Error Probability Bound for Reduced Complexity Multiuser Decoding Using Orthogonal Decomposability

Eric A. Fain and Mahesh K. Varanasi
Dept. of Electrical Engineering
University of Colorado, Boulder, CO 80309-0425 USA
Email: eric.fain@colorado.edu varanasi@boulder.colorado.edu

Abstract — **We develop an upper bound on the error probability of the reduced-complexity Group-Metric multiuser convolutional decoder [1]. This error bound results from discarding a class of redundant error sequences which are orthogonally *decomposable*. Indecomposable sequences for uncoded multiuser detection were introduced in [2] for the non-fading channel, and their enumeration is computationally intensive. By restricting attention to orthogonally decomposable error sequences for the fading channel, we can characterize sequences whose decomposability is independent of the fading parameters. Furthermore, we can easily compute this bound by evaluating the generating function of the encoder of the user in question.**

I. Introduction

We consider the group-metric (GM) decoder, introduced in [1]. This is a reduced-complexity multiuser convolutional decoder which exploits a unique feature of the multiuser decoding problem by assuming (falsely) that only one user's code is known. Our goal is to obtain a tractable bound on error probability for this decoder. The key to obtaining a good bound is to eliminate redundant error sequences. To this end, we identify a large number of orthogonally decomposable (hence redundant) sequences in a simple way, and thus obtain a closed-form upper bound expression which depends only on the single-user code generating function for the decoded user. In contrast, most other reduced-complexity decoding schemes are not easily analyzed.

II. System Model

The GM decoder and the orthogonally decomposable sequence analysis we present are characterized for the fading channel, but they may also be adapted to the non-fading channel. The K-user synchronous fading multiple-access channel has matched filter outputs for the i^{th} symbol interval given by

$$\mathbf{q}(i) = \mathbf{R}\mathbf{C}(i)\mathbf{b}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{R}$ is the multiuser correlation matrix, $\mathbf{C}(i)$ is a diagonal matrix of complex fading amplitudes with $E(\mathbf{c}(i)\mathbf{c}^\dagger(i)) = \mathbf{\Sigma}^2$, where $\mathbf{c}(i)$ is the vector of diagonal elements of $\mathbf{C}(i)$, $\mathbf{b}(i)$ is the vector of users encoded data bits in $\{\pm 1\}$, and $\mathbf{n}(i)$ is complex Gaussian noise with $E(\mathbf{n}\mathbf{n}^\dagger) = N_0\mathbf{R}$. We assume a Rayleigh fading distribution, and perfect interleaving, so that $E(\mathbf{c}(i)\mathbf{c}^\dagger(j)) = \mathbf{0}$ for $i \neq j$. This model (and the results that follow) are easily extended to allow for fading diversity. Each user employs a single-user convolutional code. The GM decoder decodes only user m, so K parallel decoders are needed to decode all users.

III. Error Sequences

Let $\mathbf{b} = [\mathbf{b}^{\mathrm{T}}(1), \cdots, \mathbf{b}^{\mathrm{T}}(N)]^{\mathrm{T}}$ be the transmitted sequence and $\hat{\mathbf{b}}$ be the coded sequence corresponding to the decoded information sequence, then with $\mathbf{e}(i) = (\hat{\mathbf{b}}(i) - \mathbf{b}(i))/2$, $\mathbf{e} = [\mathbf{e}^{\mathrm{T}}(1), \cdots, \mathbf{e}^{\mathrm{T}}(N)]^{\mathrm{T}}$ is defined as an error sequence. The union bound on error probability, which is a sum over the entire set of error sequences, can be tightened by removing those error sequences that are redundant. Decomposable sequences were defined in [2]. For our model, we have: *An error sequence $\mathbf{e}$ is decomposable for a given fading realization into $\mathbf{e} = \mathbf{e}_1 + \mathbf{e}_2$ if $\mathbf{e}_1$ and $\mathbf{e}_2$ are error sequences, there is no cancellation between $\mathbf{e}_1$ and $\mathbf{e}_2$ and if* $\sum_i \mathrm{Re}[\mathbf{e}_1^{\mathrm{T}}(i)\mathbf{C}^\dagger(i)\mathbf{R}\mathbf{C}(i)\mathbf{e}_2(i)] \geq 0$. If a sequence is decomposable into $\mathbf{e} = \mathbf{e}_1 + \mathbf{e}_2$, then $\mathbf{e}$ need not be counted in the error bound. For the closed-form bound, we consider only orthogonal decompositions where $\sum_i \mathrm{Re}[\mathbf{e}_1^{\mathrm{T}}(i)\mathbf{C}^\dagger(i)\mathbf{R}\mathbf{C}(i)\mathbf{e}_2(i)] = 0$.

Without loss of generality, we will assume that the "all -1's" sequence is transmitted, so that the error sequences have elements in $\{0, 1\}$. For the GM decoder, the set of all error sequences consists of sequences whose m^{th} subsequence is a single-user error sequence for the m^{th} user's code. Because the decoder does not know the other users' codes, the remaining error symbols are unconstrained.

Note that a sufficient (deterministic) condition for orthogonal decomposability is that $\mathbf{e}_1(i) = \mathbf{0}$ for all i where $\mathbf{e}_2(i) \neq \mathbf{0}$, or vice-versa. Let us consider an error sequence that is non-zero in at least one time-interval where the m^{th} user's error symbol is zero. Such a sequence can be orthogonally decomposed into the sum of two sequences, one which is zero in every time-interval where the m^{th} user's error symbol is zero, and one which does not affect the m^{th} user. The result is that for our upper bound on error probability, we count only those error sequences whose m^{th} subsequence is a single-user error sequence, and is zero in the intervals where the single-user error sequence is zero. The simplicity of the characterization of such sequences allows us to obtain the bound in terms of the single-user generating function for user m.

IV. Error Probability Bound

Let the m^{th} user's rate k/n convolutional code have generating function $T(X, Y)$ defined in [3]. For the GM decoder, our generating-function upper bound can be shown to be

$$P_e^m < \frac{1}{2k} \left. \frac{\partial}{\partial Y} T(X, Y) \right|_{X=p, Y=1} \tag{2}$$

with

$$p = \sum_{i=1}^{2^{K-1}} \det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{U}_i\mathbf{\Sigma}\mathbf{R}\mathbf{\Sigma}\mathbf{U}_i\right)^{-1} \tag{3}$$

where $\mathbf{U}_i$ for $i = 1, \cdots, 2^{K-1}$ are $K \times K$ diagonal matrices with diagonal elements $\in \{0, 1\}$ with the m^{th} diagonal element fixed to be one.

References

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