Signal Design for Bandwidth Efficient Multiple Access with Guaranteed Bit Error Rate

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Abstract— **The problem of signal design for bandwidth efficient multiple access is addressed under quality of service (QoS) constraints specified by (possibly different) BER. Indeed, BER more accurately measures the performance than the signal-to-interference ratio (SIR) for uncoded communication. Furthermore, we argue that the asymptotic effective energy (AEE) faithfully characterizes BER, and we translate the BER-specified QoS into AEE-specified ones. We then propose a recursive, greedy algorithm for joint signal design to minimize the system bandwidth while ensuring that each user achieves its desired AEE (and hence BER). This algorithm successfully converts excess power into bandwidth savings under reliability constraints, and significantly improves spectral efficiency over full-rank (e.g. orthogonal) signaling and SIR-based approaches.**

I. Introduction

The problem of designing signals for efficient and reliable multiuser systems is receiving growing interest, an interest that draws on analytical results available about the performance of multiuser detectors. Two recent approaches [1, 2], based on the linear and decision feedback minimum mean-squared error (MMSE) detectors with and without power control, respectively, use the strict bandwidth measure and QoS specified in terms of desired signal-to-interference ratios (SIRs). These designs yield not only noise- (or SNR-) dependent signaling, but also full-rank signaling for sufficiently high SNR, and hence no bandwidth savings over orthogonal signaling in these regimes. Moreover, the SIR measure does not directly characterize the BER of the corresponding user.

In this paper, we consider the framework of Bandwidth Efficient Multiple Access (BEMA) introduced in [1, 3], where the base station employs an MMSE decision feedback detector (MMSE-DFD), designs signals to minimize strict bandwidth while ensuring that each user achieves its desired QoS, and allocates these signals to, and for use by, uplink transmitters. Within this framework, we consider the problem that specifies the QoS in terms of BER, which we characterize, in turn, by the asymptotic effective energy (AEE) [4]. We then propose a new algorithm for signal design that meets the QoS requirements with as small a bandwidth as possible. This algorithm converts excess received power into bandwidth savings, and consists of a recursive and greedy construction of the signal matrix, where, at each stage, the new signal is designed by appropriately sharing power between two directions in the already designed signal space to exactly meet the QoS and preserve bandwidth. We illustrate that our approach is substantially more spectrally efficient than full-rank signaling and SIR-based approaches.

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The structure of the paper is as follows. The channel model is presented in Section II. Section III summarizes results in [5] on the asymptotic form and performance of, and signal design for, linear MMSE detection. In Section IV, we formulate the joint signal design problem, present our constructive algorithm, and illustrate its performance. Section V concludes this paper.

II. CWMA System Model

We consider a Correlated Waveform Multiple Access (CWMA) system wherein K users communicate independently and synchronously over an additive white Gaussian noise (AWGN) channel, by linearly modulating a signature waveform, in each signaling interval, with a symbol from an M-ary Quadrature Amplitude Modulation (QAM). The signature waveforms satisfy the generalized Nyquist criterion [6] and are normalized to have unit-energy. The users data streams are independent, and each stream is an i.i.d. sequence of symbols drawn for a common M-ary QAM alphabet denoted by $\boldsymbol{\mathcal{A}} = \{\alpha_1, \ldots, \alpha_M\}$, which is also normalized to have unit-energy. Coherent detection is assumed.

A discrete model is given by the complex N-dimensional vector $\mathbf{r} = \mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n}$, where N is the dimension of the signal space, denoted by $\mathcal{S}$, and $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$ is the $N \times K$ matrix of signals. The span of $\mathbf{S}$ is $\mathcal{S}$, it has full row-rank N, and its columns have unit-norm. $\mathbf{A} = \mathrm{diag}\,(A_1, \ldots, A_K)$, where $A_k = \sqrt{E_k}\, \mathrm{e}^{j\varphi_k}$ is the complex received amplitude of user k, $\mathbf{b}^{\mathsf{T}} = [b_1, \ldots, b_K] \in \boldsymbol{\mathcal{A}}^K$ represents the data of all users (the superscript T denotes the transpose), and $\mathbf{n}$ a zero-mean, proper, complex Gaussian random vector with known covariance $\sigma^2 \mathbf{I}_N$. The span of any matrix, denoted by a bold upper-case letter (say $\mathbf{X}$), is denoted by the corresponding calligraphic letter ($\mathcal{X}$), and the projection orthogonal to this span by $\mathbf{P}_{\mathcal{X}^\perp}$. Finally, we denote by $\mathbf{I}_x$ the $x \times x$ identity matrix for any positive integer x.

III. Linear MMSE Detection, its Asymptotic Performance, and Signal Design

The (unbiased) linear MMSE detector for any user (say user 1) consists of the MMSE filter $\mathbf{f}_{1-\mathrm{M}} = A_1 \mathbf{H}^{-1} \mathbf{s}_1$, where $\mathbf{H} = \mathrm{E}[\mathbf{r}\mathbf{r}^{\mathsf{H}}]$ (the superscript H denotes the hermitian transpose), that produces a soft estimate of the data as $y_1 = \mathbf{f}_{1-\mathrm{M}}^{\mathsf{H}} \mathbf{r}$, followed by the minimum Euclidean distance rule

$$\hat{b}_1 \in \arg\min_{\alpha \in \boldsymbol{\mathcal{A}}} \left| y_1 - E_1\, \mathbf{s}_1^{\mathsf{H}} \mathbf{H}^{-1} \mathbf{s}_1\, \alpha \right|^2 . \tag{1}$$

Its asymptotic form and performance (in the low noise limit) depend on the signal space geometry [5]. Specifically, defining $\bar{\mathbf{S}}_1 = [\mathbf{s}_2, \ldots, \mathbf{s}_K]$, $\bar{\mathbf{A}}_1 = \operatorname{diag}(A_2, \ldots, A_K)$, and $\bar{\mathbf{E}}_1 = \operatorname{diag}(E_2, \ldots, E_K)$, the limit of the MMSE filter and the AEE, denoted by e_1, are given in the linear independent ($\mathbf{s}_1 \notin \bar{\mathcal{S}}_1$) and dependent cases ($\mathbf{s}_1 \in \bar{\mathcal{S}}_1$) by

$$\begin{aligned} \text{if } \mathbf{s}_1 \notin \bar{\mathcal{S}}_1, \quad \lim_{\sigma\to 0} \mathbf{f}_{1-\mathrm{M}} &= A_1 \left(\mathbf{s}_1^{\mathsf{H}} \mathbf{P}_{\bar{\mathcal{S}}_1^{\perp}} \mathbf{s}_1\right)^{-1} \mathbf{P}_{\bar{\mathcal{S}}_1^{\perp}} \mathbf{s}_1, \\ e_1 &= E_1 \, \mathbf{s}_1^{\mathsf{H}} \mathbf{P}_{\bar{\mathcal{S}}_1^{\perp}} \mathbf{s}_1. \\ \text{if } \mathbf{s}_1 \in \bar{\mathcal{S}}_1, \quad \lim_{\sigma\to 0} \mathbf{f}_{1-\mathrm{M}} &\propto A_1 \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-1} \mathbf{s}_1, \end{aligned}$$

$$e_1 = \frac{E_1}{\mathbf{s}_1^{\mathsf{H}} \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-2} \mathbf{s}_1} \times \Bigg[\mathbf{s}_1^{\mathsf{H}} \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-1} \mathbf{s}_1 - \alpha_{\mathcal{A}} \sqrt{\frac{2}{E_1}} \sum_{k=1}^{K-1} |x_k| \Big(|\cos\theta_k| + |\sin\theta_k|\Big)\Bigg]_+^2,$$

where $[a]_+ = \max\{0, a\}$, $\mathbf{x} = \left(\bar{\mathbf{S}}_1 \bar{\mathbf{A}}_1\right)^+ \mathbf{s}_1$ (the superscript $+$ denotes the pseudo-inverse), $x_k = |x_k| e^{j \arg(x_k)}$ is its k^{th} component, $\theta_k = \varphi_1 + \arg(x_k)$, and $\alpha_{\mathcal{A}} = \frac{\max_{\alpha\in\mathcal{A}} |\alpha|}{\min_{i\neq j} |\alpha_j - \alpha_i|}$ (for square M-QAM constellations, i. e., when $M = 2^{2m}$, $\alpha_{\mathcal{A}} = \frac{\sqrt{M}-1}{\sqrt{2}}$).

Consequently, in the linear independent case, the linear MMSE detector converges to the decorrelator as $\sigma \to 0$, and hence asymptotically cancels the interference, whereas in the linear dependent case, it converges to the pseudo-decorrelator, which, as detailed in [5], only partially cancels the interference.

When the received energies and the interfering users are given, the AEE of user 1 can be maximized in the linear dependent case by choosing

$$\tilde{\mathbf{s}}_1 \in \arg \max_{\mathbf{s}_1 \in \bar{\mathcal{S}}_1,\ ||\mathbf{s}_1||=1} e_1(\mathbf{s}_1), \tag{2}$$

where the dependence of the AEE on $\mathbf{s}_1$ has been made explicit. Because this problem does not appear to be tractable, we consider instead the following problem of maximizing a lower bound on the AEE

$$\hat{\mathbf{s}}_1 \in \arg \max_{\mathbf{s}_1 \in \bar{\mathcal{S}}_1,\ ||\mathbf{s}_1||=1} \mathrm{L}(\mathbf{s}_1), \tag{3}$$

$$\text{and } e_1(\mathbf{s}_1) \geq \mathrm{L}(\mathbf{s}_1) = \frac{E_1}{\mathbf{s}_1^{\mathsf{H}} \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-2} \mathbf{s}_1} \times \left[\mathbf{s}_1^{\mathsf{H}} \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-1} \mathbf{s}_1 - 2\beta_1 \sqrt{\mathbf{s}_1^{\mathsf{H}} \left(\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}\right)^{-1} \mathbf{s}_1}\right]_+^2, \tag{4}$$

where $\beta_1 = \sqrt{\frac{K-1}{E_1}}\, \alpha_{\mathcal{A}}$. This problem is solved by the following proposition.

Proposition 1: The lower bound in (4) is maximized by the direction of least interference, i. e., by choosing $\hat{\mathbf{s}}_1$ proportional to $\boldsymbol{\phi}_1$, the eigenvector of $\bar{\mathbf{S}}_1 \bar{\mathbf{E}}_1 \bar{\mathbf{S}}_1^{\mathsf{H}}$ corresponding to its minimum eigenvalue (denoted by $\lambda_{\min}$), and the maximum lower bound is

$$\mathrm{L}(\hat{\mathbf{s}}_1) = E_1 \left[1 - 2\beta_1 \sqrt{\lambda_{\min}}\right]_+^2. \tag{5}$$

These signals are known to also maximize SIR under linear MMSE detection, and that with such a signal allocation, linear MMSE detection degenerates to matched-filter detection [7]. Moreover, we conjecture that they also maximize the AEE.

IV. Joint Signal Design under Decision Feedback Detection

Here, we consider that the base station employs the nonlinear yet computationally attractive MMSE decision feedback detector (MMSE-DFD). A solution to the problem of joint signal design to minimize bandwidth under BER-based QoS is proposed. We assume that the "worst" user achieves or exceeds its QoS, and, consequently, that other users have extra received energy, relative to that needed to achieve their respective QoS. This excess energy is converted into bandwidth savings through signal design at the base station, which, using a downlink feedback channel, allocates these signals to the users. This idea motivates Bandwidth Efficient Multiple Access (BEMA), which was proposed in [1, 3] with the SIR criterion (for coded and uncoded communications) for strict and RMS bandwidth, respectively.

A. Problem formulation

The K BER-based QoS constraints specify K desired E_b/N_0 values, depending on the modulation. These values translate, in turn, into desired effective energies, as a function of the noise at the base station, that are smaller than the actual received energies. Because effective energy is not analytically tractable in general, we define instead, for each user, a target AEE equal to the desired effective energy [4]. We have thus converted the BER-based QoS into AEE-based ones. Let $\hat{\mathbf{e}}$ be the K-length vector that denotes these QoS requirements. Since strict bandwidth is proportional to the rank of the signal matrix, the problem can be formulated analytically as

$$\hat{\mathbf{S}} \in \arg \min_{\mathbf{e}(\mathbf{S}) \geq \hat{\mathbf{e}}} \operatorname{rank}(\mathbf{S}), \tag{6}$$

where $\mathbf{e}(\mathbf{S})$ denotes the actual AEEs achieved by $\mathbf{S}$, and the vector inequality is component-wise. Obviously, the minimum rank achievable under the QoS constraints is upper bounded by K, which corresponds to linear independent (full-rank) signaling.

In general, the optimization in (6) is not analytically tractable, even if we replace the AEEs $\mathbf{e}(\mathbf{S})$ by their lower bound $\mathrm{L}(\mathbf{S})$. Instead, we propose a recursive, greedy algorithm to design signals that attempts to conserve bandwidth at each stage. The dimension of the resulting signal space thus provides an upper bound on the minimum bandwidth attainable under QoS constraints. Nevertheless, the bandwidth savings over full-rank signaling can be significant.

B. Ordering rule and asymptotic form of the MMSE-DFD

The problem of error propagation, wherein erroneous decisions are fed back, is avoided by arranging and detecting users in the decreasing order of their desired AEE (i. e., $\hat{e}_1 \geq \ldots \geq \hat{e}_K$). From [4, Th. 3], this ordering ensures that each user achieves "genie-aided" performance (the genie ensures that past users' decisions are correct), and hence that the perfect feedback assumption is valid in analyzing the asymptotic performance. Consequently, to compute AEEs, we can assume that at stage k, users 1 to $k-1$ (past users) have been successfully detected, so that user k sees a *user-expurgated* channel with only users $k+1$ to K (future users) active. Its AEE, denoted by $e_k(\mathbf{s}_k)$, thus depends only on the user-expurgated signal space geometry.

The MMSE-DFD and its asymptotic form and performance for overloaded (low-rank) systems are discussed in [5]. It is comprised of the feedforward matrix $\mathbf{F}_{\text{M}} = [\mathbf{f}_{1-\text{M}}, \ldots, \mathbf{f}_{K-\text{M}}]$, where each column represents the MMSE filter of the corresponding user in the user-expurgated channel, and of the feedback matrix $\mathbf{B}_{\text{M}}$, which is the strictly lower triangular part of $\mathbf{F}_{\text{M}}^{\mathsf{H}}\mathbf{S}\mathbf{A}$. If $\mathbf{S}_k = [\mathbf{s}_k, \mathbf{S}_{k+1}]$ denotes the user-expurgated signal matrix at stage k, the asymptotic form of $\mathbf{f}_{k-\text{M}}$ is as follows:

$$\begin{aligned}
&\text{if } \mathbf{s}_k \notin \mathcal{S}_{k+1}, \quad \lim_{\sigma\to 0} \mathbf{f}_{k-\text{M}} = A_k \left(\mathbf{s}_k^{\mathsf{H}} \mathbf{P}_{\mathcal{S}_{k+1}^{\perp}} \mathbf{s}_k\right)^{-1} \mathbf{P}_{\mathcal{S}_{k+1}^{\perp}} \mathbf{s}_k, \\
&\text{if } \mathbf{s}_k \in \mathcal{S}_{k+1}, \quad \lim_{\sigma\to 0} \mathbf{f}_{k-\text{M}} \propto A_k \left(\mathbf{S}_{k+1}\mathbf{E}_{k+1}\mathbf{S}_{k+1}^{\mathsf{H}}\right)^{+} \mathbf{s}_k.
\end{aligned}$$

The statistic at stage k, denoted by z_k, is then

$$z_k = A_k b_k \mathbf{f}_{k-\text{M}}^{\mathsf{H}} \mathbf{s}_k + \sum_{j=k+1}^{K} A_j b_j \mathbf{f}_{k-\text{M}}^{\mathsf{H}} \mathbf{s}_j + \mathbf{f}_{k-\text{M}}^{\mathsf{H}} \mathbf{n},$$

where, given our ordering rule, past users' contribution to the MAI have been perfectly canceled. z_k is fed to the decision rule (1) to yield the detected symbol for user k.

C. Joint signal design algorithm

The signal design algorithm consists of a recursive construction that starts with the last user, user K, which sees a single-user Gaussian channel.

The signal for user k is designed at stage $K-k+1$ (for $k = K-1, \ldots, 2, 1$). Let $\hat{\mathbf{S}}_{k+1}$ denote the matrix of signals for users $k+1, \ldots, K$ that have already been designed. If we choose the signal for user k such that $\mathbf{s}_k \notin \hat{\mathcal{S}}_{k+1}$, then its AEE is $e_k(\mathbf{s}_k) = E_k\, \mathbf{s}_k^{\mathsf{H}} \mathbf{P}_{\mathcal{S}_{k+1}^{\perp}} \mathbf{s}_k$, and has maximum value E_k (which corresponds to $\mathbf{s}_k \perp \hat{\mathcal{S}}_{k+1}$). On the other hand, if we select $\mathbf{s}_k \in \hat{\mathcal{S}}_{k+1}$, its AEE is given in Section III for the linear dependent case. Since this expression is not analytically practical, we instead characterize its performance by the lower bound $\mathrm{L}(\mathbf{s}_k)$, which is given in (4) after substituting for $\hat{\mathbf{S}}_{k+1}\mathbf{E}_{k+1}\hat{\mathbf{S}}_{k+1}^{\mathsf{H}}$ and $\beta_k = \sqrt{\frac{K-k}{E_k}}\, \alpha_{\boldsymbol{\mathcal{A}}}$. Denote by L_k the maximum achievable lower bound on the AEE of user k when its signal lies in $\hat{\mathcal{S}}_{k+1}$. It is given as in (5) by

$$\mathrm{L}_k = E_k \left[1 - 2\beta_k \sqrt{\lambda_{\min}^{(k+1)}}\right]_+^2,$$

where $\lambda_{\min}^{(k+1)}$ denotes the minimum eigenvalue of the matrix $\hat{\mathbf{S}}_{k+1}\mathbf{E}_{k+1}\hat{\mathbf{S}}_{k+1}^{\mathsf{H}}$ ($\lambda_{\min}^{(k+1)} > 0$ as explained later).

The algorithm is then as follows. At stage $K-k+1$, compute and compare L_k with $\hat{e}_k$ to determine whether the QoS can be met with or without incrementing the signal space dimension. In either case, infinitely many signals will satisfy the QoS requirement. The final selection should maximize the likelihood of maintaining the signal space dimension constant at the next stage, stage $K-k+2$. Maintaining the dimension constant at the next stage only depends on L_{k-1} and hence on $\lambda_{\min}^{(k)}$, the smallest eigenvalue of $\mathbf{S}_k\mathbf{E}_k\mathbf{S}_k^{\mathsf{H}}$, which in turn depends on $\mathbf{s}_k$. Therefore, the strategy is to choose the new signal at stage $K-k+1$ that maximizes L_{k-1}, or equivalently minimizes $\lambda_{\min}^{(k)}$, while satisfying the QoS constraint $e_k(\mathbf{s}_k) \geq \hat{e}_k$. The algorithm can then be summarized as follows and is solved by Proposition 2.

$$\begin{aligned}
&\hat{s}_K \longleftarrow 1 \\
&\text{for } k = K-1 \text{ to } 1, \text{ compute } \mathrm{L}_k \\
&\text{if } \mathrm{L}_k \geq \hat{e}_k, \qquad \hat{\mathbf{S}}_k \longleftarrow [\hat{\mathbf{s}}_k, \hat{\mathbf{S}}_{k+1}] \\
&\qquad \text{where } \quad \hat{\mathbf{s}}_k \in \arg\min \lambda_{\min}^{(k)} \\
&\qquad \text{subject to } \|\mathbf{s}_k\| = 1,\ \mathbf{s}_k \in \hat{\mathcal{S}}_{k+1},\ \mathrm{L}(\mathbf{s}_k) \geq \hat{e}_k \\
&\text{if } \mathrm{L}_k < \hat{e}_k, \qquad \hat{\mathbf{S}}_k \longleftarrow \left[\hat{\mathbf{s}}_k, \begin{bmatrix} \mathbf{0}^{\mathsf{T}} \\ \hat{\mathbf{S}}_{k+1} \end{bmatrix}\right] \\
&\qquad \text{where } \quad \hat{\mathbf{s}}_k \in \arg\min \lambda_{\min}^{(k)} \\
&\qquad \text{subject to } \|\mathbf{s}_k\| = 1,\ s_{k1} \neq 0,\ e_k(\mathbf{s}_k) \geq \hat{e}_k
\end{aligned}$$

where $\mathbf{0}^{\mathsf{T}}$ and s_{k1} denote a $(K-k)-$dimensional row vector of zeros and the first component of $\mathbf{s}_k$, respectively.

We refer to the two cases above as (C_1) and (C_2), respectively. Note that at stage k, the matrix $\hat{\mathbf{S}}_{k+1}$ has full row-rank (denoted by N_{k+1}) by construction, and dimensions $N_{k+1} \times (K-k)$. It follows that, for (C_1), the rank is preserved ($N_k = N_{k+1}$), while it is incremented for (C_2) ($N_k = N_{k+1} + 1$). In the latter case, we zero-pad $\hat{\mathbf{S}}_{k+1}$ with $\mathbf{0}^{\mathsf{T}}$ in order to form the new signal matrix $\hat{\mathbf{S}}_k$. In either case, the matrix $\hat{\mathbf{S}}_{k+1}\mathbf{E}_{k+1}\hat{\mathbf{S}}_{k+1}^{\mathsf{H}}$ is positive definite.

Proposition 2: The solutions to the constrained optimization problems that ensure each user meets its target AEE are

$$\begin{aligned}
(C_1):& \qquad \text{when} \quad \lambda_{\min}^{(k+1)} \leq \frac{1}{4\beta_k^2}\left(1 - \sqrt{\frac{\hat{e}_k}{E_k}}\right)^2, \\
& \qquad \hat{\mathbf{s}}_k \longleftarrow \begin{cases} \boldsymbol{\phi}_{N_{k+1}} & \text{if } j = N_{k+1} \text{ or does not exist} \\ \hat{s}\, \boldsymbol{\phi}_j + \sqrt{1-\hat{s}^2}\, \boldsymbol{\phi}_{j+1} & \text{else} \end{cases} \\
(C_2):& \qquad \text{when} \quad \lambda_{\min}^{(k+1)} > \frac{1}{4\beta_k^2}\left(1 - \sqrt{\frac{\hat{e}_k}{E_k}}\right)^2, \\
& \qquad \hat{\mathbf{s}}_k \longleftarrow \begin{bmatrix} \sqrt{\hat{e}_k/E_k} \\ \sqrt{1-\hat{e}_k/E_k}\, \boldsymbol{\phi}_1 \end{bmatrix},
\end{aligned}$$

where $\hat{\mathbf{S}}_{k+1}\mathbf{E}_{k+1}\hat{\mathbf{S}}_{k+1}^{\mathsf{H}} = \boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^{\mathsf{H}}$ is its spectral decomposition, with $\boldsymbol{\Lambda} = \operatorname{diag}\left(\lambda_{\min}^{(k+1)}, \lambda_2, \ldots, \lambda_{N_{k+1}}\right)$ the diagonal matrix of eigenvalues, arranged in non-decreasing order, and $\boldsymbol{\Phi} = \left[\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_{N_{k+1}}\right]$ the unitary matrix of eigenvectors. j is the largest integer such that $\lambda_j \leq (4\beta_k^2)^{-1}\left(1 - \sqrt{\hat{e}_k/E_k}\right)^2 \leq \lambda_{j+1}$, and $\hat{s} = \tilde{\mathrm{L}}^{-1}(\hat{e}_k/E_k)$, where

$$\tilde{\mathrm{L}}(s) = \sqrt{\frac{\delta\, s^2 + (\omega-\delta)/2}{\delta\omega\, s^2 + (\omega-\delta)^2/4}}\left(\sqrt{\delta s^2 + (\omega-\delta)/2} - 2\beta_k\sqrt{\pi}\right),$$

and $\omega,\ \delta,\ \pi$ represent, respectively, the sum, difference, and product of λ_{j+1} and λ_j.

D. Comments on the joint signal design algorithm

In (C_1), the QoS is met without incrementing the bandwidth, a case for which the performance is characterized by the lower bound. Consequently, the AEE achieved by the designed signal is actually greater than the target, i. e., $e_k(\hat{\mathbf{s}}_k) \geq \mathrm{L}(\hat{\mathbf{s}}_k) = \hat{e}_k$. Furthermore, note that preserving the bandwidth at a given stage results in a penalty for the following stage. Indeed, since $\hat{\mathbf{S}}_k \mathbf{E}_k \hat{\mathbf{S}}_k^{\mathsf{H}} = E_k \, \hat{\mathbf{s}}_k \hat{\mathbf{s}}_k^{\mathsf{H}} + \hat{\mathbf{S}}_{k+1} \mathbf{E}_{k+1} \hat{\mathbf{S}}_{k+1}^{\mathsf{H}}$, it follows that $\lambda_{\min}^{(k)} \geq \lambda_{\min}^{(k+1)}$. Consequently, the penalty consists of a potential reduction of L_{k-1}. This does not improve the likelihood that $\hat{e}_{k-1}$, the QoS requirement at stage $K-k+2$, is met without incrementing the bandwidth, but it need not degrade it either: if $j \neq 1$, then it is easy to see that $\lambda_{\min}^{(k)} = \lambda_{\min}^{(k+1)}$. For most cases, however, $j = 1$, so that the degradation depends on the power along the direction of least interference $\boldsymbol{\phi}_1$, and the solution consists in appropriately distributing power to limit this degradation while satisfying the QoS.

On the other hand, in (C_2), the QoS can only be met at the price of incrementing the bandwidth. The benefit, however, is a guaranteed reduction of the smallest eigenvalue of the new matrix $\hat{\mathbf{S}}_k \mathbf{E}_k \hat{\mathbf{S}}_k^{\mathsf{H}}$. Indeed, it follows from the eigenvalue-interlacing theorem [8, Th. 4.3.4] that $\lambda_{\min}^{(k)} \leq \lambda_{\min}^{(k+1)}$ regardless of the choice for new signal. Consequently, we are ensured that incrementing the bandwidth at stage $K-k+1$ results in a higher value for L_{k-1}, and hence a greater likelihood of meeting the QoS at stage $K-k+2$ without incrementing the bandwidth.

In either case, the signal design is such that the QoS is just met (in (C_1) this is in terms of the lower bound) while maximizing the likelihood of preserving bandwidth at the next stage.

Note that the joint signal design in Proposition 2 yields orthogonal signals, i. e., $\mathbf{S} = \mathbf{I}_K$, and Identical Waveform Multiple Access, i. e., $\mathbf{S} = [1, \cdots, 1]$, if $\hat{e}_k = E_k = 1$ and $\hat{e}_k \leq E_k \left(1 - 2\beta_k \sqrt{\sum_{j=k+1}^{K} E_j}\right)^2$ for all k, respectively.

From a practical point of view, the base station needs to update the signals with changing system conditions (users entering or leaving, significant variations in the received energies, etc.). The algorithm relies on spectral decompositions and can therefore be implemented with low complexity.

E. Numerical examples

Consider a 100-user system with three different power distributions: equal, linear and quadratic, wherein user k has received energy, respectively, $E_k = 1,\ K-k+1$ and $(K-k+1)^2$. We compute the rank of the designed signal matrix as a function of the symmetric target AEE value $\hat{e} \in (0,1]$ (which corresponds to all users having the same BER QoS) for two different modulations, namely QPSK and 256-QAM. Users are arranged in the decreasing order of their received energies. The problem of optimal ordering (in the sense of minimizing bandwidth) for the symmetric QoS case with equal powers is still open, but simulations indicate that improvements are marginal. Fig. 1 illustrates the bandwidth savings. In the equal-power case with $\hat{e} = 1$, i. e., when all users must achieve single-user performance with no extra received energy, the algorithm yields orthogonal signals. In this case, the QoS can only be achieved by effectively decoupling the users into K orthogonal single-user channels (as in TDMA or FDMA). On the other hand, power disparities and/or a lower symmetric QoS offer a considerable improvement in spectral efficiency, where spectral efficiency is defined as $\eta = K/N \log_2(M)$. For example, consider that all 100 users desire a BER under QPSK modulation of 10^{-5}, and that they have a quadratic power distribution, such as arises from their natural geographical dispersion in the cell (because channel gain is inversely proportional to the square of the distance). Assuming that the noise at the base station is such that the weakest user has $E_b/N_0 = 9.6\ \mathrm{dB}$, our joint signal design guarantees the desired BER for all users with spectral efficiency $\eta = 5.88\ \mathrm{bits/chip}$ (or $\eta = 11.76\ \mathrm{bps/Hz}$). In contrast, for full-rank signaling, the spectral efficiency is $\eta = 2\ \mathrm{bits/chip}$, and all users, except the weakest one, have BERs that are strictly lower than the desired one. Note that for the unequal-power cases, the rank varies little for $\hat{e} \in [0.25, 1]$, so that there is little gain in lowering the symmetric QoS constraint. Furthermore, because the AEE depends on $\alpha_{\boldsymbol{\mathcal{A}}}$, the performance depends on the constellation size and reduces as M increases. For the example above, the spectral efficiency of our algorithm is $\eta = 8.89$ and $14.81\ \mathrm{bits/chip}$ for 16-QAM and 256-QAM, respectively.

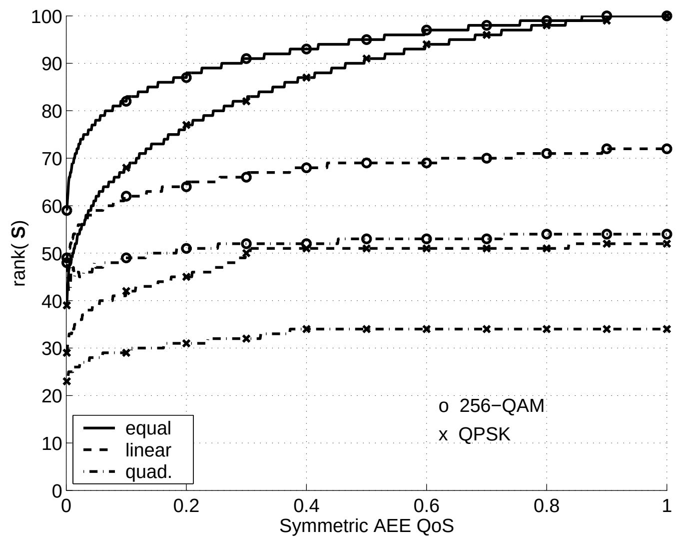

Fig. 1. Rank of the signaling matrix versus symmetric AEE QoS requirement under different power distributions for QPSK and 256-QAM.

Next, we compare the performance in terms of bandwidth reduction of our AEE-based algorithm with the SIR-based signal designs in [1, 2], which we refer to as the VG and VAT methods, respectively. A symmetric QoS is chosen as follows: for the AEE-based signal design, we fix the desired AEE to one (assuming the weakest received energy is one) and consider different modulation sizes, whereas for the VG and VAT designs (which are SNR dependent), we fix the desired SIR to $(N_0/2)^{-1}$ and consider a range of E_b/N_0 values, where the ratio is defined for the weakest user as $E_b/N_0 = 10\log_{10}(1/N_0)$. For the AEE-based and VG methods, we consider the different power distributions described above. On the other hand, for the VAT design, we assume the optimum power allocation specified in [2]. Fig. 2 plots the minimum rank N of the signal matrix that accommodates K users so that each user achieves its QoS constraint, for $E_b/N_0 = 5\ \mathrm{dB}$, quadratic powers, and $M = 2$. The three curves are quasi-linear, a behavior that was observed for varied power distributions, modulation sizes and noise densities. Defining this slope as the load factor $\beta = K/N$, we summarize its values for several scenarios in the following table.

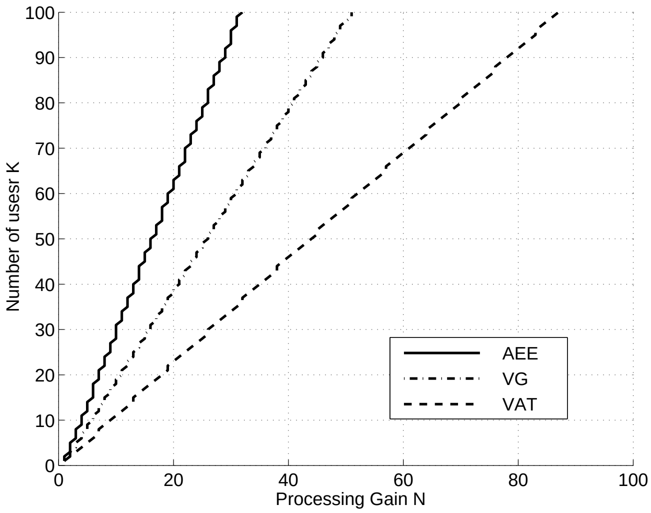

Fig. 2. Processing gain vs. number of users for the AEE-based, and SIR-based VG and VAT signal design strategies for $E_b/N_0 = 5$ dB, a quadratic power distribution, and BPSK modulation.

E_b/N_0	VAT	linear-power			quad.-power		
		VG	AEE		VG	AEE	
			M=2	M=64		M=2	M=64
0.0	1.5	2.0			2.1		
5.0	1.16	2.0	2.0	1.48	2.02	3.24	2.0
10.0	1.05	2.0			2.01		
20.0	1.0	1.6			2.0		

Finally, we illustrate the BER performance of our design with that of VG, and thereby address the important questions of the finite SNR performance of the AEE-based signal design and of the faithfulness of the SIR criterion to characterize BER. We consider the 15-user example of [1, Ex. 2] with BPSK modulation, quadratic power distribution, and symmetric constraints. The E_b/N_0 is again defined as that of the weakest user, i. e., $E_b/N_0 = 10\log_{10}(1/N_0)$. For the AEE-based approach, we fix $\hat{e} = 1$, compute the signal matrix $\hat{\mathbf{S}}_{\mathrm{AEE}}$ (we find rank$(\hat{\mathbf{S}}_{\mathrm{AEE}}) = 6$), and simulate the BER of the worst user (always user 15) for all E_b/N_0. For the SIR-based approach, the symmetric SIR constraint is $1/N_0$, so that each E_b/N_0 specifies a different QoS constraint, and hence a different signal matrix $\hat{\mathbf{S}}_{\mathrm{VG}}$ (for $E_b/N_0 \in [3, 11]$ dB, we find rank$(\hat{\mathbf{S}}_{\mathrm{VG}}) = 8$). For each E_b/N_0, we numerically simulate the BER of the worst user (always user 15, except for very low SNR). We also consider the BER of an SIR-based signal matrix designed for $E_b/N_0 = 8$ dB but used for all SNRs (hence it is optimal for $E_b/N_0 = 8$ dB only). Fig. 3 illustrates the performance along with the single-user lower bound. The AEE-based signals uniformly outperforms the optimal SIR-based signals, but the difference is small (≤ 0.5 dB). Therefore, even though they are designed for a different criterion (namely SIR), the signals in $\hat{\mathbf{S}}_{\mathrm{VG}}$ achieve good BER performance, albeit with a poorer spectral efficiency. Finally, the SIR-based signals designed for $E_b/N_0 = 8$ dB perform well for low SNRs but yield the worst performance for high SNRs. This behavior was observed for different modulation sizes and power distributions.

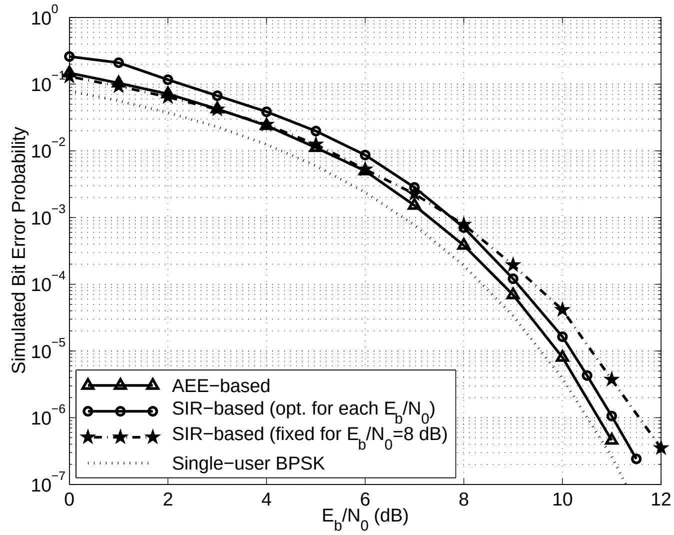

Fig. 3. BER performance comparison of different signal designs for a 15-user system with quadratic power distribution and BPSK modulation

V. Conclusion

We have proposed a new paradigm based on the BER criterion for efficient multiple access that guarantees reliable communication. In so doing, we have used a well known but little exploited performance measure of multiuser detectors, namely the AEE, which faithfully characterizes BER for high and low/medium SNR. In addition, we have introduced a new constructive, recursive, and greedy algorithm for joint signal design. At each stage, this algorithm relies on designing a new signal by distributing its power between two directions in the already constructed signal space to simultaneously meet the QoS and conserve bandwidth. This algorithm can also be adapted to SIR-based QoS [9].

References

[1] M. K. Varanasi and T. Guess, "Bandwidth-efficient multiple-access (BEMA): A new strategy based on signal design for multiuser receivers under quality-of-service constraints," *IEEE Trans. Commun.*, vol. 49, no. 5, pp. 844–854, May 2001.

[2] P. Viswanath, V. Anantharam, and D. Tse, "Optimal sequences, power control, and user capacity of synchronous CDMA systems with linear MMSE receivers," *IEEE Trans. Inform. Theory*, vol. 45, no. 6, pp. 1968–1983, Sept. 1999.

[3] T. Guess and M. K. Varanasi, "Signal design for bandwidth efficient multiple-access communications based on eigenvalue optimization," *IEEE Trans. Inform. Theory*, vol. 46, no. 6, pp. 2045–2058, Sept. 2000.

[4] M. K. Varanasi, "Decision feedback multiuser detection: A systematic approach," *IEEE Trans. Inform. Theory*, vol. 45, no. 1, pp. 219–240, Jan. 1999.

[5] A. Kapur and M. K. Varanasi, "Multiuser detection for overloaded CDMA systems," to appear *IEEE Trans. Inform. Theory*, 2003.

[6] E. A. Lee and D. G. Messerschmitt, *Digital Communication*, Kluwer Academic Publishers, Boston, MA, 2nd edition, 1994.

[7] C. Rose, S. Ulukus, and R. D. Yates, "Wireless systems and interference avoidance," *IEEE Trans. Wireless Commun.*, vol. 1, no. 3, pp. 415–428, July 2002.

[8] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 1993.

[9] A. Kapur and M. K. Varanasi, "Joint signal design for bandwidth efficient signaling in CWMA systems," in *Proc. IEEE Inform. Theory Workshop*, Paris, France, Apr. 2003.