A Comparison of Signal Design Methods for Correlated Waveform Multiple-Access Communications

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Abstract

There have been several recent papers in the literature that deal with the design of signature waveforms for use by the transmitters for up-link, single-cell, multiple-access communications. In particular, we consider the approach introduced in [1] and [2] where the signature waveforms are specifically designed for the centralized multiuser receiver at the base so that each transmitter can be guaranteed a pre-assigned Quality-of-Service (QoS) requirement in terms of the received Signal-to-Interference Ratio (SIR). The resulting strategy is called Bandwidth Efficient Multiple Access (BEMA) [2]. When all users employ Pulse Amplitude Modulation (PAM) and a common signaling rate, the key question in BEMA is how the waveforms must be designed to occupy as little bandwidth as possible and still meet the QoS objectives. For a strict measure of bandwidth, and for a given set of received powers, this question was addressed by the authors for the Maximum Signal-to-Interference Ratio-Decision Feedback (MSIR-DF) receiver in [3]. A similar question was addressed in [4] where, for a sum constraint on the received powers, optimal signature signals and transmit powers were obtained for the *linear* MSIR receiver (without decision feedback). A somewhat different but related non-QoS approach proposes the design of signature signals that maximize the total capacity of the multiple-access channel under a spreading gain constraint (cf. [5]-[6]). This paper undertakes a comparison of the minimum bandwidth required to achieve the QoS requirements for the signals designed for the MSIR-DF receiver [3], the linear MSIR receiver [4], and for sum-capacity maximization as in [6]. We show that the bandwidth required for multiuser receivers with decision feedback can be significantly less than that required for linear receivers or for sum-capacity maximization.

I. The CWMA System Model

A Correlated-Waveform Multiple Access (CWMA) system is modeled as one where K users transmit PAM waveforms simultaneously and the received signal is a symbol-synchronous superposition of those waveforms and additive noise so that

$$y(t) = \sum_n \sum_{k=1}^{K} X_k(n) u_k(t - nT) + n(t), \tag{1}$$

where $u_k(t)$ and $\{X_k(n)\}_n$ are the signature waveform and the sequence of transmitted symbols of the k^{th} user, respectively, and $n(t)$ is an additive white Gaussian noise process with a two-sided power spectral density of $N_0/2$. The signature waveforms are normalized to have unit energy so that the received power of the k^{th} user in units of energy/symbol is given by $p_k = E[X_k(n)^2]$. Define the diagonal power matrix $\mathbf{P} = \text{diag}(p_1, \cdots, p_K)$. It is assumed that the signature waveforms satisfy the Multiuser Nyquist Condition (MNC) which we define as:

$$\int_{-\infty}^{\infty} u_i(t) u_j(t + nT) dt = R_{ij} \delta_n \,, \tag{2}$$

where δ_n is the Kronecker delta (equal to 0 when $n \neq 0$ and 1 when $n = 0$). Define the signal correlation matrix $\mathbf{R} = \{R_{ij}\}$ (note the diagonal elements are equal to unity).

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An equivalent discrete-time model can be obtained by passing the received signal through a bank of filters matched to the users' signature waveforms and then sampling the outputs at integer multiples of T. The sufficient statistics thus obtained can be expressed as

$$\underline{Y}(n) = \mathbf{R}\underline{X}(n) + \underline{N}(n), \tag{3}$$

where $\underline{X}(n)$ is the vector of information symbols of the users at time n and $\{\underline{N}(n)\}$ is a sequence of independent, identically-distributed, zero-mean, Gaussian random vectors, each with covariance $\frac{N_0}{2}\mathbf{R}$. Note that the CWMA channel is memoryless as a consequence of the MNC constraint. We drop the time index n and work with the model $\underline{Y} = \mathbf{R}\underline{X} + \underline{N}$.

The MNC is trivially satisfied if the signature waveforms are time-limited to a symbol duration but for a finite value of the strict measure of bandwidth, these signals can not be finitely supported. It can be shown that the minimum (strict) bandwidth required to satisfy the MNC is equal to $M/(2T)$ where M is the rank of $\mathbf{R}$. Given $\mathbf{R}$, it is possible to construct the signature wavforms that satisfy the MNC as linear combinations of M orthonormal basis functions. These basis functions can be the M time-translates of a single sinc pulse of bandwidth $M/(2T)$ (translated by integer multiples of T/M), or M modulations of the sinc function of bandwidth $1/4T$ (modulated by frequencies that are positive and odd integer multiples of $1/4T$) [7]. With these choices for the basis functions, note that when $\mathbf{R}$ is equal to the identity matrix, the CWMA model reduces to TDMA or FDMA, respectively.

II. Bandwidth-Efficient Multiple Access

In this section, we briefly summarize the signal design method for multiuser receivers with decision feedback as obtained by the authors in [3].

A decision feedback multiuser receiver detects (or decodes in a coded system) the users successivly and is shown in Figure 1. It is parameterized by a feedforward matrix $\mathbf{F}$ and a

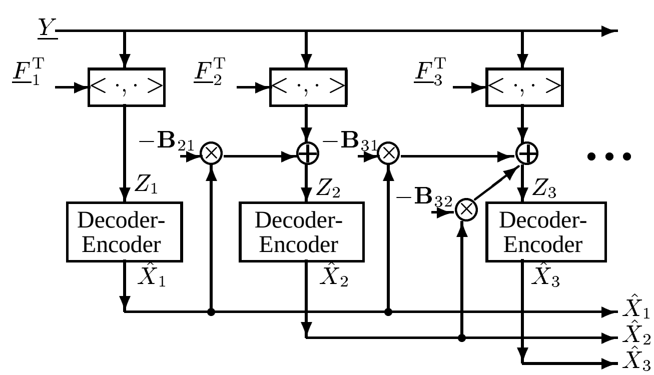

Fig. 1. The decision-feedback receiver.

strictly lower-triangular feedback matrix $\mathbf{B}$. The first user is decoded based on the decision statistic $Z_1 = \underline{F}_1^T\underline{Y}$ where $\underline{F}_1^T$ is the first row of $\mathbf{F}$ (the superscript T denotes transpose of a matrix). The resulting symbol decision is denoted as $\hat{X}_1$. The k^{th} user is decoded based on the decision statistic $Z_k = \underline{F}_k^T\underline{Y} - \sum_{i=1}^{k-1} B_{ki}\hat{X}_i$. Such processing effectively provides for each user a single-input single-output channel. For a given order in which the users are processed, and under the assumption of perfect feedback, $\mathbf{F}$ and $\mathbf{B}$ are obtained to maximize every users' signal-to-interference ratios (SIR) in [8] [9]. This receiver will be refered to as the MSIR-DF receiver and is specified next.

Define the weighted correlation matrix $\mathbf{H} = \mathbf{P}^{1/2}\mathbf{R}\mathbf{P}^{1/2}$. Let $\mathbf{H}_{(k+1)}$ denote the principal submatrix of $\mathbf{H}$ formed by the indices k through K, and $\underline{H}_{(k)}$ the first column of $\mathbf{H}_{(k+1)}$ after striking out its first element.

The feedforward matrix for the MSIR-DF receiver is given by $\mathbf{F} = (\mathbf{L}^{\mathsf{T}})^{-1}\mathbf{P}^{1/2}$ where $\mathbf{H} + \frac{N_0}{2}\mathbf{I} = \mathbf{L}^{\mathsf{T}}\mathbf{L}$ is a Cholesky decomposition with $\mathbf{L}$ being lower triangular, and where $\mathbf{I}$ is the identity matrix. The feedback matrix is the strictly lower triangular part of the matrix product $\mathbf{FR}$. The SIR of the k^{th} user for the MSIR-DF receiver, γ_k, can be expressed as [7]

$$\gamma_k = (\tfrac{N_0}{2})^{-1}\left(p_k - \underline{H}_{(k)}^{\mathrm{T}}(\mathbf{H}_{(k+1)} + \tfrac{N_0}{2}\,\mathbf{I})^{-1}\underline{H}_{(k)}\right) . \tag{4}$$

We are now in a position to state the signal design problem: given received powers $\mathbf{P}$, and the QoS constraints that user k must achieve an SIR that is no less than β_k, find a correlation matrix $\mathbf{R}$ (or equivalently $\mathbf{H}$) with as small a rank (and hence bandwidth) as possible that will result in $\gamma_k \geq \beta_k$ for each $k = 1, 2, \cdots, K$ where γ_k is given in (4). We choose β_k to be some fraction $\eta_k \in (0, 1]$ of the maximum achievable SIR (that of a single-user channel), i.e., $\beta_k = 2\eta_k \frac{p_k}{N_0}$.

In [7] we obtain an algorithm for designing a correlation matrix that satisfies the QoS constraints and has small, if not minimal, rank. The algorithm is very simple to describe. Assume that the users are decoded in the increasing order of their indices (1 to K). The matrix $\mathbf{H}$ is constructed recursively as follows. Set $H_{KK} = p_K$. Given that $\mathbf{H}_{(k+1)}$ has been determined, $\mathbf{H}_{(k)}$ is obtained by specifying $\underline{H}_{(k)}$ to be

$$\underline{H}_{(k)} = \sqrt{\min\{(1-\eta_k)(\lambda_k + \tfrac{N_0}{2}), \lambda_k\}}\, p_k^{1/2}\underline{V}_k, \tag{5}$$

where λ_k is the smallest non-zero eigenvalue of $\mathbf{H}_{(k+1)}$ and $\underline{V}_k$ is its corresponding eigenvector.

There is, of course, the issue of choice of order of decoding. In a coded system erroneous feedback can be reduced to an acceptable level with good single-user codes so that, in principle, any ordering is fine [10]. Relative to some other orderings, it has been found that processing the users in decreasing order of $(1 - \eta_k)$ consistently does well at conserving bandwidth. The idea is to detect the users with more power to spare before those with little or no extra power.[1] Under a symmetric SIR constraint (i. e., $\gamma_k \geq \beta_{\text{sym}}$ for all k), this ordering rule is equivalent to decoding the users in descending order of p_k.

The algorithm is suboptimal in that it does not guarantee the minimal possible rank. However, it is shown in [7] that the above algorithm often yields a correlation matrix with a rank

[1] In an uncoded system, an ordering rule that directly mitigates error propagation effects, such as the decreasing order of the QoS requirements, is more appropriate.

that is close to a (generally unachievable) lower bound on minimum rank (to be described below). In the best-case scenario, the minimum rank and our signal design strategy would yield the minimum processing gain of 1. In this case, it suffices for all users to employ the same signature waveform. In the worst-case scenario, the minimum rank required may be $M = K$ (e.g. when $\eta_k = 1$ for all k) in which case it is not possible to improve on the bandwidth required for Time or Frequency Division Multiple-Access (TDMA or FDMA).

The minimum rank is a function of the received powers and the SIR constraints but the problems of finding it and/or the corresponding correlation matrices (with minimum rank) that satisfy the QoS constraints are as yet unsolved. However, in [7] we find a (generally unachievable) lower bound by allowing a variable signaling rate $1/T$ (that depends on the received powers) and joint maximum-likelihood decoding at the receiver or rate-splitting multiple-access [11]. These strategies are however extremely difficult, if not impossible, to implement in practice.

Henceforth, the CWMA system that specifically uses the MSIR-DF receiver and the signals derived from the correlation matrix obtained according to the above recursive algorithm will be refered to as the BEMA system. The corresponding signal design will be referred to as the BEMA design.

III. Signal Designs for Linear Receivers and Sum-capacity Maximization

Recently, two other methods of signal design have been proposed in the literature.

In [4], Viswanath, Anantharam and Tse consider the linear MSIR receiver. For a given processing gain (and with or without a constraint on the average received powers), they obtain necessary and sufficient conditions under which there would exist powers and signature sequences that will allow the linear MSIR receiver to achieve a given set of target SIR constraints.

In [6], Viswanath and Anantharam consider the CWMA model with fixed users' received powers. For any fixed processing gain, they obtain signature sequences that maximize the sum capacity of the channel (this problem was solved earlier for the special case of equal fixed receive powers in [5]). As in BEMA, this approach does not employ power control, but unlike BEMA it focuses on a single design criterion (i. e., sum capacity) instead of a set of QoS criteria.

This section describes ways of interpreting the results of [4] and [6], optimal though they are in their respective contexts, within the context of a QoS-based approach to a combined design of signature waveforms and receiver to achieve bandwidth efficient multiple-access communications.

In addition to the notation used so far, we let the signature sequences (coefficients of the linear combination of basis function waveforms) of the users be arranged as unit-norm columns in an $M \times K$ matrix $\mathbf{A}$ (note that $\mathbf{R} = \mathbf{A}^\top \mathbf{A}$), and define $\mathbf{S} = \mathbf{I} + (\frac{N_0}{2})^{-1}\mathbf{P}^{1/2}\mathbf{A}^\top\mathbf{A}\mathbf{P}^{1/2}$.

A. Optimum Signal Design and Power Control for the Linear MSIR Receiver

Given a set of SIR constraints, the Viswanath-Anantharam-Tse (VAT) method of signal design determines whether or not there exists a set of signature sequences with processing gain M and a set of received powers (with or without an average-power constraint) such that each user meets its SIR constraint with a linear MSIR receiver. In this case, the SIR of the $k^{\rm th}$ user, given by $\gamma_k = 1/(\mathbf{S}^{-1})_{kk} - 1$, is required to be no less than β_k. Without loss of generality, assume that $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_K$, and define the *effective bandwidth* of the $k^{\rm th}$ user as $e(\beta_k) \triangleq \beta_k(1+\beta_k)^{-1}$. There is a unique $k^* \in \{0, 1, \cdots M-1\}$ such that $(M-k^*)e(\beta_{k^*}) > \sum_{j=k^*+1}^{K} e(\beta_j) \geq (M-k^*)e(\beta_{k^*+1})$. A key result in [4] is as follows.

For a given processing gain $M < K$, there exists a $(\mathbf{P}, \mathbf{A})$ pair such that the users meet the SIR constraints and the average-power constraint, $K^{-1}\sum_{k=1}^{K} p_k \leq \overline{P}$, if and only if

$$M > \sum_{k=1}^{K} e(\beta_k) \quad \text{and} \quad \overline{P} \geq \frac{N_0/2}{K}\left(\sum_{i=1}^{k^*}\beta_i + \frac{(M-k^*)\sum_{i=k^*+1}^{K} e(\beta_i)}{M-k^*-\sum_{i=k^*+1}^{K}\beta_i}\right). \qquad (6)$$

Without the power constraint, these two conditions reduce to only the first condition.

It is straightforward to view this design within the context of BEMA. In the BEMA design, we are given the users' received powers so that their average power is $\overline{P} = \frac{1}{K}\sum_{k=1}^{K} p_k$. Suppose that the BEMA signal design described Section II yields a correlation matrix with rank $M_{\rm BEMA}$. Similarly, let $M_{\rm VAT}$ (subject to the average power constraint) and $M_{\rm VAT-U}$ (unconstrained powers) denote the minimum required processing gains for the VAT signal design to meet the SIR constraints. In contrast to BEMA, the VAT and VAT-U designs have the additional freedom of choosing the most favorable power distribution, the former within the confines of an average power constraint and the latter without any such constraints. Consider now the following two examples.

Example 1: In this example the target SIR values of all users are assumed to be identical (i. e., we have a symmetric SIR constraint). Figure 2 plots the required processing gain versus SNR for fifteen users for several power distributions (see [12] for a justification of these distributions). The SNR is defined to be that of the weakest user, whose power is set at unity, so that $\text{SNR} = 10\log_{10}(1/N_0)$. The signaling interval is fixed at $T = 1.0$, and the symmetric SIR constraint is given by $\beta_{\rm sym} = (\frac{N_0}{2})^{-1}$. Additionally, the figure also plots the FDMA upper bound and the lower bound mentioned in Section II.

It is clear from the figure that both BEMA and VAT/VAT-U lie within the upper and lower bounds, and that BEMA uniformly and significantly outperforms the VAT and VAT-U designs. The performance gains of BEMA are especially evident as the disparity in the users' powers is increased. Note also that there are situations where BEMA actually achieves the lower bound and is hence optimal in such cases.

It can be easily shown that for large enough SNR both VAT-U and BEMA are forced to require the same processing gain as FDMA (i. e., $M = K$). For VAT-U, this occurs whenever

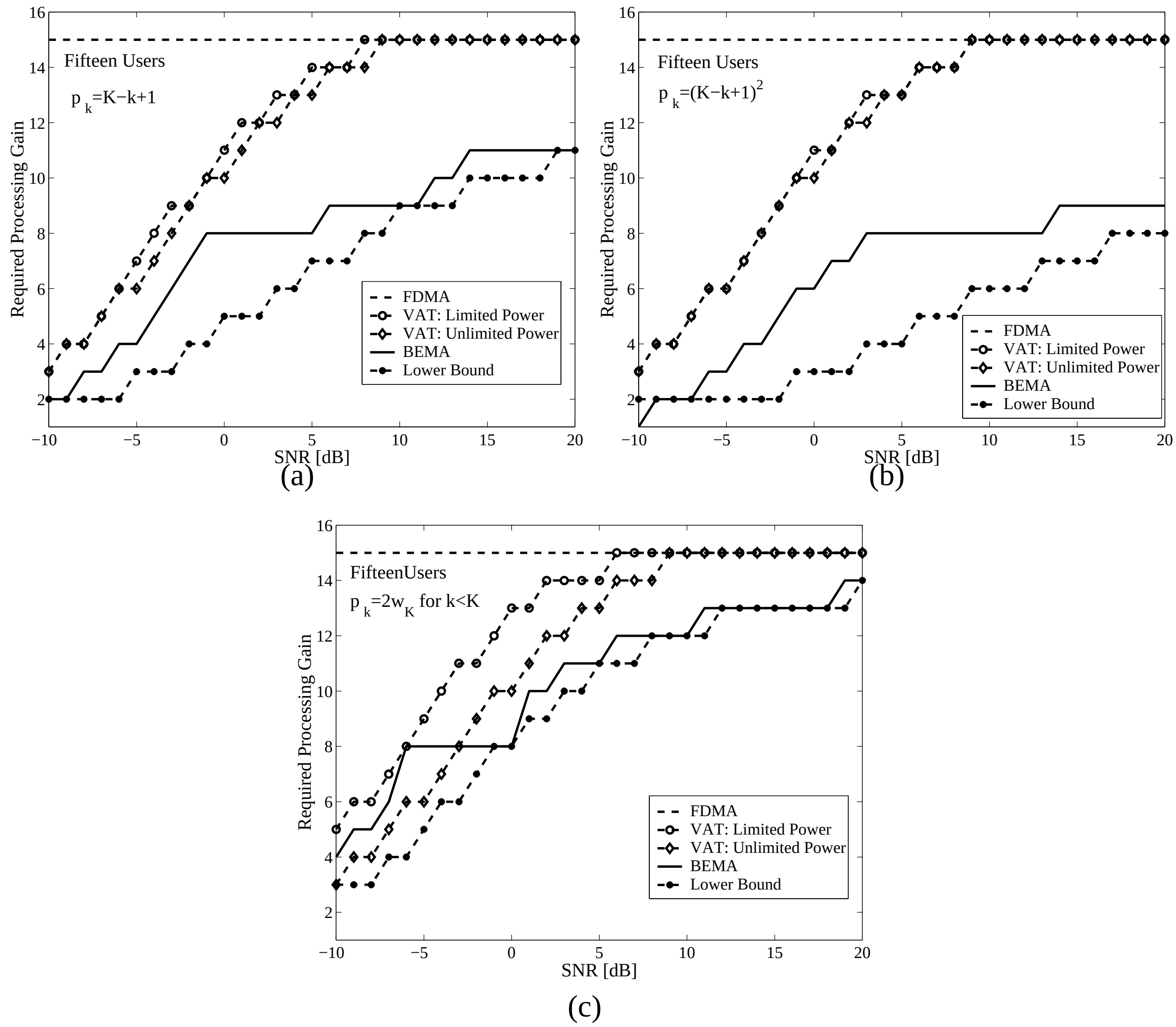

Fig. 2. Required processing gain versus SNR for fifteen users and several power disparities.

$\beta_{\rm sym} > K-1$. Thus, for the example at hand the processing gain of VAT-U is 15 whenever the SNR is greater than 8.451 dB. In contrast, we find experimentally that the processing gain of BEMA does not reach 15 in Figure 2 (a) until the SNR reaches 120.3 dB. Moreover, at this SNR, the lower bound on the processing gain is 14 which is itself close to that required by FDMA. Similarly, in Figure 2 (c), the processing gain of BEMA reaches 15 at an SNR of 41.2 dB, at which point the corresponding lower bound on the processing gain is again 14.

Example 2: In this example the SIR constraints are taken to be asymmetric. There are five users whose powers, in the case of BEMA, are fixed, and the signaling interval is once again set at $T = 1.0$. To establish a set of asymmetric SIR constraints, we randomly choose the SIR constraint of the $k^{\rm th}$ user, β_k, from the interval $[0, p_k/(N_0/2)]$ according to a uniform distribution. Given these constraints, the required processing gain of both BEMA and VAT/VAT-U are evaluated. In Figure 3 we plot a histogram of the processing gain of BEMA versus the processing gain of VAT/VAT-U over one thousand samples of the SIR constraints for users whose powers exhibit linear disparity; several values of SNR are considered. As was the case in the previ-

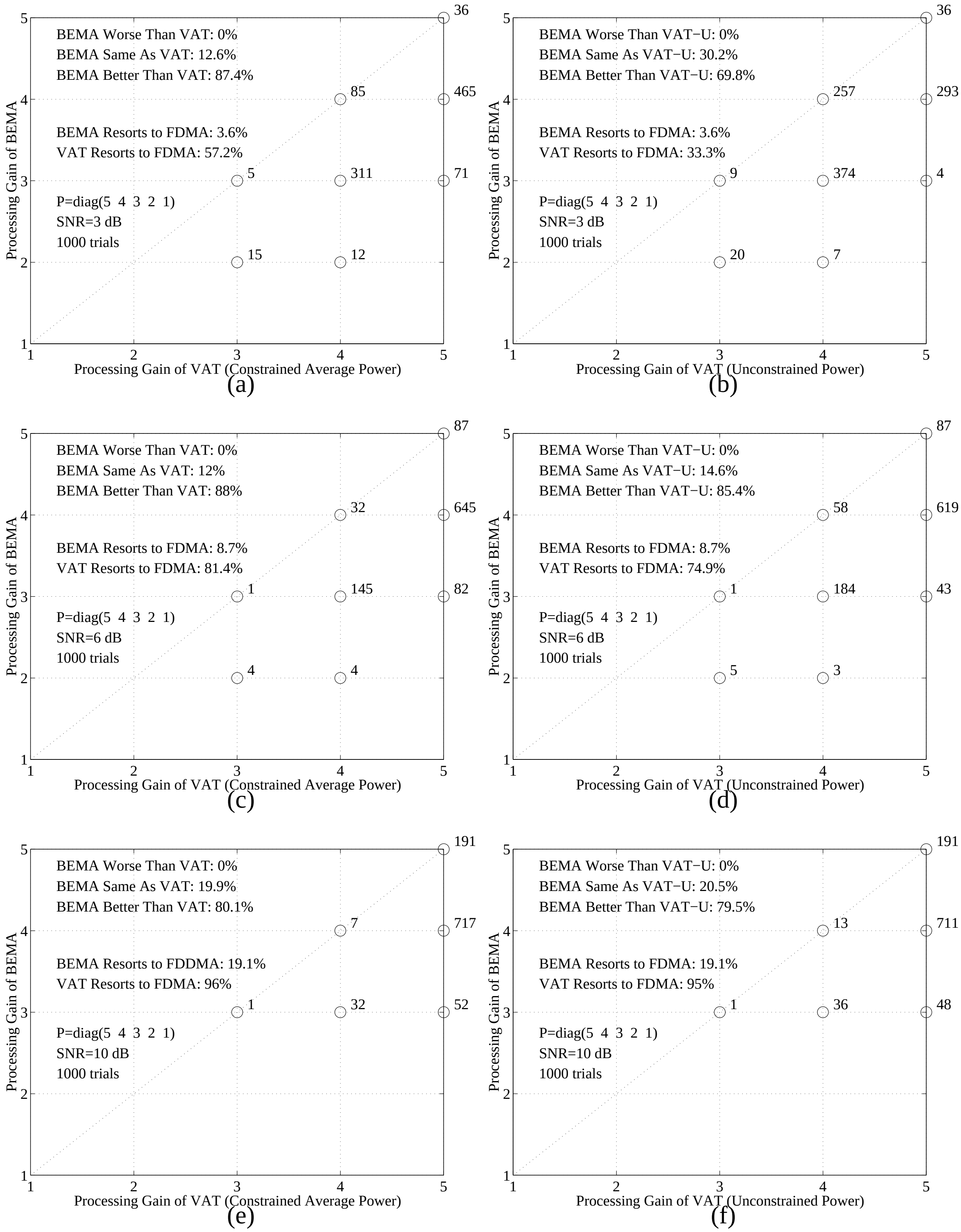

Fig. 3. Random non-symmetric QoS constraints with linearly disparate users.

ous example, BEMA once again dramatically outperforms the VAT and the VAT-U designs. Nowhere do VAT and VAT-U perform better than BEMA, while BEMA is strictly better than VAT/VAT-U anywhere from 69% to 88% of the time. Also observe that the VAT and VAT-U signal designs often degenerate into FDMA, thereby requiring maximum processing gain. For instance, this happens 81.4% of the time for VAT and 74.9 % of the time for VAT-U when the SNR is 6 dB. In stark contrast, BEMA's performance degenerates to orthogonal signals only 8.7% of the time under the same conditions.

We conclude from the above two examples that for many scenarios the structure of linear multiuser receivers is too restrictive to allow any significant improvements over the simple FDMA system in spite of using optimum signal and power allocations. In stark contrast, nonlinear receivers with successive decoding are able to extract much of the gain in spectral efficiency that is theoretically possible with variable signaling rate and multiuser coding.

B. Maximum Sum-Capacity Signal Design

Given a processing gain M and a set of received powers, the Viswanath and Anantharam (VA) method of signal design chooses the set of signature sequences that maximizes the sum capacity of the channel [6]. The sum capacity of the channel is given by $\frac{1}{2T}\log|\mathbf{S}|$ in nats/sec. Assume without loss of generality that $p_1 \geq p_2 \geq \cdots \geq p_K$, and let $P_{\mathrm{tot}} = \sum_{k=1}^{K} p_k$. There is a unique $k^* \in \{0, 1, \cdots, M-1\}$ that satisfies $(M-k^*)p_{k^*} > \sum_{j=k^*+1}^{K} p_j \geq (M-k^*)p_{k^*+1}$. A central result in [6] is that for $M < K$, the maximal sum capacity is

$$C_{\mathrm{sum}} = \frac{M-k^*}{2T}\log\left(1 + \frac{\sum_{j=k^*+1}^{K} p_j}{(M-k^*)\frac{N_0}{2}}\right) + \frac{1}{2T}\sum_{j=1}^{k^*}\log\left(1 + \frac{p_j}{\frac{N_0}{2}}\right). \tag{7}$$

See [6] for the construction of the corresponding signature-sequence matrix $\mathbf{A}$.

We now make a comparison of this signal design to BEMA. Suppose we are given a set of SIR constraints and that the BEMA design yields a correlation matrix with processing gain M_{BEMA}. For the VA signal design we start with a processing gain of $M = 1$ and find the set of signature sequences that maximizes the sum capacity. To place this in the BEMA framework we assume that the MSIR-DF receiver is used for this sum-capacity signal design. This allows the users to achieve the sum capacity with single-user coding [9]. However, we require that all of the QoS constraints be met. Therefore, if the MSIR-DF receiver can meet the users' SIR constraints for *some* ordering of the users, we let $M_{\mathrm{VA}} = 1$, otherwise we repeat the process for $M = 2$, etc. The minimum processing gain such that the MSIR-DF receiver can meet the SIR objectives is denoted by M_{VA}. Note that while both the BEMA and VA designs use the same type of receiver, the VA design has the extra freedom relative to BEMA of choosing the best permutation from all $K!$ orders in which users can be decoded.

Example 3: Consider a five-user channel where the received powers are fixed and the signaling interval is $T = 1.0$. To establish a set of asymmetric SIR constraints, we randomly choose the QoS of the k^{th} user to be the rate r_k [nats/sec] from the interval $[0, \frac{1}{2T}\log(1 + p_k/(N_0/2)]$ according to a uniform distribution.[2] From r_k it is easy to obtain the corresponding QoS constraint expressed as a SIR value, β_k. Given these constraints, the required processing gains of both BEMA and the VA design are evaluated. Figure 4 contains a histogram of the processing gain of BEMA versus the processing gain of VA over one thousand samples of the QoS constraints. Several different distributions of received powers are considered, and the weakest user's SNR is either 3 dB or 6 dB. Note that the VA design rarely performs better than BEMA (less than two percent of the time in any one of the six histograms in Figure 4), whereas BEMA strictly outperforms the VA design anywhere from 37% to 78% of the time depending on the situation. Note also that as the disparity in the users' powers and/or the SNR increases, the VA design tends to resort to orthogonal signals with maximum bandwidth (as much as 46% of the time in part (f)), whereas BEMA degenerates into FDMA no more than 0.3% of the time.

It should be noted that for BEMA the users were always decoded in decreasing order of $1 - \eta_k$ in accordance with the signal design given in Section II. By considering all possible permutations instead, as was done for the VA design in this example, the value of M_{BEMA} can sometimes be further reduced, although usually not significantly.

One must of course not lose sight of the fact that the VA design is optimal in the sense that it maximizes sum capacity. So if BEMA is evaluated in terms of sum capacity, it would be suboptimal. The above examples, though, do illustrate the point though that the sum-capacity is too coarse a design metric to be used in the BEMA framework where the problem is one of bandwidth conservation under QoS constraints with a decision-feedback receiver.

IV. Conclusions

Bandwidth-efficient multiple-access (BEMA) is a strategy for uplink communication that combines multiuser decision-feedback equalization and signal design under quality-of-service constraints. Within this QoS-based framework, we have shown that the BEMA signal design method requires much less bandwidth than the recently proposed optimum signal-designs methods for linear MSIR receivers and sum-capacity maximization (with a decision-feedback receiver). The constraint that the receiver be linear in the former approach is too restrictive, and the single-quantity maximization of the latter may be too coarse a design metric.

[2] Here we choose the QoS uniformly over the capacity of a single-user channel without any multiple-access interference, rather than uniformly over the valid range of SIR values. This measure of QoS is more in keeping with the original idea of BEMA as well as the sum-capacity maximization of the VA design. The conclusions are however similar for both cases.

REFERENCES

[1] T. Guess and M. K. Varanasi, " Onion Peeling for CDMA: Symmetric Capacity Under RMS-Bandwidth Constraints ," in *Proc. Thirtieth Annual Conference on Information Sciences and Systems,* Princeton, NJ, pp. 584–589, Mar. 1996.

[2] M. K. Varanasi and T. Guess, " Bandwidth-Efficient Multiple-Access via Signal Design for Decision-Feedback Receivers: Towards an Optimal Spreading-Coding Trade-Off ," in *Proc. IEEE Global Telecommun. Conf.: Commun. Theory Mini-Conf.*, Phoenix, AZ, pp. 159–165, Nov. 3-8 1997.

[3] M. K. Varanasi and T. Guess, " Bandwidth-Efficient Multiple-Access (BEMA): Signal Design with a Strict Bandwidth Criterion Under Quality-of-Service Constraints ," in *Proc. Intl. Symposium on Signals, Systems, and Electronics (ISSSE'98)* Pisa, Italy, Sep. 29-Oct. 2 1998.

[4] P. Viswanath, V. Anantharam, and D. Tse, "Optimal Sequences, Power Control and User Capacity of Synchronous CDMA Systems with Linear MMSE Multiuser Receivers," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1968–1983, Sept. 1999.

[5] M. Rupf and J. L. Massey, "Optimum Sequence Multisets for Synchronous Code-Division Multiple-Access Channels," *IEEE Trans. Inform. Theory*, vol. IT-40, pp. 1261–1266, Jul. 1994.

[6] P. Viswanath and V. Anantharam, "Optimal Sequences and Sum Capacity of Synchronous CDMA Systems," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1984–1991, Sept. 1999.

[7] M. K. Varanasi and T. Guess, " Bandwidth-Efficient Multiple Access (BEMA): A New Strategy Based on Signal Design for Multiuser Receivers Under Quality-of-Service Constraints ," Submitted *IEEE Trans. Commun.*, May 1998.

[8] T. Guess and M. K. Varanasi, " Multiuser Decision-Feedback Receivers for the General Gaussian Multiple-Access Channel ," in *Proc. Thirty-fourth Allerton Conf. Commun., Contr. and Comput.,* Allerton, IL, pp. 190–199, Oct. 1996.

[9] M. K. Varanasi and T. Guess, " Optimum Decision Feedback Multiuser Equalization with Successive Decoding Achieves the Total Capacity of the Gaussian Multiple-Access Channel ," in *Proc. Thirty-First Asilomar Conf. Signals, Systems, and Computers* , pp. 1405–1409, Nov. 1997.

[10] T. Guess and M. K. Varanasi, "Error Exponents for Maximum-Likelihood and Successive Decoders for the Gaussian CDMA Channel," *IEEE Trans. Inform. Theory*, vol. 46, pp. 1683–1691, Jul. 2000.

[11] B. Rimoldi and R. Urbanke, " A Rate-Splitting Approach to the Gaussian Multiple-Access Channel ," *IEEE Trans. Inform. Theory*, vol. IT-42, pp. 364–375, Mar. 1996.

[12] T. Guess and M. K. Varanasi, "Signal Design for Bandwidth-Efficient Multiple-Access Communications Based on Eigenvalue Optimization," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2045–2058, Sept. 2000.

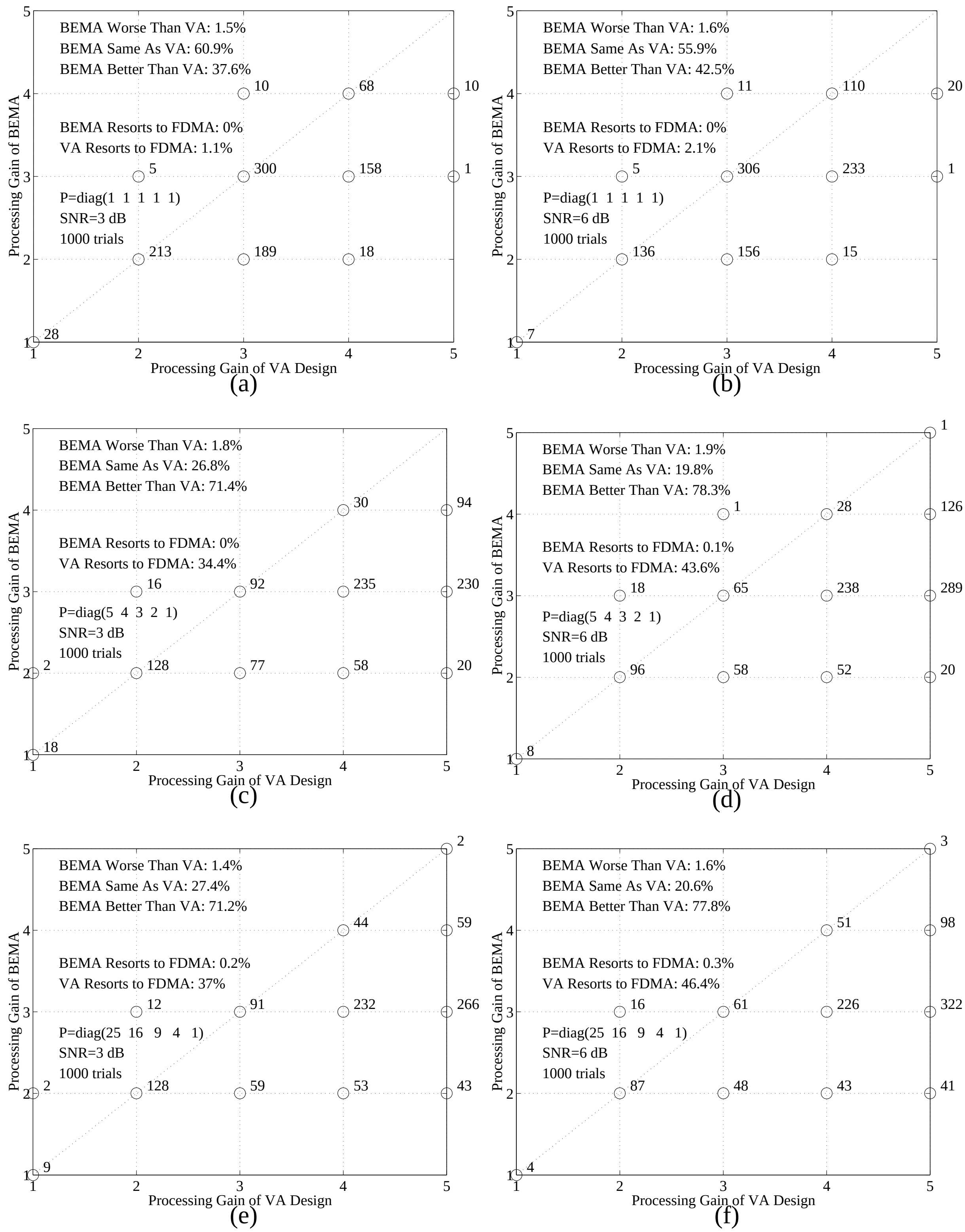

Fig. 4. Random non-symmetric QoS constraints for several user disparities and several SNR values.