y}_k = \sqrt{\overline{\gamma}}\mathbf{S}_k\mathbf{H}_k + \mathbf{N}_k, 1 \le k \le K$. The entries of $\mathbf{N}_k$ and $\mathbf{H}_k$ consist of zero-mean, unit variance complex normal random variables. The entries of $\mathbf{N}_k$ are independent across time and space. The average energies of $\mathbf{T}$ and $\mathbf{B}_k$ are normalized to one. The matrix $\mathbf{T}$ in $\mathbf{S}_k$ is the training symbol, $\{\mathbf{B}_k\}_{k=1}^K$ denotes the underlying space-time codeword, and τ represents the fraction of energy of $\mathbf{S}$ alloted to training. Therefore, $\overline{\gamma}$ represents the average received signal-to-noise ratio (SNR). Define $\boldsymbol{\mathcal{T}} = \mathbf{I}_{KN_{\rm R}} \otimes \mathbf{T}$ and $\boldsymbol{\mathcal{B}} = {\rm diag}(\mathbf{I}_{N_{\rm R}} \otimes \mathbf{B}_1, \ldots, \mathbf{I}_{N_{\rm R}} \otimes \mathbf{B}_K)$. The matrix $\boldsymbol{\mathcal{B}}$ is therefore the information bearing part of the codeword $\mathbf{S}$. The K $(i,j)^{th}$ entries of $\{\mathbf{H}_k\}_{k=1}^K$ are correlated with the $K \times K$ temporal correlation given by $\boldsymbol{\Sigma}_t$ (independent of (i,j)). The spatial correlation is specified by the matrix $\boldsymbol{\Sigma}_s = {\rm E}[{\rm vec}(\mathbf{H}_k){\rm vec}(\mathbf{H}_k)^\dagger], \forall k$. Define $\mathbf{h} = [{\rm vec}(\mathbf{H}_1)^T {\rm vec}(\mathbf{H}_2)^T \ldots {\rm vec}(\mathbf{H}_K)^T]^T$ so that the overall covariance matrix $\boldsymbol{\Sigma} = {\rm E}[\mathbf{h}\mathbf{h}^\dagger] = \boldsymbol{\Sigma}_t \otimes \boldsymbol{\Sigma}_s$. The channel outputs during the training and data phases can be written as

$$\mathbf{y}_{\rm T} = \sqrt{\overline{\gamma}\tau}\boldsymbol{\mathcal{T}}\ \mathbf{h} + \boldsymbol{\eta}_{\rm T}\,, \quad \mathbf{y}_s = \sqrt{\overline{\gamma}\frac{1-\tau}{K}}\boldsymbol{\mathcal{B}}\mathbf{h} + \boldsymbol{\eta}_s\,. \qquad (1)$$

We propose an estimator-decoder receiver structure that first forms an MMSE estimate $\widehat{\mathbf{h}}$ of the fading coefficients from $\mathbf{y}_{\rm T}$, and then uses the estimate as if it were correct to decode $\boldsymbol{\mathcal{B}}$. The decoder is thus

$$\begin{aligned} \Phi^{\rm E-D}:\ \widehat{\mathbf{h}} &= \sqrt{\frac{\overline{\gamma}\tau}{K}}\boldsymbol{\Sigma}\boldsymbol{\mathcal{T}}^\dagger\left(\frac{\overline{\gamma}\tau}{K}\boldsymbol{\mathcal{T}}\boldsymbol{\Sigma}\boldsymbol{\mathcal{T}}^\dagger + \mathbf{I}\right)^{-1}\mathbf{y}_{\rm T}, \qquad (2)\\ \widehat{\boldsymbol{\mathcal{B}}} &= \arg\min_{\boldsymbol{\mathcal{B}}}\left\|\mathbf{y}_s - \sqrt{\overline{\gamma}\frac{1-\tau}{K}}\boldsymbol{\mathcal{B}}\widehat{\mathbf{h}}\right\|. \end{aligned}$$

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Hence, if there is an efficient decoder for the underlying coherent code, then the corresponding training code also has an efficient decoder, namely, the $\Phi^{\rm E-D}$ decoder. For example, one can exploit the efficient sphere-decoding algorithm when the underlying coherent code of the training code is chosen to be a linear space-time code. Note that the $\Phi^{\rm E-D}$ decoder does not, in general, coincide with the optimum noncoherent decoder [2]. Its diversity order under i.i.d. fading is summarized in the following proposition.

Proposition 1: For i.i.d. fading $(\boldsymbol{\Sigma} = \mathbf{I})$, $N_{\rm T}\boldsymbol{\mathcal{T}}^\dagger\boldsymbol{\mathcal{T}} = \mathbf{I}$ and $0 < \tau < 1$, the diversity order of the multiple block training code with the $\Phi^{\rm E-D}$ receiver is given by

$$N_{\rm R} \times \min_{\{\mathbf{B}_k\} \neq \{\mathbf{B}'_k\}} \sum_{k=1}^K {\rm rank}\left[\mathbf{B}_k - \mathbf{B}'_k\right]. \qquad (3)$$

The design criterion implied by Proposition 1 for the signals $\{\mathbf{B}_k\}_{k=1}^K$ in the training code is the same as the *sum of ranks* criterion [5] known for the design of coherent space-time codes for the multiple block i.i.d. fading channel. Hence, the existing efficiently decodable coherent codes that achieve full space and time/frequency diversity can be incorporated into the training codes structure to guarantee full spatial and temporal diversity with low complexity decoding in the noncoherent setting. The proof of Proposition 1 and a discussion on the choice of τ for i.i.d. fading is provided in [2].

In a general correlated fading scenario, however, the diversity order of the training code with the $\Phi^{\rm E-D}$ receiver need not match the diversity order of the underlying coherent code. Let $\widehat{\boldsymbol{\Sigma}} = {\rm E}[\widehat{\mathbf{h}}\widehat{\mathbf{h}}^\dagger]$ so that

$$\widehat{\boldsymbol{\Sigma}} = \boldsymbol{\Sigma}\left(\boldsymbol{\Sigma} + \left(\frac{\overline{\gamma}\tau}{K}\boldsymbol{\mathcal{T}}^\dagger\boldsymbol{\mathcal{T}}\right)^{-1}\right)^{-1}\boldsymbol{\Sigma}\,. \qquad (4)$$

The exact diversity order results for the correlated case are given in the following proposition.

Proposition 2: When $\boldsymbol{\mathcal{T}}$ is of full rank and $0 < \tau < 1$, the diversity order of a training code with the $\Phi^{\rm E-D}$ receiver is given by

$$\min_{\boldsymbol{\mathcal{B}} \neq \boldsymbol{\mathcal{B}}'} {\rm rank}\left[\widehat{\boldsymbol{\Sigma}}(\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}')^\dagger(\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}')\right]\,, \qquad (5)$$

while that of the coherent code (with perfect channel knowledge) is

$$\min_{\boldsymbol{\mathcal{B}} \neq \boldsymbol{\mathcal{B}}'} {\rm rank}\left[\boldsymbol{\Sigma}(\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}')^\dagger(\boldsymbol{\mathcal{B}} - \boldsymbol{\mathcal{B}}')\right]\,. \qquad (6)$$

Proposition 2 and equation (4) imply that if $\boldsymbol{\Sigma}$ has full rank, then $\widehat{\boldsymbol{\Sigma}}$ also has full rank and the diversity order of the training scheme is equal to that of the underlying coherent code. However, this need not be true if $\boldsymbol{\Sigma}$ is rank deficient.

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