On Maximizing Symmetric Capacity via Signal Design in CDMA Systems

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Abstract — **The maximization through signal design of the symmetric capacity of CDMA systems with optimal and minimum mean-square error decision feedback (MMSE-DF) decoding is considered. In particular, lower bounds on maximum symmetric capacity (and the signal set that achieves them) are derived for the two decoders. A common sufficient condition is obtained for both decoders under which the signal design is optimal in that the lower bounds coincide with an upper bound on the maximum achievable symmetric capacity.**

I. Introduction

The Welch bound equality (WBE) signals maximize both sum capacity and symmetric capacity (defined as the maximum equal rate) of the synchronous CDMA system with users of equal power [1]. For the more general case where the users have unequal powers, the so-called generalized WBE signals maximize sum-capacity [2]. However, these signals can be far from maximizing symmetric capacity. In fact, they can result in rate reductions for the very users that may be least able to tolerate them, namely, the weak users.

In this paper, we obtain constructive lower bounds on the symmetric capacity for the optimal as well as the (single-user coding based) MMSE-DF decoder [3]. The associated signal set achieves high values of symmetric capacity for both decoders, and for sufficiently large dimensions per channel use, coincides with the maximum achievable symmetric capacity (by an optimum decoder). Consequently, in such cases, the restriction of the decoder to the low-complexity MMSE-DF decoder does not entail a loss in achievable symmetric capacity.

In particular, the signal design algorithm employed here is the one presented recently by the authors in [5] in the context of Bandwidth Efficient Multiple-Access (BEMA) [4]. The goal in [4] [5] was to obtain, for a given set of received powers and quality-of-service (QoS) requirements for each user in terms of rate (in bits/sec) to be met by the MMSE-DF decoder (or equivalently, signal-to-interference ratio (SIR)), the signal set with the smallest overall bandwidth.

Notation: K, N, $\mathbf{p} = [p_1, \ldots, p_K]^\mathsf{T}$, and σ^2 denote, respectively, the number of users, processing gain, power constraints, and the noise variance. The users' symbols are assumed independent and Gaussian distributed, and signals are unit-norm.

II. Main Results

For SIR constraints specified by the K-length vector $\boldsymbol{\gamma}$, let $\hat{N}(\boldsymbol{\gamma})$ denote the rank of the signal set obtained from the algorithm in [5].

Proposition 1 *Given K, N, $\mathbf{p}$, and σ^2, the symmetric capacity (in bits/chip) under MMSE-DF decoding is lower bounded by $C^{\mathrm{L}}_{\mathrm{sym}} = 1/(2N)\log_2(1+\gamma_{\mathrm{L}})$, where γ_{L} is the largest value such that $\hat{N}(\gamma_{\mathrm{L}}\mathbf{1}) \le N$ ($\mathbf{1}$ denotes a K-dimensional vector of ones). This lower bound is achieved by the signal matrix $\hat{\mathbf{S}}$ constructed from the algorithm of [5] for the symmetric SIR requirement $\gamma_{\mathrm{L}}\mathbf{1}$.*

Proposition 2 *An upper bound on the maximum achievable symmetric capacity (by any decoder, and hence the optimal decoder) is*

$$C^{\mathrm{U}}_{\mathrm{sym}} = \min\left\{\min_{\substack{J\subseteq\{1,\ldots,K\}\\|J|>N}} \frac{C^{\mathrm{U}}_{\mathrm{sum}}(J)}{|J|}, \frac{1}{2N}\log_2\left(1+\frac{N}{\sigma^2}\min_{k=1,\ldots,K} p_k\right)\right\}$$

where $C^{\mathrm{U}}_{\mathrm{sum}}(J)$ denotes the maximum sum-capacity when only users with indices in J are active and is given as in [2].

Define γ_{U} implicitly via $C^{\mathrm{U}}_{\mathrm{sym}} = 1/(2N)\log_2(1+\gamma_{\mathrm{U}})$.

Proposition 3 *If $N \ge \hat{N}(\gamma_{\mathrm{U}}\mathbf{1})$, the symmetric capacity achieved by the MMSE-DF decoder coincides with the upper bound $C^{\mathrm{U}}_{\mathrm{sym}}$.*

The restriction of the decoder to be the MMSE-DF decoder does not result in a loss of optimality in symmetric capacity when N is sufficiently large.

Corollary 1 *The symmetric capacity achieved by the optimal decoder with the signals $\hat{\mathbf{S}}$ of Proposition 1 coincides with $C^{\mathrm{U}}_{\mathrm{sym}}$ (and is hence optimal) whenever $N \ge \hat{N}(\gamma_{\mathrm{U}}\mathbf{1})$.*

Interestingly, even if the condition of the corollary does not hold, the signal set $\hat{\mathbf{S}}$ when employed with the optimal decoder still yields values of symmetric capacity close to the upper bound $C^{\mathrm{U}}_{\mathrm{sym}}$.

The results of this paper are illustrated in Fig. 1 for a 15-user system, with SNR = 10 dB and quadratic power distribution. A similar behavior was observed for different sets of values of $K, \mathbf{p}$ and SNR.

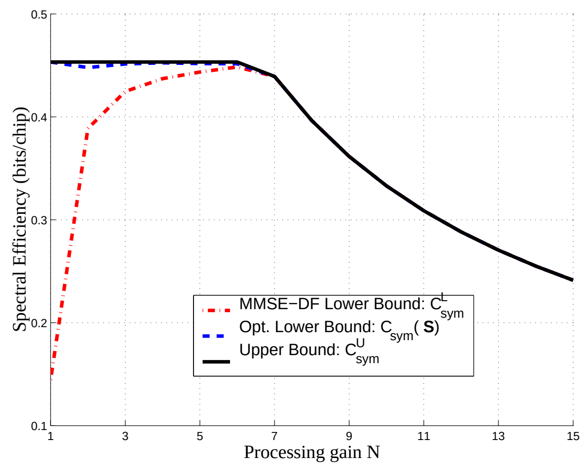

Figure 1: Bounds on symmetric capacity for a 15-user system at SNR = 10 dB for weakest user and with $p_k = (K-k+1)^2$.

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References

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