A New Successively Decodable Coding Technique for Intersymbol-Interference Channels[1]

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Abstract — **For the Gaussian channel with intersymbol-interference (ISI), it is known that there is no loss in channel capacity if the receiver is an ideal minimum mean-squared error (MMSE) decision-feedback equalizer (DFE) with error-free feedback. However, combining the DFE with channel coding is problematic. Transmitter precoding and reduced-state sequence estimation are two common approaches (cf. [1] and references therein). This paper introduces a new successively-decodable coding technique that effectively combines channel coding with decision-feedback that is housed in the receiver.**

I. The Channel Model

Consider the real-valued discrete-time Gaussian channel with intersymbol interference (ISI) represented by

$$y_k = \sum_{j=0}^{M-1} h_j x_{k-j} + n_k, \qquad (1)$$

where $\{x_k\}$ is a sequence of zero-mean, independent identically-distributed (i.i.d.) transmitted symbols with power $E[x_k^2] = p$, $\{h_k\}_{k=0}^{M-1}$ is the finite-tap discrete-time, post-cursor channel response, and $\{n_k\}$ is an i.i.d. sequence of zero-mean Gaussian noise samples with variance $E[n_k^2] = \sigma^2$. The average mutual information of the channel (bits per channel use) is maximized when the symbol distributions are zero-mean Gaussian random variables with power p.

II. Successively Decodable Coding Technique

We describe the two-level successive decoder for the ISI channel from which the corresponding coding technique is easily inferred. Begin by blocking the channel output sequence into vectors of length L. We view this vector output sequence as N distinct vector channels, the n-th of which is given by

$$\left\{ \left[\begin{array}{ccc} y_{(kN+n-1)L+1} & \cdots & y_{(kN+n)L} \end{array} \right]^{\mathsf{T}} \right\}_{k=-\infty}^{\infty}. \qquad (2)$$

Note that y_k is statistically independent of y_{k-M}. Therefore, if $N \geq \left\lceil \frac{M+L-1}{L} \right\rceil$, then the output sequence of the n-th channel is the output sequence of a memoryless vector channel. Thus, we have decomposed the ISI channel into N memoryless vector channels that are statistically related to each other.

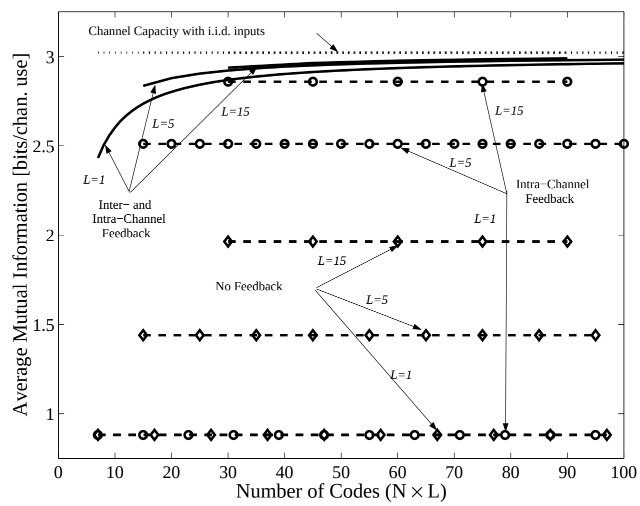
Outer-level coding allows the N vector channels to be decoded one at a time, starting with channel 1 and ending with channel N. If, when decoding the n-th channel, we make use of symbol decisions from the channels that have already been decoded (i. e., vector channels 1 through $n-1$), we refer to this as inter-channel feedback. Clearly, the potential advantage of inter-channel feedback increases with N, the number of vector channels.

Inner-level coding addresses each vector channel by viewing it as consisting of L scalar sub-channels that are successively decoded with single-user coders and decoders. If, when decoding the l-th sub-channel of a particular vector channel, we make use of symbol decisions from sub-channels that have already been decoded (i. e., sub-channels 1 through $l-1$), we refer to this as intra-channel feedback. Since this vector channel can be cast as a memoryless multiple-access channel, the optimal successive-decoding technique developed in [2] can be implemented. For any given vector channel, performance potential will improve as L, the block size, increases.

Hence, the original ISI channel is treated as a composition of NL sub-channels which are to be coded and decoded successively using single-user codes, with or without inter- and intra-channel feedback.

III. Example

Consider the response of the 2 kft-AWG26 channel (i. e., $h_0 = 1$, $h_1 = -0.6$, $h_2 = -0.15$, $h_3 = -0.12$, $h_4 = -0.05$, $h_5 = 0.00$, and $h_6 = 0.05$) [1], which is operating at a coded-symbol signal-to- noise ratio of $p/\sigma^2 = 18.0$ dB. The following figure compares the theoretical rate of information transmission for several schemes. The average mutual information is plotted as a function of the total



number of sub-channels, NL. It is evident that increasing the vector length L can provide substantial gains for each scheme presented and that there is an advantage in implementing inter-channel feedback in addition to intra-channel feedback.

References

[1] D. Yellin, V. Vardy, and O. Amrani, "Joint Equalization and Coding for Intersymbol Interference Channels," *IEEE Trans. Inform. Theory*, vol. IT-43, pp. 409–425, Mar. 1997.

[2] M. K. Varanasi and T. Guess, " Optimum Decision Feedback Multiuser Equalization with Successive Decoding Achieves the Total Capacity of the Gaussian Multiple-Access Channel ," in *Proc. Thirty-First Asilomar Conf. Signals, Systems, and Computers* , pp. 1405–1409, Nov. 1997.

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