Efficient Multiuser Cooperation Strategies Using QAM Space-Time Block Codes

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Abstract — **Practical space-time coding strategies are proposed for multiple users cooperating to communicate to a destination in a wireless channel. For a system with m single antenna users and a destination with N receive antennas, a two-phase cooperative scheme is proposed that achieves a diversity order of $m-1+N$ for the error probability. This effectively proves the achievability of the diversity order implied by the outage probability for this model. This two-phase scheme employs a short full diversity space-time block code over QAM symbols. Even though the suboptimum decoder used by the destination assumes that the inter-user communication is successful, the corresponding analysis does account for the decoding errors in the inter-user communications phase. The key to the diversity order analysis is an appropriate modification of the rule for deciding whether or not a user acts as a relay for another user. The performance analysis technique presented here also extends to another practical cooperation strategy, proposed specifically for the case of $m = 2$ and $N = 1$, that is inspired from an optimal diversity-multiplexing tradeoff curve achieving protocol.**

I. Introduction

In a wireless system with m users, denoted by $\mathcal{M} = \{1, \ldots, m\}$, that attempt to communicate to their destinations, a cooperative strategy can improve the reliability of each user's information at the receiver. Several such cooperation strategies have been proposed recently.

In the strategies such as in [1–4], the cooperative scheme is essentially divided into two phases. The first phase consists of users communicating within themselves to share information. In the second phase, the users collectively transmit their information to the destination. In [4], the advantage of such a cooperative strategy is shown by computing the diversity order of the resulting outage probability. In other works such as [1, 2], the error probability analysis is done by making the assumption that the users can always detect whether or not the inter-user communication is successful. The recent work [5] addresses performance issues for a practical two-phase cooperative scheme but it relies on approximations and only considers the case for $m = 2$.

A different cooperation strategy was proposed in [6] for two users that does not consider the division of the scheme into two distinct phases. This protocol was shown to achieve the optimal diversity-multiplexing tradeoff curve but again coding for a practical scenario was not considered.

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In the first part of this paper, we propose a practical two-phase cooperative scheme for arbitrary number of users. In the first phase of the new scheme, users transmit uncoded QAM information symbols. In the second phase, a full rate full diversity space-time code over QAM symbols is employed by the cooperating users. The performance analysis of the proposed cooperative scheme is presented in detail for a practical scenario when the users cannot always detect a decoding error during the inter-user communications phase. This analysis shows that the diversity order promised by the outage probability is indeed achievable by a specific coding scheme.

In the second part of the paper, we propose a practical cooperation scheme inspired by the protocol presented in [6] for $m = 2$ and $N = 1$. This scheme also consists of QAM information symbols for each user. However, the symbol transmitted by each user is a linear combination of its own information symbol and an estimate of the other user's information. This estimate is obtained through a process of *self-information* cancellation and maximum-likelihood decoding. The concept of self-information cancellation was not considered in [6] but is essential from a practical point of view. The performance analysis techniques presented in the first part of the paper are also applicable to this new two user scheme. The analysis accounts for the possibility of an error in the estimation of the other user's information and shows that the full diversity order of 2 is still achievable with the proposed scheme. This scheme is shown to provide better performance although at a higher complexity compared to the two-phase scheme.

This paper is organized as follows. The two-phase system model is described in Section II. The new cooperative strategy for this model is introduced in Section II-A. The performance and outage analysis of the proposed two-phase strategy is presented in Section II-B. The alternative scheme for the two user cooperation strategy is presented in Section III. Numerical results are reported in Section IV and the conclusions are given in Section V.

II. Two-phase Multiuser Cooperative System

Consider the system model proposed in [4] wherein each user is equipped with one antenna and the entire bandwidth is divided into m non-overlapping sub-bands. In the time domain, communication occurs in two phases, the first being an inter-user communications phase and the second a cooperative communications phase. The spectral efficiency of each user is R bits/s/Hz. In the following, the letters α and $\mathbf{n}$ denote fading and noise coefficients, respectively, which are i.i.d. zero-mean, unit variance complex normal random variables. In the first phase, each user transmits its information in a separate sub-band and simultaneously receives the information from the other users in the remaining sub-bands. One may write the received statistic from the k-th band at the r-th user as

$$\mathbf{y}_{r,k} = \alpha_{k,r}\mathbf{x}_k + \mathbf{n}_{r,k}, \quad (1)$$

where $\mathbf{x}_k$ is the message of user k with $E[|\mathbf{x}_k|^2] = P$ (we have replaced the term $2P/m$ in [4] by P). The user r decides to decode the data of k if the channel from k to r is not in outage, i. e., , if

$$\frac{1}{2}\log(1 + P|\alpha_{k,r}|^2) \geq R. \quad (2)$$

Hence, at the end of the first phase, for each source s, there exists a set of relays $\mathcal{D}(s)$ among the $\mathcal{M} - \{s\}$ users that are deemed to possess the correct information of s. For simplicity, consider a destination $d(s)$ for s with $N = 1$ receive antenna so that the statistic at $d(s)$ in this phase is

$$\mathbf{y}_{d(s),1} = \alpha_{s,d(s)}\mathbf{x}_s + \mathbf{n}_{d(s),1}. \quad (3)$$

In the second phase, the relays in $\mathcal{D}(s)$ cooperate to send the information of s in the s-th band for each s. The destination $d(s)$ therefore receives a signal of the form

$$\mathbf{y}_{d(s),2} = \sum_{r\in\mathcal{D}(s)} \alpha_{r,d(s)}\mathbf{x}_{r,d(s)} + \mathbf{n}_{d(s),2}, \quad (4)$$

where $\mathbf{x}_{r,d(s)}$ is the message about s relayed from r to $d(s)$ with $E[|\mathbf{x}_{r,d(s)}|^2] = P$. It is assumed that the destination knows the decoding sets $\mathcal{D}(s)$ for each s. The mutual information $\mathrm{I_{stc}}$ between the source s and $d(s)$ conditioned on $\mathcal{D}(s)$ is given by [4]

$$\mathrm{I_{stc}} = \frac{1}{2}\log(1 + P|\alpha_{s,d(s)}|^2) + \frac{1}{2}\log(1 + P\sum_{r\in\mathcal{D}(s)}|\alpha_{r,d(s)}|^2),$$

and the outage probability of this scheme is defined as

$$P_{out} = \sum_{\mathcal{D}(s)} P(\mathrm{I_{stc}} < R/\mathcal{D}(s))P(\mathcal{D}(s)). \quad (5)$$

The division of system resources into orthogonal frequency bands and time slots can alternatively be performed purely in the time domain. This interpretation is used subsequently to compare with another cooperative strategy based only on time multiplexing. The average signal-to-noise ratio of a user depends on the outage rule (2) itself and can be written as

$$SNR = \frac{P}{2m}\left(1 + (m-1)\exp\left(\frac{-(2^{2R}-1)}{P}\right)\right). \quad (6)$$

For large values of P, it is easily seen that[1] $P \doteq SNR$ and $SNR \approx \frac{P}{2}$. Therefore, the diversity order of the outage probability P_{out} can be defined with respect to either SNR or P.

For a fixed rate R, it was proved in [4] that the diversity order of P_{out} is equal to the total number of users m. A generalization of the model described above to $N > 1$ number of receive antennas was considered in [7] wherein it was shown that for a fixed rate the diversity order of the outage probability is $m + N - 1$. While the outage probability analysis provides a limit on the achievable performance of the cooperative scheme in terms of diversity order, it does not address the issue of performance when improper estimates at the relays and practical space-time coding schemes are considered.

In the following subsection, we propose a new cooperative strategy based on the above model for which we will be able to prove the achievability of the maximum diversity order implied by the outage probability at a fixed rate.

A. Proposed Two-Phase Cooperative Scheme

We utilize the LPST code construction proposed in [8]. Let $\mathcal{I}$ represent the standard 2^{2R}-QAM constellation scaled such that its average energy is P. For $M = 2^\nu, \nu \geq 1$, the 1-layer LPST code consists of $M \times M$ codewords of the form

$$\begin{bmatrix} z_1 & z_2 & z_3 & \dots & z_M \\ iz_M & z_1 & z_2 & \dots & z_{M-1} \\ iz_{M-1} & iz_M & z_1 & \dots & z_{M-2} \\ \dots & \dots & \dots & \dots & \dots \\ iz_2 & iz_3 & \dots & iz_M & z_1 \end{bmatrix}, \quad (7)$$

where the information symbols $z_1, \dots, z_M$ are drawn from $\mathcal{I}$. The LPST code given above exhibits the full transmit diversity of M [8]. Therefore, the space-time code of size $p \times M (1 \leq p \leq M)$ obtained by using any p fixed rows of the LPST code in (7) will exhibit a transmit diversity of p.

In order to use the LPST code in a cooperative strategy for m users, we require the coherence time T of the channel to be of the form $T = 2 \cdot 2^\nu, \nu \geq 1$, with $m - 1 \leq \frac{T}{2}$ if $m > 2$. If $m = 2$, then no constraint on T is needed. We set $M = \frac{T}{2}$ and define a frame as consisting of $T = 2M$ time slots. Each frame is divided into two equal length phases. Consider the following sequence of steps.

- In the first phase, each source s sends a stream of independent information symbols $x_{s,1}, \dots, x_{s,M}$ from $\mathcal{I}$.
- Each relay $r \in \mathcal{M} - \{s\}$ decides to be in the decoding set $\mathcal{D}(s)$ if the following condition is satisfied,

$$\frac{1}{2}\log(1 + P|\alpha_{s,r}|^2) \geq c(P) \cdot R, \quad (8)$$

where $c(P) > 0$ is a function of P. Setting $c(P) = 1$ gives the outage based rule in (2). In general, the function $c(P)$ is the key to achieving the maximum possible diversity order for the cooperative system model. Specific choices of $c(P)$ will be discussed in Section II-B.

- Each relay $r \in \mathcal{D}(s)$ makes a maximum-likelihood (ML) estimate $E_l^{r,s}$ of the data of $\mathbf{x}_{s,l}$ of s using the received signal $\mathbf{y}_{r,s}^l = \alpha_{s,r}\mathbf{x}_{s,l} + \mathbf{n}_{r,s}^l$ as

$$E_l^{r,s} = \arg\min_{u\in\mathcal{I}} |\mathbf{y}_{r,s}^l - \alpha_{s,r}u|^2, \quad 1 \leq l \leq M. \quad (9)$$

- In the second phase, each relay in $\mathcal{D}(s)$ creates a predetermined row of the space-time code matrix in (7) based on its own estimate of the data of the source s. Specifically, let $P(r,s)$ denote the position of r in the index set $(1, \dots, s-1, s+1, \dots, m)$ for $r \neq s$. Then, the stream sent

[1] $f \doteq g$ means that $\lim_{SNR\to\infty} \frac{\log(f)}{\log(SNR)} = \lim_{SNR\to\infty} \frac{\log(g)}{\log(SNR)}$

by the relay $r \in \mathcal{D}(s)$ in the s-th band in the second phase is given by

$$[iE^{r,s}_{M-p+2}, \dots, iE^{r,s}_{M}, E^{r,s}_1, \dots, E^{r,s}_{M+1-p}], \qquad (10)$$

where $p = P(r, s)$.

• The destination processes the received statistics from both phases with the assumption that the estimates $E^{r,s}_l$ are the actual data symbols $x_{s,l}$ and applies the sphere-decoding algorithm [9] with a rearranged channel matrix. Note that this does not correspond to the optimum maximum-likelihood detection rule at the destination because we do not consider the average likelihood obtained from all possible values of the estimates of the relays. Our simplified rule, however, is easier to implement and additionally enables a performance analysis of the proposed scheme. Moreover, the proposed scheme is sufficient to achieve the maximum possible diversity order implied by the outage probability. The outage probability of the proposed scheme is given by (5) but with the probability $P(\mathcal{D}(s))$ evaluated according to the decoding set rule defined by (8).

Note that the average signal-to-noise ratio SNR is now given by (6) with R replaced by $c(P)R$. Thus, if

$$\lim_{P\to\infty} \frac{2^{2c(P)R} - 1}{P} = 0, \qquad (11)$$

then we get that $P \doteq SNR$ and the diversity order of the outage probability and the error probability can be defined with respect to either SNR or P. The total number of bits per frame with this scheme is given by mRT bits.

The proposed cooperative scheme and the corresponding error probability analysis to follow differ from the other strategies proposed recently [1, 2] due to two reasons. Firstly, we do not use any convolutional or binary outer code in the proposed scheme and concentrate on a short length space-time block code. Secondly, for the performance analysis, we will not assume that the relays know the source symbols exactly, irrespective of the channel state. This is in contrast to [1, 2, 4] where a CRC check or an outage based rule was argued to be sufficient to decide whether the inter-user communication is successful.

The advantage of using the LPST code in the proposed strategy instead of orthogonal designs is that the LPST code exhibits a rate of 1 symbol per channel use even for $M > 2$. The 1-layer LPST code transmits QAM information symbols and exhibits the least possible value of the Peak-to-mean Power Ratio among the class of linear space-time codes [8]. The proposed scheme is also easily generalized to the case when the destination is equipped with $N > 1$ receive antennas.

B. Performance and Outage Analysis

In this subsection, we consider specific choices of the function $c(P)$ and analyze the resulting diversity order of the frame error probability and the outage probability for the cooperative scheme proposed in Section II-A. In each case, we will see that (11) is satisfied so that $P \doteq SNR$.

The following theorem provides a set of functions with which the diversity order of the error probability with the proposed scheme can be made arbitrarily close to $m - 1 + N$.

Theorem 1: For the cooperative scheme proposed in Section II-A, if $c(P)$ is made to depend on P as

$$c(P) = \begin{cases} \frac{\delta}{2(m-1)R} \log_2(P) & \text{if } P > 1, \\ 1 & \text{otherwise}, \end{cases} \qquad (12)$$

for some $0 < \delta < m - 1$, then the diversity order of the frame error probability for each user is at least $m - 1 + N - \delta$. Also, in this case, $P \doteq SNR$.

Proof: With the choice of $c(P)$ stated in this theorem, the condition (11) is met so that $P \doteq SNR$ and the diversity order can be defined with respect to P. Consider the frame error probability of user 1. Let $\mathbf{x}_1, \dots, \mathbf{x}_M$ be a sequence of information symbols sent by the first user in the first phase of a frame. Then, the pairwise error probability between the sequence $\{\mathbf{x}_i\}_{i=1}^M$ and another distinct sequence $\{\mathbf{y}_i\}_{i=1}^M$ can be written as

$$P(\mathbf{x} \to \mathbf{y}) = \sum_{\mathcal{D}(1) \subset \{2,\dots,m\}} P(\mathbf{x} \to \mathbf{y}/\mathcal{D}(1))P(\mathcal{D}(1)). \quad (13)$$

Let $\mathcal{O}$ denote one possible decoding subset $\mathcal{D}(1)$ in the summation given above and consider the product $P(\mathbf{x} \to \mathbf{y}/\mathcal{O})P(\mathcal{O})$. Let $P > 1$ so that $c(P)$ is given by the first line in (12). The probability of $\mathcal{O}$ is

$$\begin{aligned} P(\mathcal{O}) &= \prod_{k\in\mathcal{O}} P\left(|\alpha_{1,k}|^2 \geq \frac{2^{2c(P)\cdot R} - 1}{P}\right) \times \\ &\quad \prod_{k\notin\mathcal{O}} P\left(|\alpha_{1,k}|^2 < \frac{2^{2c(P)\cdot R} - 1}{P}\right) && (14) \\ &= \left[\exp\left(\frac{-(P^{\frac{\delta}{m-1}} - 1)}{P}\right)\right]^{|\mathcal{O}|} \times \\ &\quad \left[1 - \exp\left(\frac{-(P^{\frac{\delta}{m-1}} - 1)}{P}\right)\right]^{m-1-|\mathcal{O}|} && (15) \\ &\leq \left(\frac{P^{\frac{\delta}{m-1}} - 1}{P}\right)^{m-1-|\mathcal{O}|} && (16) \\ &\leq \frac{1}{P^{(1-\frac{\delta}{m-1})\cdot(m-1-|\mathcal{O}|)}}, && (17) \end{aligned}$$

where the upper bounds are obtained by using $0 \leq 1 - e^{-x} \leq x, \forall x \geq 0$, and by disregarding the 1 in the last step.

If $|\mathcal{O}| = 0$, then no space-time codeword for the information of user 1 is sent in the second phase and the diversity order of $P(\mathbf{x} \to \mathbf{y}/\mathcal{O})$ is N. Otherwise, if $|\mathcal{O}| \neq 0$, then we upper bound $P(\mathbf{x} \to \mathbf{y}/\mathcal{O})$ by distinguishing between the two possible scenarios depending on whether the symbols decoded at the relays in $\mathcal{O}$ are in error or not. Let $\mathcal{E}$ denote the event $\{E^{1,k}_l = \mathbf{x}_l, \forall k \in \mathcal{O}, 1 \leq l \leq M\}$ and let $\overline{\mathcal{E}}$ be the complement of $\mathcal{E}$. Then, we write

$$\begin{aligned} P(\mathbf{x} \to \mathbf{y}/\mathcal{O}) &= P(\mathbf{x} \to \mathbf{y}/\mathcal{O}, \mathcal{E})P(\mathcal{E}/\mathcal{O}) + \\ &\quad P(\mathbf{x} \to \mathbf{y}/\mathcal{O}, \overline{\mathcal{E}})P(\overline{\mathcal{E}}/\mathcal{O}) && (18) \\ &\leq P(\mathbf{x} \to \mathbf{y}/\mathcal{O}, \mathcal{E}) + P(\overline{\mathcal{E}}/\mathcal{O}), && (19) \end{aligned}$$

wherein we have upper bounded the omitted probabilities by 1. We shall now show that due to the choice of the function $c(P)$

given by (12), the term $P(\overline{\mathcal{E}}/\mathcal{O})$ decays exponentially with P. For any information symbol $\mathbf{z}$ in the QAM constellation $\mathcal{I}$, let the pair $(\xi_{1,\mathbf{z}}, \xi_{2,\mathbf{z}})$ be defined as $(1,1)$ if $\mathbf{z}$ is a corner point, $(1,2)$ if $\mathbf{z}$ is a point on the edge but not a corner point and $(2,2)$ if $\mathbf{z}$ is any other point in the interior. Let $Q(.)$ denote the standard Q-function corresponding to a standard normal random variable, namely, $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$. Then, evaluating the expectation over all the independent noise variates involved, we get that

$$P(\overline{\mathcal{E}}/\mathcal{O}) = \mathrm{E}\left[1 - \prod_{k\in\mathcal{O}}\prod_{i=1}^{M}\left(1 - \xi_{1,\mathbf{x}_i} Q\left(\frac{|\alpha_{1,k}|\sqrt{\kappa P}}{2}\right)\right) \times \left(1 - \xi_{2,\mathbf{x}_i} Q\left(\frac{|\alpha_{1,k}|\sqrt{\kappa P}}{2}\right)\right)\right],$$

where the expectation is over the conditional density of the fading coefficients $\alpha_{1,k}, k \in \mathcal{O}$, conditioned on $\mathcal{O}$, i. e., conditioned on $|\alpha_{1,k}|^2 \geq \frac{P^{\frac{\delta}{m-1}}-1}{P}$. The constant κ only depends on R. Now, using the inequality $1 - \prod_{i=1}^{A}(1-r_i) \leq \sum_{i=1}^{A} r_i$ when $0 \leq r_i \leq 1, \forall i$, we get the upper bound

$$\begin{aligned} P(\overline{\mathcal{E}}/\mathcal{O}) &\leq \mathrm{E}\left[\sum_{k\in\mathcal{O}}\sum_{i=1}^{M}(\xi_{1,\mathbf{x}_i} + \xi_{2,\mathbf{x}_i}) Q\left(\frac{|\alpha_{1,k}|\sqrt{\kappa P}}{2}\right)\right] \\ &= \left(\sum_{i=1}^{M}(\xi_{1,\mathbf{x}_i} + \xi_{2,\mathbf{x}_i})\right)\cdot \sum_{k\in\mathcal{O}} \mathrm{E}\left[Q\left(\frac{|\alpha_{1,k}|\sqrt{\kappa P}}{2}\right)\right] \\ &\leq \left(\sum_{i=1}^{M}(\xi_{1,\mathbf{x}_i} + \xi_{2,\mathbf{x}_i})\right)\cdot \sum_{k\in\mathcal{O}} \mathrm{E}\left[\exp\left(\frac{-|\alpha_{1,k}|^2\kappa P}{8}\right)\right], \end{aligned} \quad (20)$$

where the last step follows from the Chernoff bound $Q(x) \leq \exp(\frac{-x^2}{2})$. The conditional density of $|\alpha_{1,k}|^2$ conditioned on $\mathcal{O}$ is $\frac{e^{-|\alpha_{1,k}|^2}}{\exp(-\frac{P^{\frac{\delta}{m-1}}-1}{P})} \cdot \chi_{[\frac{P^{\frac{\delta}{m-1}}-1}{P},\infty)}(|\alpha_{1,k}|^2)$, where $\chi_I(.)$ is the indicator function for the set I. Therefore, the required expectation in (20) can be evaluated as

$$\begin{aligned} &\sum_{k\in\mathcal{O}} \mathrm{E}\left[\exp\left(\frac{-|\alpha_{1,k}|^2\kappa P}{8}\right)\right] = \\ &\sum_{k\in\mathcal{O}} \frac{\int_{|\alpha|^2 \geq \frac{P^{\frac{\delta}{m-1}}-1}{P}} \exp(-|\alpha|^2(1+\frac{\kappa P}{8}))d|\alpha|^2}{\exp(-\frac{P^{\frac{\delta}{m-1}}-1}{P})} \quad (21) \\ &= |\mathcal{O}| \cdot \frac{\exp\left(-\frac{P^{\frac{\delta}{m-1}}-1}{8\kappa^{-1}}\right)}{1+\frac{\kappa P}{8}}, \quad (22) \end{aligned}$$

which decays exponentially with P because $\delta > 0$. Hence, it follows from (20) that $P(\overline{\mathcal{E}}/\mathcal{O})$ also decays at least exponentially with P. The other term $P(\mathbf{x} \to \mathbf{y}/\mathcal{O}, \mathcal{E})$ in (19) corresponds to the case wherein all relays possess the correct information symbols of user 1. This, along with the fact that we are using sub-matrices of a full diversity space-time code in the second phase, implies that the diversity order of the pairwise error probability $P(\mathbf{x} \to \mathbf{y}/\mathcal{O}, \mathcal{E})$ is at least $(1 + |\mathcal{O}|)N$.

Hence, the diversity order of $P(\mathbf{x} \to \mathbf{y}/\mathcal{O})$ is at least $(1 + |\mathcal{O}|)N$ for all possible decoding sets $\mathcal{O}$. Combining this with (17), we see that the diversity order of $P(\mathbf{x} \to \mathbf{y}/\mathcal{O})P(\mathcal{O})$ is at least

$$\begin{aligned} (1 - \frac{\delta}{m-1}) \cdot (m - 1 - |\mathcal{O}|) + (1 + |\mathcal{O}|)N &= \\ m - 1 + N - \delta + |\mathcal{O}|(N - 1 + \frac{\delta}{m-1}). & \quad (23) \end{aligned}$$

Thus, from (13), the diversity order of $P(\mathbf{x} \to \mathbf{y})$ is at least

$$\begin{aligned} &\min_{\mathcal{D}(1)\subset\{2,\ldots,m\}} [m - 1 + N - \delta + |\mathcal{D}(1)|(N - 1 + \frac{\delta}{m-1})] \\ &= m - 1 + N - \delta. \quad (24) \end{aligned}$$

From the union bound to the frame error probability, we conclude that the diversity order for user 1, and by symmetry for each user, is at least $m - 1 + N - \delta$. ■

Thus, by an appropriate choice of δ, any diversity order close to $m - 1 + N$ can be guaranteed by the proposed scheme in spite of the data estimation errors at the relays. Let us denote the function given by (12) as $h_1(P)$. We show next that the lower bound on the diversity order given by Theorem 1 matches the diversity order of the outage probability when $h_1(P)$ is the function used for $c(P)$.

Theorem 2: When $c(P) = h_1(P)$, the diversity order of the outage probability of the proposed scheme is $m - 1 + N - \delta$.

Proof: The diversity order of $P(\mathrm{I_{stc}} < R/\mathcal{D}(s))$ is obtained in [7] to be $N(|\mathcal{D}(s)| + 1)$. When $c(P) = h_1(P)$, the diversity order of $P(\mathcal{D}(s))$ is $(m - |\mathcal{D}(s)| - 1)(1 - \frac{\delta}{m-1})$ as seen from (17). Thus, the diversity order of $P(\mathrm{I_{stc}} < R/\mathcal{D}(s))P(\mathcal{D}(s))$ matches the term in (23). Taking the minimum over all $\mathcal{D}(s)$, we get that the diversity order of the outage probability $\sum_{\mathcal{D}(s)} P(\mathrm{I_{stc}} < R/\mathcal{D}(s))P(\mathcal{D}(s))$ is also $m - 1 + N - \delta$. ■

Even though the diversity order of the outage probability of the proposed scheme is δ-smaller than the previously known result (i. e., for $c(P) = 1$), we have been able to show the achievability of this diversity order by a specific coding scheme due to Theorem 1. In the next theorem, we propose another function for $c(P)$ that bridges even the above mentioned diversity gap of δ and achieves the diversity order of $m - 1 + N$ exactly.

Theorem 3: For the cooperative scheme proposed in Section II-A, if $c(P)$ is made to depend on P as

$$c(P) = \begin{cases} \frac{1}{R}\log_2(\log_e(P)) & \text{if } P > e, \\ 1 & \text{otherwise} \end{cases} \quad (25)$$

then the diversity order of the frame error probability for each user is at least $m - 1 + N$. Also, in this case, $P \doteq SNR$.

Proof: For $P > e$, we can write $c(P)$ as

$$c(P) = \frac{\delta(P)}{2(m-1)R}\log_2 P, \quad (26)$$

where $\delta(P) = 2(m-1)(\log_P(\log_e(P)))$. We now note that the expression for $c(P)$ is similar to the one in Theorem 1, except that the constant δ has been replaced by the function $\delta(P)$. Tracing the steps in the proof of Theorem 1, we only need to prove the following facts.

• The term corresponding to $\exp\left(-\frac{P^{\frac{\delta}{m-1}}-1}{8\kappa^{-1}}\right)$ in (22) has infinite diversity order, i. e.,

$$\lim_{P\to\infty} \frac{P^{\frac{\delta(P)}{m-1}}}{\log_e P} = \infty. \quad (27)$$

• The diversity order expression $m-1+N-\delta$ becomes $m-1+N$, i. e., $\lim_{P\to\infty}\delta(P)=0$. This fact is also needed to satisfy $0<\delta(P)<m-1$ for large enough P.

• (11) is satisfied so that $P\doteq SNR$.

Each of these facts follows directly from the expression for $\delta(P)$. We mainly need to note that $P^{\frac{\delta(P)}{m-1}}=(\log_e P)^2$ and that $\delta(P)=2(m-1)\frac{\log_e(\log_e P)}{(\log_e P)}$. ■

Let us denote the function given by (25) as $h_2(P)$. The following result on the outage probability with $c(P)=h_2(P)$ follows from the proof of Theorem 2 because $\lim_{P\to\infty}\delta(P)=0$.

Theorem 4: When $c(P)=h_2(P)$, the diversity order of the outage probability of the proposed scheme is $m-1+N$.

In each of the above theorems, one can replace the function $c(P)$ by $\max(1,c(P))$, without changing the conclusions, so that the multiplicative factor in (8) is always greater than 1.

III. Alternative Cooperative Scheme for $m=2$

Another practical cooperative scheme for the special case of $m=2$ and $N=1$ is described now. We assume that the inter-user fading coefficient $\alpha_{1,2}(=\alpha_{2,1})$ and the source-destination fading coefficients $\alpha_{1,d}$ and $\alpha_{2,d}$ remain fixed during a frame of length n time slots. Only one user transmits in a given time slot. Let $\mathbf{t}_k$ denote the signal transmitted in the k-th time slot with $\mathbf{t}_k$ transmitted by user 1 for odd k and by user 2 for even k. Let $\mathbf{x}_{1,k}$ represent the information symbol for user 1 in time slot k for odd k and $\mathbf{x}_{2,k}$ that of user 2 for even k. Each information symbol is drawn from a 2^R-QAM constellation $\mathcal{I}$ with average energy P. Denote the noise coefficients and the received statistics at any user by the letters $\mathbf{n}$ and $\mathbf{r}$, respectively. All noise and fading coefficients are i.i.d zero-mean unit variance complex normal random variables.

The users cooperate in the frame if the inter-user channel is sufficiently good, i. e., if

$$\log(1+P|\alpha_{1,2}|^2)\ge c(P)\cdot R, \quad (28)$$

where $c(P)$ is some function of P to be specified later.

If (28) is met, then the following sequence of steps is followed. Let a and b be fixed scalars with $|a|^2+|b|^2=1$.

• User 1 sends $\mathbf{t}_1=a\mathbf{x}_{1,1}$ in the first slot. User 2 receives $\mathbf{y}_1=\alpha_{1,2}a\mathbf{x}_{1,1}+\mathbf{n}_1$ in this slot and forms the ML estimate $\hat{\mathbf{x}}_{1,1}=\arg\min_{\mathbf{u}\in\mathcal{I}}|\mathbf{y}_1-\alpha_{1,2}a\mathbf{u}|^2$ of the first user's data.

• In time slot 2, user 2 transmits $\mathbf{t}_2=a\mathbf{x}_{2,2}+b\hat{\mathbf{x}}_{1,1}$. Now, user 1 receives $\mathbf{y}_2=\alpha_{1,2}\mathbf{t}_2+\mathbf{n}_2$ and it subtracts its own information from time slot 1 to form $\tilde{\mathbf{y}}_2=\mathbf{y}_2-\alpha_{1,2}b\mathbf{x}_{1,1}$. The modified statistic is now used to form an estimate of the data $\mathbf{x}_{2,2}$ of user 2 as $\hat{\mathbf{x}}_{2,2}=\arg\min_{\mathbf{u}\in\mathcal{I}}|\tilde{\mathbf{y}}_2-\alpha_{1,2}a\mathbf{u}|^2$. This process of *self-information* cancellation and subsequent ML decoding is similarly performed in the time slots to follow.

• For each odd time slot k, $3\le k<n$, user 1 sends $\mathbf{t}_k=a\mathbf{x}_{1,k}+b\hat{\mathbf{x}}_{2,k-1}$. User 2 receives $\mathbf{y}_k=\alpha_{1,2}\mathbf{t}_k+\mathbf{n}_k$ in this slot and forms the estimate $\hat{\mathbf{x}}_{1,k}=\arg\min_{\mathbf{u}\in\mathcal{I}}|\mathbf{y}_k-\alpha_{1,2}b\mathbf{x}_{2,k-1}-\alpha_{1,2}a\mathbf{u}|^2$.

• For each even time slot k, $4\le k<n$, user 2 sends $\mathbf{t}_k=a\mathbf{x}_{2,k}+b\hat{\mathbf{x}}_{1,k-1}$. User 1 receives $\mathbf{y}_k=\alpha_{1,2}\mathbf{t}_k+\mathbf{n}_k$ in this slot and forms the estimate $\hat{\mathbf{x}}_{2,k}=\arg\min_{\mathbf{u}\in\mathcal{I}}|\mathbf{y}_k-\alpha_{1,2}b\mathbf{x}_{1,k-1}-\alpha_{1,2}a\mathbf{u}|^2$.

• In the last time slot, the appropriate user does not send any new information and the transmission is either $b\hat{\mathbf{x}}_{2,n-1}$ or $b\hat{\mathbf{x}}_{1,n-1}$ depending on whether n is odd or even.

• The destination assumes that all the estimates for both the users are error-free and employs a sphere decoder of size $n-1$ complex dimensions to perform a joint decoding of the information symbols $\{\mathbf{x}_{1,2i+1}\}$ and $\{\mathbf{x}_{2,2i}\}$. Note again that this does not correspond to the optimum detection rule at the destination. Nevertheless, this detector is easier to implement and its performance can also be characterized.

If (28) is not met, then the transmitted signals are simply given by

$$\mathbf{t}_k=\mathbf{x}_{[k+1 \bmod 2]+1,k}, \quad 1\le k\le n,$$

which means that each user transmits only its own information in their alloted time slots. The destination can easily decode each information symbol separately in this case. Note that the destination is assumed to know whether (28) is met or not but not the exact value of $\alpha_{1,2}$.

The scheme described above will be referred to as the Self-information Cancelling Linear (SCL) cooperative scheme. The average signal-to-noise ratio SNR for each user in this scheme is equal to $P/2$ for reasonably large n. The effective number of bits per frame is given by $R(n-1)\approx Rn$ bits for large n. While comparing with the two-phase scheme of Section II-A, we would need to use $n=2T$ time slots of half the interval length to obtain the same rate within the same coherence time.

The analytic tools described in the previous section can be applied to a performance analysis of the SCL scheme as well. The coefficients (a,b) play an important role in the performance of the SCL scheme. The properties of Full Modulation Diversity (FMD) matrices (for e.g. from [10]) are invoked to obtain the following result.

Theorem 5: For the SCL scheme proposed in this section, let the row vector $[\ a \quad b\ ]$ correspond to any appropriately scaled row of a two dimensional FMD generator matrix. Then, with $c(P)=2h_1(P)$, the diversity order of the frame error probability is equal to $2-\delta$. Also, if $c(P)=2h_2(P)$, then the frame error probability exhibits the full diversity order of 2.

Compared to the two-phase strategy, one should expect the SCL cooperative scheme to have a better performance due to a smaller QAM constellation size. However, a sphere decoder of size $n-1$ complex dimensions needs to be applied for the SCL scheme compared to just 1 complex dimensions for the two-phase strategy for two users. For larger coherence times, one may use several independent blocks of length n within a frame to limit the complexity of the scheme.

IV. Numerical Results

The advantages of the proposed cooperative strategy are now shown by simulations. We also compare the various cooperative schemes with the non-cooperative scheme of [4] wherein each user transmits in its own sub-band for all time. For the non-cooperative scheme at a spectral efficiency of R, a QAM constellation of size 2^R and average energy P is used so that the signal-to-noise ratio becomes $SNR = \frac{P}{m}$.

Consider the example with $m = 5$ users, framelength of $T = 10$, rate $R = 2$ bits/s/Hz and $N = 1$ receive antenna. In Figure 1, we simulate the outage probability of the proposed cooperative scheme with $c(P)$ chosen as $h_1(P)$ (from Theorem 1) and for $\delta = 1, 2$ and 3. The outage probability of the outage based rule and that of the non-cooperative scheme are also shown in the same figure. As predicted by Theorem 2, the diversity order of outage probability of the proposed scheme increases as δ decreases and it approaches the diversity order for the outage based rule. More importantly, we see that the value of the outage probabilities are significantly smaller for the proposed strategy as compared to the outage based rule for practical SNR values. In Figure 2, we compare the simulated frame error probability of the cooperative scheme with the outage based rule and the proposed rules given by Theorems 1 and 3. We see that the outage based rule performs the worst and it does not even improve upon the non-cooperative scheme. This shows the serious limitation of the outage based rule for a practical coding scheme. The rules given by Theorems 1 and 3 considerably improve upon the performance of the outage based rule and provide a gain of $6-8$ dB in the figure shown.

In Figure 3, we compare the performance of the two-phase scheme with the SCL scheme for $m = 2$ and $N = 1$. Both schemes are implemented in time multiplexed form with a coherence time of 20 time slots. For the SCL scheme, two blocks with $n = 10$ are used within a frame. For the two-phase scheme, we set $T = 20$. To achieve the same spectral efficiency, we use the constellation 4-QAM with the SCL scheme and 16-QAM with the two-phase scheme. The SCL scheme is of higher complexity but provides up to a 5 dB improvement in performance compared to the two-phase scheme.

V. Conclusions

In order to device practical multiuser cooperative coding schemes to realize the promise of outage analysis, one must use good cooperation rules that are practical counterparts of the outage based rule and analyze their performance by not making the assumption of perfect inter-user communication. Towards that end, new multiuser cooperation strategies are proposed based on QAM information symbols. A modification in the rule to decide the extent of user cooperation not only aids the performance analysis of the proposed schemes but is a practical necessity to get good performance.

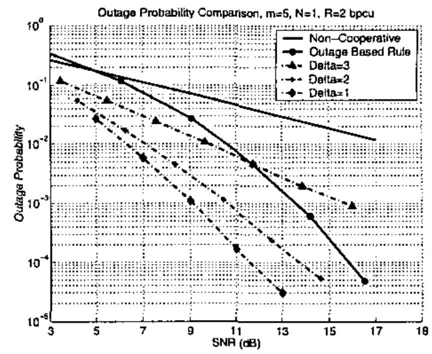

Fig. 1. Outage Probability Comparison

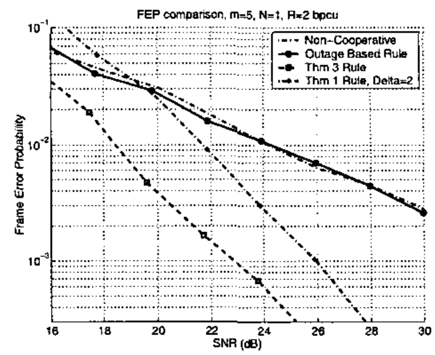

Fig. 2. Frame Error Probability Comparison

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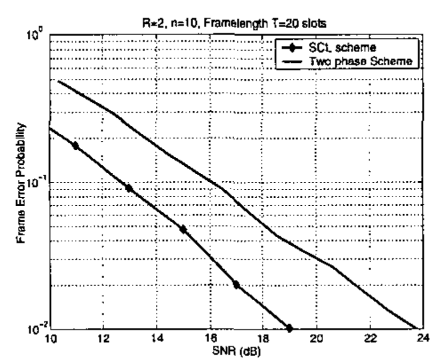

Fig. 3. Frame Error Probability for $m = 2$ users