Group Metric Decoding for Synchronous Frequency-Selective Rayleigh Fading Multiple-Access Channels

Eric A. Fain and Mahesh K. Varanasi
ECE Dept., University of Colorado, Boulder, CO 80309-0425, USA.

Abstract—**We propose the new Group Metric (GM) decoder for convolutionally coded synchronous multiple-access channels. The GM decoder makes decoding decisions for one user, but incorporates multiuser information in its metrics. This decoder will have a reduced complexity which is exponential in the sum of encoder memory and the number of users. The soft-decision maximum-likelihood (ML) joint decoder can be readily described as in [1]. This optimal decoder suffers from a high complexity requirement which is exponential in the *product* of encoder memory and the number of users. Numerical results show that the performance gap between the GM and ML decoders can be small.**

I. Introduction

THIS paper considers the problem of joint detection-decoding for the synchronous multiple-access channel. For synchronous transmission, the optimal detectors are well known ([2]), and have a complexity which is exponential in the number of users. This high complexity demand has motivated sub-optimal strategies such as group detection ([2][3]).

For convolutionally-coded Gaussian channels, the optimal joint detector-decoder has been considered in [1], and various sub-optimal combinations have been considered in [4]. Because the optimal joint decoder has a complexity which is exponential in the product of the number of users and encoder memory, it is natural to look for reduced complexity decoding schemes.

In this paper, we present a new reduced complexity algorithm, the Group-Metric (GM) decoder. This complexity reduction exploits the particular nature of the multiuser decoding problem, and results in a complexity which is exponential in the sum of the number of users and the encoder memory.

For convolutionally-coded systems, the soft-decision decoding gain on fading channels with coherent detection can be quite high [5]. This is the motivation for developing a soft-decision decoder for the frequency-selective Rayleigh-fading multiple-access channel. However, the principle of the GM decoder can easily be applied to the non-fading, Gaussian channel.

This paper will be organized as follows. First, we present a description of the channel model, which will be a straightforward extension of the model in [2]. Secondly, we specify the two decoding schemes, the joint soft-decision decoder and the group-metric decoder. Additional applications of the GM decoder are discussed in section IV. Lastly, numerical results are presented in Section V.

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II. Multipath-Fading/Diversity Model

A. Overview

The channel model presented here is for Rayleigh-fading multipath channels where coherent detection with a RAKE front-end is used. This is simplified by a tapped-delay line (discrete FIR) channel model. This model generalizes and unifies the models given in [2],[6], and [7].

B. Model

In the synchronous multiple-access channel with K active users, the received signal is the superposition of K signature waveforms, each with bandwidth B_T and digitally modulated. Each of these waveforms has passed through a frequency-selective fading channel. The k^{th} user's channel will have L_k resolvable paths, inversely proportional to the channel's multipath spread. A tapped-delay line model for the received signal in the k^{th} channel with tap spacing $1/B_T$ is given by ([5])

$$h_k(t) = \sqrt{w_k}\mathbf{u}_k^T(t)\mathbf{c}_k(t), \qquad (1)$$

where $\mathbf{c}_k(t)$ is a vector of the complex fading coefficients for the L_k paths in the k^{th} channel, w_k is the k^{th} user's average symbol energy, and $\mathbf{u}_k(t)$ is a vector of the time-translates of the k^{th} user's signature waveform $u_k(t)$.[1] Despite the synchronous nature of this system, intersymbol interference (ISI) will still occur at the edges of the symbol interval, for a duration of $(L_k-1)/B_T$ per symbol period. We will therefore assume a guard-time of $t_{ISI} = (L_{max}-1)/B_T$ is inserted between symbols.

We assume that each of the K users employs a binary code, which produces a sequence of encoded binary digits. For BPSK transmission, these bits are mapped into $\{\pm 1\}$. We let b_{ki} denote the k^{th} user's i^{th} such encoded symbol.

We find the sum of the received signals over $[t_{ISI}, T]$ is

$$r(t) = \sum_{k=1}^{K} b_k h_{ki}(t) + n(t) = \mathbf{h}^T(t)\mathbf{b}(i) + n(t), \qquad (2)$$

where the symbol vector $\mathbf{b}(i) = [b_{1i}, \cdots, b_{Ki}]^T$ and $\mathbf{h}(t) = [h_1(t), h_2(t), \cdots, h_K(t)]^T$. The complex noise process $n(t)$ is circularly symmetric, additive, white, and Gaussian with power spectral height N_0.

Consider now a $(\rho \times 1)$ vector $\mathbf{g}(t)$ whose entries form an orthonormal basis for the signal space spanned by the shifts of the received waveforms. For simplicity, let $L = \sum_k L_k$.

[1] As a matter of notation, all vectors will be taken to be column vectors, and denoted with boldface type.

Invoking a matrix notation, we define the $(L \times K)$ channel fading information (CFI) matrix $\mathcal{C}$ as

$$\mathcal{C}(i) = \begin{bmatrix} \mathbf{c}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{c}_K \end{bmatrix}, \quad (3)$$

so that the received signal can be expressed as

$$r(t) = \mathbf{g}^T(t)\mathcal{F}\mathcal{C}(i)\mathbf{W}^{1/2}\mathbf{b}(i) + n(t), \quad (4)$$

where $\mathbf{W} = diag\{w_1, \cdots, w_K\}$, and $\mathcal{F}$ is a $\rho \times L$ matrix mapping the basis functions into the received waveforms. Let $\mathbf{q}(i)$ be the ρ-dimensional vector of sufficient statistics obtained by matched-filtering the received signal with respect to each orthonormal basis function over the i^{th} symbol interval, so that $\mathbf{q}(i)$ can be written succinctly as

$$\mathbf{q}(i) = \mathcal{F}\mathcal{C}(i)\mathbf{W}^{1/2}\mathbf{b}(i) + \mathbf{n}(i), \quad (5)$$

where $\mathbf{n}(i)$ is a complex, circularly symmetric Gaussian noise vector with covariance $N_0\mathbf{I}$.

In the case of Rayleigh fading, the $\mathbf{c}_k$'s are zero-mean complex normal vectors with covariance $\mathbf{\Sigma}_{kk}$. Under the assumption that each user fades independently, $\mathcal{C}(i)$ will be completely described by the block diagonal matrix

$$\mathbf{\Sigma} = E\left(\mathcal{C}(i)\mathcal{C}^\dagger(i)\right) = \mathrm{diag}\left(\mathbf{\Sigma}_{11}, \cdots, \mathbf{\Sigma}_{KK}\right). \quad (6)$$

where † denotes conjugate (Hermitian) transpose.)

III. Soft-Decision Decoding

We consider now the case where each user employs a single-user rate k/n convolutional code, and we assume perfect interleaving.

Our new decoding scheme will decode the users individually, but use decoding metrics that are a function of the entire group of users. We call this method *Group Metric* (GM) decoding. For simplicity, we consider the case where every user's code is the same with memory m, so that each encoder is a finite-state machine with 2^{km} states.

For completeness, we first specify the joint decoder, then present the GM decoder. These parallel descriptions will emphasize the similarities and differences between the two decoders.

A. The Maximum-Likelihood Joint Decoder

The maximum-likelihood decoding rule requires a minimization for which we may naturally turn to the Viterbi algorithm. This decoder will specialize that of [1] to the synchronous fading diversity channel.

Let $\mathbf{q}(i)$ denote the output at time instance i. We define the block diagonal matrices $\tilde{\mathcal{F}} = diag\{\mathcal{F}, \cdots, \mathcal{F}\}$, $\tilde{\mathbf{W}} = diag\{\mathbf{W}, \cdots, \mathbf{W}\}$, and let $\mathbf{q} = [\mathbf{q}^T(1), \cdots, \mathbf{q}^T(I)]^T$, $\mathbf{b} = [\mathbf{b}^T(1), \cdots, \mathbf{b}^T(I)]^T$, and $\mathcal{C} = diag\{\mathcal{C}(1), \cdots, \mathcal{C}(I)\}$, where I is the message length.[2]

[2]The tilde (˜) notation is used to indicate a block extension of the single-symbol parameters to message-length parameters.

Definition 1: The maximum-likelihood joint decoding rule for all the users, given that the CFI is known for the entire sequence is given by

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{x} \in \mathcal{A}} \|\tilde{\mathcal{F}}\mathcal{C}\tilde{\mathbf{W}}^{1/2}\mathbf{x} - \mathbf{q}\|^2. \quad (7)$$

where $\mathcal{A}$ is the set of all possible coded K-user sequences.

If each user's outputs are synchronized, then the above decoding rule can be implemented with a Viterbi decoder that has 2^{kmK} states, 2^{kK} branches to and from every state, with a state transition every n bits. The branch metric between two adjacent states x and y at time t is given by

$$d_{xy}(t) = \|\tilde{\mathcal{F}}_t\mathcal{C}_t\tilde{\mathbf{W}}_t^{1/2}\mathbf{p}_{xy} - \mathbf{q}_t\|^2. \quad (8)$$

where we define $\mathcal{C}_t = diag\{\mathcal{C}(t-n+1), \cdots, \mathcal{C}(t)\}$ and the other matrices accordingly. The nK-dimensional vector $\mathbf{p}_{xy}$ is formed from the K n-bit words generated by transition from state x to state y.

In order to estimate the decoder's computational complexity, we count the number of L-dimensional quadratic forms evaluated per coded bit, and we thus find the joint decoder to have complexity of order $\mathcal{O}(2^{(m+1)kK})$.

An identical decoder structure would be obtained if we considered a decoder for a single "super-code" having kK inputs, nK outputs and total memory mK [1]. The super-code encoder would be a parallel combination of the K users' encoders, with i^{th} output $K-tuple$ corresponding to symbol vector $\mathbf{b}(i)$.

B. Group Metric Decoder

The Group Metric decoder will reduce the complexity of the ML-decoder by considering only one user to be coded. The GM-decoder is thus a single-user decoding scheme that will use information from all the users' matched-filter outputs for the decoding metrics, and will operate in a bank of K parallel units.

Definition 2: Under the restriction that only the g^{th} user is coded, or alternatively that the other users' codes are not known, the conditionally maximum-likelihood rule for user g is as follows

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{p} \in \mathcal{A}_g} \|\tilde{\mathcal{F}}\mathcal{C}\tilde{\mathbf{W}}^{1/2}\mathbf{p} - \mathbf{q}\|^2 \quad (9)$$

$$= \arg\min_{\mathbf{p}_1 \in \mathcal{X}_g} \left[\min_{\mathbf{p}_2 \in \mathcal{X}_{\bar{g}}} \|\tilde{\mathcal{F}}\mathcal{C}\tilde{\mathbf{W}}^{1/2}(\mathbf{p}_1 + \mathbf{p}_2) - \mathbf{q}\|^2\right] \quad (10)$$

where we replace $\mathcal{A}$ in (7) with $\mathcal{A}_g$, the set of all possible transmitted sequences assuming only the g^{th} user is coded, and further decompose this set by letting $\mathcal{X}_g$ be the set of all vectors that are coded outputs for user g and are zero in all other positions, and let $\mathcal{X}_{\bar{g}}$ be the set of all vectors in $\{\pm 1\}$ which are zero in the positions for user g.

With this scheme, a Viterbi decoder for the g^{th} user can be implemented with 2^{km} states, and 2^k branches to and from each state. The branch metric from state x to state y at time t is given by

$$d_{xy}(t) = \min_{\substack{\mathbf{p}: p_i \in \{\pm 1\} \\ \forall i \neq \{k, 2k, \ldots, nk\}}} \|\tilde{\mathcal{F}}_t\mathcal{C}_t\tilde{\mathbf{W}}_t^{1/2}\mathbf{p} - \mathbf{q}_t\|^2. \quad (11)$$

The entries $p_k, p_{2k}, \cdots, p_{nk}$ make up the k^{th} user's encoded n-bit word generated from the transition to state y from state x. Due to the block diagonal structure of the matrix $\tilde{\mathcal{F}}_t$, each branch metric requires a minimization over only $n2^{K-1}$ possibilities. Counting the number of L-dimensional quadratic forms evaluated per coded bit, we find a bank of K GM-decoders has a complexity of order $\mathcal{O}(K2^{(m+1)k+K-1})$, generally much less than that for the ML-decoder.

IV. Applications

A. Antenna Diversity

The channel model chosen in this paper is easily extended to provide for multiple antennas to be used at the receiver, when the correlation between the fading parameters at each antenna is known. In making this extension, only the notation changes, with the effect of multiplying the number of resolvable paths by the number of antennas. The analysis and specification of the decoders stay the same whether we include antennas or not.

B. Group Detection and GM Decoding

An additional important application of the GM decoder will be in group detection, in which only subsets of the users are detected jointly [2][3]. While the details are omitted in this paper, the application of the GM decoder (as well as the ML-decoder) to either a pre-combining or post-combining group detection strategy will closely parallel the preceding development. In this way, complexity can be further reduced, while retaining the benefits of soft-decision decoding.

V. Numerical Results and Conclusions

In this section, we illustrate the performance of the ML and GM decoders by simulation. For simulation, we consider a two-user equal energy situation with no multipath, and the cross-correlation between the users is .5. Each user is given a rate 1/2 convolutional code of constraint length 2, with free distance of 5. Figure 1 shows simulated bit error rates for one user. The ML and GM decoders are seen to be very close to one another.

In Figure 2, the simulation parameters are the same except the cross-correlation is .9. Here, the GM decoder is further removed from the ML decoder, but the performance gain over the soft-decision decorrelator-decoder is impressive. (This decoder uses a linear transformation to separate the users, then uses a single-user soft-input ML decoder.)

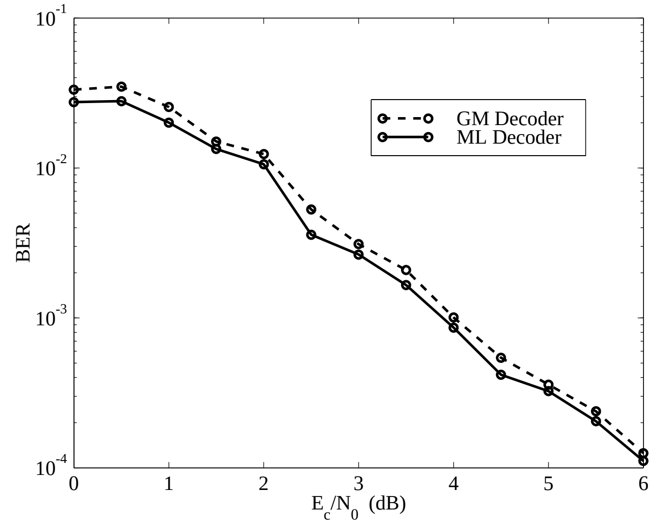

Fig. 1. Simulation results for medium cross-correlation.

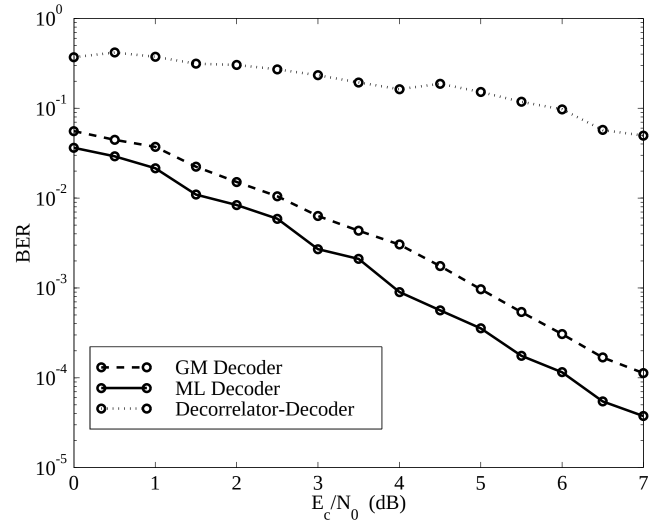

Fig. 2. Simulation results for high cross-correlation

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