Generalized Multiuser QAM based Cooperation Strategy and an Optimal Two-User Scheme*

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Abstract

A two user cooperation strategy based on QAM information symbols, known as the self-information canceling linear (SCL) scheme, was recently proposed by the authors. The SCL scheme achieves the full diversity order of 2 *and the full multiplexing gain of* 1 *inherent in the system. However, the diversity versus multiplexing gain tradeoff (DMT) of the SCL scheme was not shown to be optimal. In the first part of this work, we propose a new two user cooperation strategy that achieves the optimal DMT curve for the two user cooperative diversity system. This scheme is obtained by a suitable modification of the original SCL scheme. The modified SCL scheme is also a practical cooperation strategy based on QAM information symbols and the present analysis accounts for the inter-user communications errors. The modified SCL scheme is, thus, established as a specific coding scheme to achieve the information theoretic limits within the DMT framework. In the second part of this work, we generalize the original SCL scheme to the case of* m *users for any* $m > 2$*. The generalization leads to a practical multiuser cooperative scheme based on QAM symbols that provides the full diversity order of* m*, fair rate allocation among the users and potential for achieving the full multiplexing gain.*

1 Introduction

Multiuser cooperation strategies have recently appeared as a way to increase the reliability of the information of several users that attempt to communicate to a destination over a wireless channel. We assume here that all users and the destination are equipped with only one antenna. In recent works by the authors [1, 2], practical cooperative coding schemes based on quadrature amplitude modulation (QAM) symbols have been proposed and their diversity order analysis have been performed. The analysis accounts for the inter-user communication errors in a systematic manner. The diversity order analysis of these schemes for fixed rate was performed in [1]. The diversity order for the rate of the QAM constellation increasing with the signal-to-noise ratio (SNR), referred to as a diversity versus multiplexing gain tradeoff (DMT) analysis, was performed in [2].

A two user cooperative strategy known as the self-information canceling linear (SCL) scheme was proposed in [1] and proved to have the maximum diversity order of 2 for a fixed rate. A DMT analysis of this scheme done in [2] showed that the SCL scheme also achieved the maximum multiplexing gain of 1 inherent in the system. However, the DMT curve was not shown to be the best possible.

The first major contribution of this work is a modification of the SCL scheme which allows it to achieves the optimal DMT curve. In the SCL scheme of [1], when the two users cooperate, they transmit a linear combination of their information symbol and an estimate of the other user's information from the previous time slot. In the modified SCL scheme, when the two users cooperate, one of the users transmits a linear combination of its information symbol and an appropriate conjugate of the entire signal transmitted by the other user in the previous time slot. The appropriate choice of this conjugate improves a certain product distance between the signals of the two users. This new structure of the signals sent by the users also simplifies the DMT analysis of the modified SCL scheme compared to that of the SCL scheme in [2].

The second major contribution of this work is the extension of the SCL scheme for an arbitrary number m of users with $m > 2$. This extension is derived by first abstracting the technical details from the proofs of [1, 2] and providing certain design rules for building cooperative schemes based on QAM symbols. An inductive approach is presented for the construction of the SCL scheme for more than two users. The proposed construction provides the maximum diversity order of m and is fair in terms of rate allocation among the users. Also, the generalized construction is such that almost one new information symbol is transmitted in every time slot. This potentially allows this scheme to achieve the maximum multiplexing gain of one inherent in the system.

This paper is organized as follows. In Section 2, we quickly review the SCL scheme and the corresponding

*This work was supported in part by NSF grants CCR-0112977 and CCF-0431170

DMT result derived in [2]. In Section 3, the modified SCL scheme is proposed and the optimality in terms of the DMT curve is proved. In Section 4, the generalization of the SCL scheme for more than two users is presented. In Section 5, we summarize our conclusions.

2 SCL Scheme

We now review the DMT analysis of the SCL scheme performed in [2]. This scheme is meant for the two user cooperative diversity system shown in Figure 1. The inter-user fading coefficient $\mathbf{h}$ and the fading coefficients $\mathbf{g}_1$ and $\mathbf{g}_2$ from the two users to the destination are all independent zero-mean unit-variance complex normal random variables.

Let the coherence time $T = n$ time slots with $n >> 1$. Each of the two users in the SCL scheme are active in alternate time slots. The information symbol for each user is drawn from a 2^R-QAM constellation of average energy P, where R is the spectral efficiency of the scheme. The information symbols are scaled by a factor θ. The two users cooperate if

$$\log(1 + \theta^2 P|\mathbf{h}|^2) \geq cR, \qquad (1)$$

where $c > 1$ is a constant. When (1) is met, the scheme is described as follows. In the first time slot, the first user transmits its information symbol $\mathbf{x}_1$. Subsequently, for time slot $t \in [2, n-1]$, the signal sent by a user is a linear combination of its own information symbol $\mathbf{x}_t$ and an estimate $\hat{\mathbf{x}}_{t-1}$ of the other users information of time slot $t-1$. This estimate of the other users data is obtained using the statistic obtained in time slot $t-1$, by first subtracting its own information symbol from time slot $t-2$ and then performing a maximum-likelihood (ML) decoding. Let (a, b) be the coefficients of the linear combination with $|a|^2 + |b|^2 = 1$. In the last time slot, no new information is transmitted and the active user transmits the estimate of the information of the other user for time slot $n-1$. The destination processes the received signal assuming that all the estimates are error free and performs an equivalent ML decoding under that assumption using the sphere-decoding algorithm [3].

A compact representation of this scheme when (1) is met is provided in the following matrix. The two rows of this matrix correspond to the signals transmitted by the two users. The columns denote the signals transmitted during any given time slot.

$$\theta \begin{bmatrix} a\mathbf{x}_1 & 0 & a\mathbf{x}_3 + b\hat{\mathbf{x}}_2 & 0 & \cdots & \cdots & a\mathbf{x}_{n-1} + b\hat{\mathbf{x}}_{n-2} & 0 \\ 0 & a\mathbf{x}_2 + b\hat{\mathbf{x}}_1 & 0 & a\mathbf{x}_4 + b\hat{\mathbf{x}}_3 & \cdots & \cdots & 0 & b\hat{\mathbf{x}}_{n-1} \end{bmatrix} \qquad (2)$$

When (1) is not met, there is no cooperation and in each time slot the active user only transmits its own information symbol.

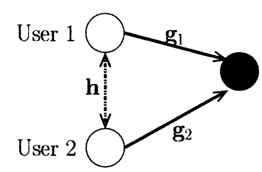

Figure 1: Two user system

For a multiplexing gain r, we have that $R = r\log(SNR)$ which means that the average energy P of the QAM constellation satisfies $P \doteq SNR^r$. The constant θ is now chosen as $\theta^2 \doteq SNR^{1-r}$ so that $\theta^2 P \doteq SNR$.

The coefficients (a, b) play an important role in the performance of the SCL scheme. These coefficients have to be chosen from any appropriately scaled row of a two dimensional Full Modulation Diversity (FMD) generator matrix $\mathbf{G}$ (for e.g. from [4]). The unnormalized minimum product distance of the lattice generated by $\mathbf{G}$ is defined as

$$d_p = \min_{\substack{\mathbf{u} \in Z[i]^2 \setminus \mathbf{0} \\ \mathbf{x} = \mathbf{G}\mathbf{u}}} |\mathbf{x}_1 \mathbf{x}_2|^2$$

When $d_p > 0$, the diversity order of the error probability with the SCL scheme was found to be at least

$$\begin{array}{ll} 2 - 3r, & 0 < r \leq 0.5, \\ 1 - r, & 0.5 \leq r < 1. \end{array}$$

Thus, the SCL scheme achieves both maximum diversity order and the maximum multiplexing gain inherent in the system. However, it is not clear if the tradeoff of diversity versus multiplexing for this scheme is optimal.

An upper bound on the DMT curve would correspond to the case that the two users can fully cooperate. In other words, the DMT curve of the cooperative two user system can not be above the DMT curve for a single user two transmit antenna system. Thus, an upper bound to the best achievable diversity order is $2(1-r)$ for $r \in (0, 1)$ [5]. In the next section, we propose a modification of the SCL scheme and prove that it achieves this upper bound on the diversity order. The modified SCL scheme is, thus, optimal in terms of the DMT curve for this system.

3 Optimal two-user cooperation protocol

In this section, we present the optimal DMT curve achieving cooperative diversity protocol for two users based on the QAM alphabet. Let the matrix $\mathbf{G}$ given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad (3)$$

be a two dimensional FMD matrix such that $|a|^2 + |b|^2 = 1$ and $|c|^2 + |d|^2 = 1$. Let c_1 and c_2 be two complex numbers such that $|c_1|^2 + |c_2|^2 = 1$. In order to get the maximum multiplexing gain, we assume that $n >> 1$ since we eventually transmit $n-1$ information symbols in n time slots. For the sake of presentation, we only consider the case that n is even. The scheme for n being odd can be derived in a similar manner. The proposed scheme is described with the following sequence of steps corresponding to each time slot $t \in [1, n]$ when the users do cooperate.

- $t = 1$: User 1 transmits its information symbol $\mathbf{x}_1$. User 2 forms a ML estimate $\hat{\mathbf{x}}_1$ of $\mathbf{x}_1$ based on the signal it receives in this time slot.
- $t = 2$: User 2 transmits a linear combination of its information symbol $\mathbf{x}_2$ and $\hat{\mathbf{x}}_1$. This linear combination is of the form $a\mathbf{x}_2 + b\hat{\mathbf{x}}_1$. User 1 subtracts the information $b\mathbf{x}_1$ from the signal it receives in this time slot before forming the ML estimate $\hat{\mathbf{x}}_2$ of $\mathbf{x}_2$.
- $t = 2l+1$ for $1 \le l \le \frac{n}{2} - 1$: User 1 transmits $a\mathbf{x}_{2l+1} + b\hat{\mathbf{x}}_{2l}$. User 2 subtracts $b\mathbf{x}_{2l}$ from the signal it receives in this slot before forming the ML estimate $\hat{\mathbf{x}}_{2l+1}$ of $\mathbf{x}_{2l+1}$.
- $t = 2l$ for $2 \le l \le \frac{n}{2}$: User 2 transmits $c_1\mathbf{x}_{2l} + c_2(c\hat{\mathbf{x}}_{2l-1} + d\mathbf{x}_{2l-2})$. User 1 subtracts $c_2(c\mathbf{x}_{2l-1} + d\hat{\mathbf{x}}_{2l-2})$ from the signal it receives in this slot before forming the ML estimate $\hat{\mathbf{x}}_{2l}$ of $\mathbf{x}_{2l}$.
- $t = n$: User 2 transmits $c\hat{\mathbf{x}}_{n-1} + d\mathbf{x}_{n-2}$.

A compact representation of this scheme, which is the counterpart of (2), is provided in the following matrix.

$$\theta \begin{bmatrix} \mathbf{x}_1 & 0 & a\mathbf{x}_3 + b\hat{\mathbf{x}}_2 & 0 & a\mathbf{x}_5 + b\hat{\mathbf{x}}_4 & \dots & a\mathbf{x}_{n-1} + b\hat{\mathbf{x}}_{n-2} & 0 \\ 0 & a\mathbf{x}_2 + b\hat{\mathbf{x}}_1 & 0 & c_1\mathbf{x}_4 + c_2(c\hat{\mathbf{x}}_3 + d\mathbf{x}_2) & 0 & \dots & 0 & c\hat{\mathbf{x}}_{n-1} + d\mathbf{x}_{n-2} \end{bmatrix}$$

The following result characterizes the optimal DMT property of the proposed scheme.

Proposition 1. *Let the FMD matrix* $\mathbf{G}$ *be such that the minimum product distance* $d_p > 0$. *Then, the diversity-multiplexing tradeoff of the error probability with the modified SCL scheme is*

$$2(1-r), \quad r \in (0,1) \tag{4}$$

which is the optimal DMT curve for this model.

Proof. Let $\mathcal{C}$ denote the event that the users cooperate and $\bar{\mathcal{C}}$ the event that they do not. Let $\mathcal{E}$ denote the event that the destination makes an error in decoding the information stream of the two users. We can write

$$P(\mathcal{E}) = P(\mathcal{E}/\mathcal{C})P(\mathcal{C}) + P(\mathcal{E}/\bar{\mathcal{C}})P(\bar{\mathcal{C}}). \tag{5}$$

Under the event $\bar{\mathcal{C}}$, the scheme is identical to the SCL scheme of [2]. Hence, the analysis of the term $P(\mathcal{E}/\bar{\mathcal{C}})P(\bar{\mathcal{C}})$ in (5) is exactly the same as in the proof of Proposition 2 in [2]. We have that for some $\delta \in (0,1)$ and $r \in (0, 1-\delta)$,

$$P(\mathcal{E}/\bar{\mathcal{C}})P(\bar{\mathcal{C}}) \dot{\le} SNR^{(c+1)r-2}, \tag{6}$$

where the constant c is chosen such that

$$1 < c < \frac{1}{1-\delta}. \tag{7}$$

Hence, by choosing δ close to 0, the diversity order of the term $P(\mathcal{E}/\bar{\mathcal{C}})P(\bar{\mathcal{C}})$ approaches the curve $2(1-r)$.

To analyze $P(\mathcal{E}/\mathcal{C})$, we condition on the event that any of the estimates by the two cooperating users are in error. Thus, we can write

$$P(\mathcal{E}/\mathcal{C}) \le P(\mathcal{E}/\mathcal{C}, \hat{\mathbf{x}}_i = \mathbf{x}_i \forall i) + P(\hat{\mathbf{x}}_i \ne \mathbf{x}_i, \text{ for any } i/\mathcal{C}).$$

As in [2], the idea is to show that the term $P(\hat{\mathbf{x}}_i \ne \mathbf{x}_i, \text{ for any } i/\mathcal{C})$ decays exponentially with SNR so that the diversity order of $P(\mathcal{E}/\mathcal{C})$ is at least the diversity order of the other term $P(\mathcal{E}/\mathcal{C}, \hat{\mathbf{x}}_i = \mathbf{x}_i \forall i)$. The probability that any of the estimates made by the cooperating users is incorrect for any given inter-user channel realization $\mathbf{h}$ is

$$P_{\mathbf{h}}(\hat{\mathbf{x}}_i \ne \mathbf{x}_i, \text{ for any } i) = P_{\mathbf{h}}(\cup_{i=1}^{n-1}(\hat{\mathbf{x}}_i \ne \mathbf{x}_i)). \tag{8}$$

Similar to [2], this event can be written as a disjoint union of events obtained from the self-information cancellation being error free until time slot $i-1$ for each i. It is easy to see that even for the modified SCL scheme proposed in this section, self-information cancellation being error free until time slot $i-1$ means that the estimate in time slot i being incorrect only depends on the noise variable corresponding to that slot. This is because the signal transmitted by the active user during time slot i is only a function of the estimates $\hat{\mathbf{x}}_j$ for $1 \le j \le i-1$ or the exact information symbols $\mathbf{x}_j$ for $1 \le j \le i$. By averaging over the conditional density of $\mathbf{h}$ conditioned on $\mathcal{C}$, it follows from the expression in [2] that $P(\hat{\mathbf{x}}_i \ne \mathbf{x}_i, \text{ for any } i/\mathcal{C})$ decays exponentially with SNR.

To find the diversity order of $P(\mathcal{E}/\mathcal{C}, \hat{\mathbf{x}}_i = \mathbf{x}_i, \forall i)$, we note that conditioned on all the estimates being error-free, the effective signal received at the destination is similar to the case wherein there is only one user but with two transmit antennas and which employs a space-time code of the form shown below.

$$\mathbf{S}(\{\mathbf{x}_i\}) = \theta \times \begin{bmatrix} \mathbf{x}_1 & 0 & a\mathbf{x}_3 + b\mathbf{x}_2 & 0 & a\mathbf{x}_5 + b\mathbf{x}_4 & \dots & a\mathbf{x}_{n-1} + b\mathbf{x}_{n-2} & 0 \\ 0 & a\mathbf{x}_2 + b\mathbf{x}_1 & 0 & c_1\mathbf{x}_4 + c_2(c\mathbf{x}_3 + d\mathbf{x}_2) & 0 & \dots & 0 & c\mathbf{x}_{n-1} + d\mathbf{x}_{n-2} \end{bmatrix} \tag{9}$$

For two valid codewords $\mathbf{S}(\{\mathbf{x}_i\})$ and $\mathbf{S}(\{\mathbf{x}'_i\})$ and a particular pair of channel realizations $\mathbf{g}_1$ and $\mathbf{g}_2$, the Euclidean distance between the two codewords in the equivalent faded constellation is

$$\theta^2\|[\mathbf{g}_1\mathbf{g}_2]\mathbf{S}(\{\Delta\mathbf{x}_i\})\|^2 \quad (10)$$

where $\Delta\mathbf{x}_i = \mathbf{x}_i - \mathbf{x}'_i, 1 \leq i \leq n-1$. Define $d_1(\{\Delta\mathbf{x}_i\})$ and $d_2(\{\Delta\mathbf{x}_i\})$ as the row norms of the first and second rows of $\mathbf{S}(\{\Delta\mathbf{x}_i\})$, respectively. We have that

$$\begin{aligned} d_1(\{\Delta\mathbf{x}_i\}) &= |\Delta\mathbf{x}_1|^2 + (\sum_{l=1}^{n/2-1} |a\Delta\mathbf{x}_{2l+1} + b\Delta\mathbf{x}_{2l}|^2) \\ d_2(\{\Delta\mathbf{x}_i\}) &= |a\Delta\mathbf{x}_2 + b\Delta\mathbf{x}_1|^2 + \\ & (\sum_{l=1}^{n/2-1} |c_1\Delta\mathbf{x}_{2l} + c_2(c\Delta\mathbf{x}_{2l-1} + d\Delta\mathbf{x}_{2l-2})|^2) + \\ & |c\Delta\mathbf{x}_{n-1} + d\Delta\mathbf{x}_{n-2}|^2. \end{aligned}$$

Then,

$$\begin{aligned} \|[\mathbf{g}_1\mathbf{g}_2]\mathbf{S}(\{\Delta\mathbf{x}_i\})\|^2 &= |\mathbf{g}_1|^2 d_1(\{\Delta\mathbf{x}_i\}) + |\mathbf{g}_2|^2 d_2(\{\Delta\mathbf{x}_i\}) \\ &\geq \left[(|\mathbf{g}_1|^2|\mathbf{g}_2|^2)(d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\}))\right]^{\frac{1}{2}}, \quad (11) \end{aligned}$$

where we have applied the Arithmetic-Mean Geometric-Mean (AM-GM) inequality in the last step.

We now show that the product $d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\})$ is, in fact, lower bounded by a constant independent of the SNR. Note that at least one of $\Delta\mathbf{x}_i$ is non-zero. In the product $d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\})$, consider the term

$$(|a\Delta\mathbf{x}_{n-1} + b\Delta\mathbf{x}_{n-2}||c\Delta\mathbf{x}_{n-1} + d\Delta\mathbf{x}_{n-2}|)^2 \quad (12)$$

which is the product of the last terms in each of the expressions for $d_1(\{\Delta\mathbf{x}_i\})$ and $d_2(\{\Delta\mathbf{x}_i\})$ given above. If either of $\Delta\mathbf{x}_{n-1}$ or $\Delta\mathbf{x}_{n-2}$ is non-zero, then the product in (12) is lower bounded by the constant d_p which would imply that $d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\})$ is lower bounded by a constant. If both $\Delta\mathbf{x}_{n-2}$ and $\Delta\mathbf{x}_{n-1}$ are zero, then consider the term in the product $d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\})$ which is of the form

$$(|a\Delta\mathbf{x}_{n-3} + b\Delta\mathbf{x}_{n-4}|c_2|c\Delta\mathbf{x}_{n-3} + d\Delta\mathbf{x}_{n-4}|)^2. \quad (13)$$

Again this term is lower bounded by $|c_2|^2 d_p$ unless both $\Delta\mathbf{x}_{n-3}$ and $\Delta\mathbf{x}_{n-4}$ are zero. Proceeding inductively in this manner, we shall arrive at the case that $\Delta\mathbf{x}_i$ is non-zero only for $i = 1$ in which case

$$d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\}) = |b|^2|\Delta\mathbf{x}_1|^4 \dot{\geq} SNR^0, \quad (14)$$

because $\Delta\mathbf{x}_1 \in Z[i]$. Thus, in any case, we have that

$$d_1(\{\Delta\mathbf{x}_i\})d_2(\{\Delta\mathbf{x}_i\}) \dot{\geq} SNR^0. \quad (15)$$

Following [5], we make the substitutions $|\mathbf{g}_1|^2 = SNR^{-\alpha_1}$, $|\mathbf{g}_2|^2 = SNR^{-\alpha_2}$. Then, the minimum Euclidean distance between the codewords in the faded constellation can be written as

$$\begin{aligned} d^2_{min}(\alpha_1, \alpha_2) &= \theta^2 \min_{\{\Delta\mathbf{x}_i\}} \|[\mathbf{g}_1\mathbf{g}_2]\mathbf{S}(\{\Delta\mathbf{x}_i\})\|^2 \quad (16) \\ &\dot{\geq} \theta^2|\mathbf{g}_1\mathbf{g}_2| \quad (17) \\ &\dot{\geq} SNR^{1-r-\frac{\alpha_1+\alpha_2}{2}}, \quad (18) \end{aligned}$$

where the first inequality is obtained by applying (11) and (15). Following [6], the sphere-bound for a particular channel realization pair (α_1, α_2) implies that

$$\begin{aligned} & P(\mathcal{E}/\mathcal{C}, \hat{\mathbf{x}}_i = \mathbf{x}_i, \forall i, \alpha_1, \alpha_2) \leq \\ & e^{-d^2_{min}(\alpha_1,\alpha_2)/4} \sum_{k=0}^{n-1} \frac{\left[d^2_{min}(\alpha_1,\alpha_2)/4\right]^k}{k!}. \quad (19) \end{aligned}$$

As in [2], we average this bound over (α_1, α_2) to get that the diversity order of $P(\mathcal{E}/\mathcal{C}, \hat{\mathbf{x}} = \mathbf{x}_i, \forall i)$ is the minimum of $\alpha_1 + \alpha_2$ over the region defined by the relations

$$\alpha_1 > 0, \alpha_2 > 0, 1 - r - \frac{\alpha_1 + \alpha_2}{2} < 0.$$

This minimum is easily seen to be equal to $2(1-r)$. Hence, the diversity order of $P(\mathcal{E}/\mathcal{C})P(\mathcal{C})$ is at least $2(1-r)$. Thus, the overall diversity order of the modified SCL scheme is also at least $2(1-r)$ for $r \in (0,1)$. This diversity order is also equal to that obtained by a single user two antenna system which is the best case scenario. Hence, the DMT curve achieved by the modified SCL scheme is indeed optimal.

□

4 SCL scheme for $m > 2$

In this section, we propose an extension of the SCL scheme to $m > 2$ users. The aim here is to design a cooperative coding scheme based on QAM information symbols that achieves a diversity order of m and also leads to equal allocation of rate among all the users. Moreover, the scheme should have the potential for achieving the maximum multiplexing gain in the system.

For a system with m users that attempt to communicate to a common destination, there exist $\binom{m}{2}$ independent fading links between the users. We first extract the following essential components from the analysis of the SCL scheme presented earlier in [1,2].

1. It is possible to choose a cooperation rule between any two users so that the probability that the link between these two users does not satisfy the cooperation rule has a diversity order of 1. This rule is provided in [1].

Therefore, for an m user system, if k links do not satisfy the cooperation rule, then the diversity order for the probability of this event is k. We say that a link between two users is good if the inter-user channel coefficient is such that the cooperation rule is satisfied. Otherwise, we say that the link is bad.

2. Suppose two users u_1 and u_2 are such that the link between them is good. Suppose u_1 transmits a signal that is a linear combination of a new information symbol and some other symbols that are known to u_2. Then, it is possible for u_2 to receive the signal transmitted by u_1, subtract the part known to itself and perform a ML decoding to obtain an estimate of the information symbol of u_2. The probability that this estimate is in error conditioned on the link being good decays exponentially with SNR. Hence, for diversity order analysis, when the link between u_1 and u_2 is good, it is sufficient to consider that the users u_1 and u_2 know the exact information symbols of each other if they do cooperate.

3. Consider a subset $\mathcal{F}$ of the m users with $|\mathcal{F}| = F$ such that all the $\binom{F}{2}$ links between the users in $\mathcal{F}$ are good. Then, a generalized SCL scheme for the users in $\mathcal{F}$ can be conceived that provides a diversity order of F. Without loss of generality, let the users in $\mathcal{F}$ be numbered such that $\mathcal{F} = \{1, 2, \ldots, F\}$. The users in $\mathcal{F}$ transmit in a round-robin fashion. For a frame length of n time slots, the scheme consists of an initial phase of length F slots, a main phase of $n - 2F$ slots and a termination phase of length F slots. To illustrate the scheme, let us consider the specific case of $F = 3$ time slots. The signals transmitted can be represented in the compact form shown below.

$$\left[\begin{array}{cccc} \mathbf{x}_1 & 0 & 0 & \cdots \\ 0 & a\mathbf{x}_2 + b\hat{\mathbf{x}}_1 & 0 & \cdots \\ 0 & 0 & a\mathbf{x}_3 + b\tilde{\mathbf{x}}_2 + c\tilde{\mathbf{x}}_1 & \cdots \end{array}\right.$$

$$\begin{array}{ccc} a\mathbf{x}_k + b\bar{\mathbf{x}}_{k-1} + c\bar{\mathbf{x}}_{k-2} & 0 & 0 \\ 0 & a\mathbf{x}_{k+1} + b\hat{\mathbf{x}}_k + c\hat{\mathbf{x}}_{k-1} & 0 \\ 0 & 0 & a\mathbf{x}_{k+2} + b\tilde{\mathbf{x}}_{k+1} + c\tilde{\mathbf{x}}_k \end{array}$$

$$\left.\begin{array}{cccc} \cdots & a\mathbf{x}_{n-2} + b\bar{\mathbf{x}}_{n-3} + c\bar{\mathbf{x}}_{n-4} & 0 & 0 \\ \cdots & 0 & b\hat{\mathbf{x}}_{n-2} + c\hat{\mathbf{x}}_{n-3} & 0 \\ \cdots & 0 & 0 & \tilde{\mathbf{x}}_{n-2} \end{array}\right]$$

As seen above, during any given time slot in the main phase, the active transmits a linear combination of a new information symbol with the estimates of the other users obtained during the previous $F - 1$ time slots. To obtain the estimate from any time slot, the user subtracts the part of the linear combination that it has already estimated before performing the ML decoding. A similar process occurs during the initial phase, when the users are just beginning to transmit. In the termination phase, the last $F - 1$ users do not transmit any new information. Based on our previous two analytical components, we can conclude that conditioned on the channel between all the users being good, the generalized SCL scheme achieves a diversity order of F.

We now show that there indeed exists a specific cooperative coding strategy such that a diversity order of m is achieved for a general $m > 2$ system. This scheme has to be fair in terms of rate allocation for each user.

The error probability $P(\mathcal{E})$ can be written as

$$P(\mathcal{E}) = \sum_{k=0}^{\binom{m}{2}} P(\mathcal{E}/k)P(k). \tag{20}$$

As stated above, the diversity order achieved for the probability of the event that k links are bad, i. e., $P(k)$, is k. Therefore, for each $0 \leq k \leq \binom{m}{2}$, we only need to design a cooperative coding strategy that guarantees a diversity order of $m - k$ for $P(\mathcal{E}/k)$ so that the overall diversity order of each term $P(\mathcal{E}/k)P(k)$ is at least m. In the following, we describe the strategies that achieve this objective for any value of m. For the description of this scheme, we first define a (k, m) cooperative system as one in which there are m users but k of the $\binom{m}{2}$ links between them are bad.

($k = 0, m > 2$): When $k = 0$, one can employ the generalized SCL scheme described above to achieve a diversity order of m.

($k \in [m-1, \binom{m}{2}], m > 2$): For this range of k, one can simply allow each user to send independent information symbols in a round-robin fashion. This scheme achieves a diversity order of 1 leading to an overall diversity of m for such k.

($k = 1, m > 2$): When $k = 1$, one can choose a subset $\mathcal{F}$ of $m - 2$ users that do not correspond to this particular one bad link as shown in Figure 2. The scheme is described in Figure 3. For the first n time slots, user 1 sends independent information symbols. Each user in $\mathcal{F}$ listens and decodes these information symbols. Then, user 2 sends independent information symbols for the next n time slots while users in $\mathcal{F}$ listen and decode these symbols. For the next $(m-1)n$ time slots, users in $\mathcal{F} \cup \{1\}$ transmit using the generalized SCL strategy with the additional difference that each user in $\mathcal{F}$ also adds the information symbol of user 1 that they decoded in the first n time slots. Similarly, for the next $(m-1)n$ time slots, users in $\mathcal{F} \cup \{2\}$ transmit using the generalized SCL strategy with each user in $\mathcal{F}$ also adding the information symbol of user 2 that they decoded in the second n time slots. This way each symbol of each user goes through $m - 1$ independent users leading to a diversity order of $m - 1$. Also, each user gets to send exactly the same number $2n$ of information symbols.

($2 \leq k \leq m-2, m > 2$): For this general case, we use an inductive approach to construct a scheme for the (k, m) system for any m. We have already described a scheme for the $(1, m)$ system for any m. Consider the inductive hypothesis that a scheme exists for a (k, M), system with diversity order of $P(\mathcal{E}/k)$ being at least $M - k$ for each $k \in [1, M-2]$. We show that one can now construct a scheme for a $(k, M+1)$ system with diversity order of $M + 1 - k$ for each $k \in [1, M-1]$.

We begin with a (K, M) system and introduce a new user into the system. Hence, M new links are created between the new user and the existing M users. We allow exactly one of these new links to be bad. In other words, we have converted a (K, M) system to a $(K+1, M+1)$ system. To construct a coding scheme for this new $(K+1, M+1)$ system, let j be the index of the existing user which is connected to the new user via the new bad link. Let I be the number of bad links between the j-th user and the existing m users. Denote the existing set of m users as $\mathcal{M}$. Consider the two sets $\mathcal{F}_1 = \mathcal{M}$ and $\mathcal{F}_2 = \mathcal{M} \backslash \{j\} \cup \{m+1\}$. The set $\mathcal{F}_1$ is the same as the existing set of m users and corresponds to a (K, M) system. The set $\mathcal{F}_2$ is obtained by switching the j-th user with the new user and corresponds to a $(K - I, M)$ system. By the inductive hypothesis, there already exists a scheme for each of the (K, M) and $(K - I, M)$ systems that leads to diversity orders of at least $M - K$ and $M - K + I$, respectively. Consider a strategy in which we first allow users in $\mathcal{F}_1$ to transmit using the scheme for the (K, M) system. Let this lead to n_1 symbols per user. Then, we allow users in $\mathcal{F}_2$ to transmit using the scheme for the $(K - I, M)$ system. Let this lead to n_2 symbols per user. This way, we have achieved a diversity order of at least $M - K$. However, users in $\mathcal{F}_1 \backslash \{j\}$ have sent $n_1 + n_2$ symbols each but users j and $m+1$ have sent only n_1 and n_2 symbols, respectively. In order to balance the rates, we modify this strategy so that user j first sends n_2 information symbols. Then, we allow users in $\mathcal{F}_1$ to transmit using the scheme for the (K, M) system but each user with a good link to j also forwarding the estimate of user j's information from the first n_2 time slots. Similarly, user $m+1$ first sends n_1 information symbols. Then, users in $\mathcal{F}_2$ transmit using the scheme for the $(K - I, M)$ system but each user with a good link to $m+1$ also forwarding the estimate of user $m+1$'s information from the initial n_1 time slots. This modified scheme now achieves equal rate of $n_1 + n_2$ symbols for each user and also achieves a diversity order of $M - K$. Hence, we have designed a scheme for the $(K+1, M+1)$ system.

Since we already have a scheme for the $(1, m)$ system for any m, our inductive design can now be used to construct a coding scheme for any (k, m) system.

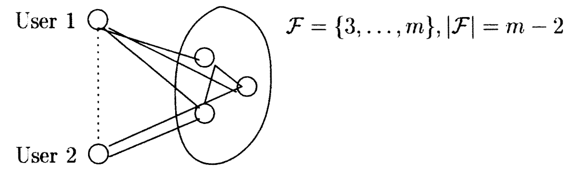

Figure 2: $(1, m)$ system

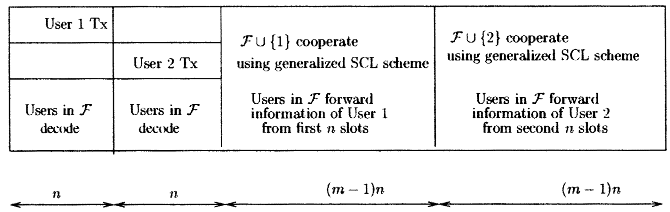

Figure 3: Coding scheme for $(1, m)$ system

5 Conclusion

A new two user practical cooperation strategy based on QAM information symbols is proposed that achieves the optimal DMT curve for this system. This is obtained from a modification of the SCL scheme proposed earlier. The SCL scheme is also generalized to the case of more than two users so that full diversity order of m and equal rates allocation is obtained for an m user cooperative communication system. The generalized scheme has the potential for achieving the full multiplexing gain of one inherent in the system.

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