ITW2003, Paris, France, March 31 – April 4, 2003

Improved Signal Design for Bandwidth Efficient Multiple Access

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Abstract — **The problem of optimizing signals for the Gaussian multiple access channel under Quality of Service (QoS) constraint is addressed. In particular, the Bandwidth Efficient Multiple Access (BEMA) approach of [1] is considered, wherein signals are designed at the base station and fed back to, and for use by, uplink transmitters, in order to minimize strict bandwidth while ensuring that each user meets a rate-specified QoS constraint. A new recursive, greedy algorithm for signal design is proposed that *exactly* meets the QoS requirements. Preliminary analysis and numerical examples suggest it is optimal.**

I. Introduction

The information theory of the Gaussian multiple access channel has been extensively studied, owing as much to this channel's practicality in modeling cellular systems, as to its relative simplicity. The capacity region of Correlated Waveform Multiple Access (CWMA) is derived in [2], and signals that maximize its sum-capacity in [3] and [4] for equal and arbitrary powers. An alternative QoS-based approach to signal design was proposed in [1] and motivated a new multiple access strategy, namely Bandwidth Efficient Multiple Access (BEMA), that assumes reception with the single-user-code-based, vertex-achieving minimum mean-squared error (MMSE) decision feedback detector [5]. The key feature of BEMA is that it converts excess received-power into bandwidth savings while meeting the QoS requirements. A QoS-based approach was also used in [6] for joint signal design and power control under linear MMSE detection.

In this paper, we revisit the problem of [1] and propose an alternative signal design algorithm. Unlike the previous one, this new algorithm provably ensures that each user *exactly* meets its target QoS, and hence that the joint QoS is achieved with a better spectral efficiency. The insight comes from optimizing signals under linear MMSE detection, for which we show that the direction of minimum interference maximizes the capacity of a bandwidth- and power-constrained user. Extending this approach to joint signal design under linear MMSE detection yields the sum-capacity maximizing signals of [3, 4] for which, however, capacity can only be achieved by using more powerful detectors. Moreover, their performance under linear detection is known to be limited (see e.g. [7]). Therefore, we consider the MMSE-DFD and assume a QoS-based approach, wherein each user specifies a desired transmission rate. To meet the QoS requirements with minimum bandwidth, we propose a recursive, greedy algorithm that designs signals, at each stage, by sharing power along appropriate directions in the already designed signal space.

[1]This work was supported by ARO Grant DADD19-99-1-0291 and by NSF Grant CCR-0112977

The channel model is presented in Section II, signal design under linear MMSE detection and MMSE-DFD are discussed in Sections III and IV, respectively. Numerical examples are given in Section V, and Section VI concludes this paper.

II. CWMA System Model

Consider a CWMA system wherein K users communicate in symbol synchronism over an AWGN channel by linearly modulating unit-energy signature waveforms that satisfy the generalized Nyquist criterion [8]. The symbols $\{X_k\}_{k=1}^{K}$ are independent and distributed according to the capacity-maximizing distributions [2], i. e., $X_k \sim \mathcal{CN}(0, E_k)$ (complex, proper normal), where $\mathbf{E} = \operatorname{diag}(E_1, \ldots, E_K)$ is the power-constraint matrix. Passing the baseband received signal through a bank of N matched filters, matched to a set of N orthonormal basis waveforms that span the signal space, yields the discrete-time model $\underline{Y} = \mathbf{S}\underline{X} + \underline{W}$, where $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_K]$ is the $N \times K$ signal matrix ($\mathbf{S}$ has full row-rank equal to N), $\underline{X}^{\mathsf{T}} = [X_1, \ldots, X_K]$ is the symbol vector of all users, and $\underline{W} \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}_N\right)$. CWMA models CDMA, orthogonal signaling such as TDMA and FDMA when $\mathbf{S} = \mathbf{I}_K$, as well as the conventional Gaussian MAC, aka. Identical Waveform Multiple Access (IWMA), when $\mathbf{S} = \mathbf{1}^{\mathsf{T}}$. Henceforth, $\mathcal{X}$ denotes the span (or range) of the matrix $\mathbf{X}$, and the superscripts $^{\mathsf{T}}$, $^{\mathsf{H}}$ and $^{+}$ denote, respectively, the transpose, hermitian transpose, and pseudo-inverse.

III. Signal Design: Linear MMSE Detection

The linear MMSE detector for any user, say user 1, consists of the MMSE filter that produces a soft estimate of the symbol, which is then fed to an appropriate decoder. Specifically, the (scaled) MMSE filter is $\mathbf{f}_{1-\mathrm{M}} = \mathbf{H}^{-1}\mathbf{s}_1$, where $\mathbf{H} = \mathrm{E}[\underline{Y}\,\underline{Y}^{\mathsf{H}}]$, and its output, which is not MAI-free, is $Z_1 = \mathbf{f}_{1-\mathrm{M}}^{\mathsf{H}}\underline{Y}$.

The capacity of user 1 under linear MMSE detection is, given the interference, $C_1 = (1/T)\log_2(1 + \mathrm{SIR}_1)$, where SIR_1 denotes its signal-to-interference ratio. Using a Woodbury identity, it can be shown that $\mathrm{SIR}_1 = E_1\,\mathbf{s}_1^{\mathsf{H}}\bar{\mathbf{H}}_1^{-1}\mathbf{s}_1$, where $\bar{\mathbf{H}}_1 = \mathbf{H} - E_1\mathbf{s}_1\mathbf{s}_1^{\mathsf{H}}$. We also introduce $\bar{\mathbf{S}}_1$ and $\bar{\mathbf{E}}_1$ to denote the interfering signals and their power constraints, respectively.

When the interfering users and their power constraints are given, C_1 can be maximized under power and bandwidth constraints (E_1 is given and $\mathbf{s}_1 \in \bar{\mathcal{S}}_1$) by choosing

$$\hat{\mathbf{s}}_1 \in \arg\max_{\mathbf{s}_1 \in \bar{\mathcal{S}}_1,\ ||\mathbf{s}_1||=1} E_1\,\mathbf{s}_1^{\mathsf{H}}\bar{\mathbf{H}}_1^{-1}\mathbf{s}_1. \tag{1}$$

The solution is $\hat{\mathbf{s}}_1 \propto \boldsymbol{\phi}_1$, where $\boldsymbol{\phi}_1$ is the left singular vector of $\bar{\mathbf{S}}_1\bar{\mathbf{E}}_1^{1/2}$ that corresponds to its minimum singular value, denoted by $\sqrt{\lambda_{\min}}$. Note that, when $\mathbf{s}_1 \in \bar{\mathcal{S}}_1$, $\lambda_{\min} > 0$ because $\bar{\mathbf{S}}_1$ has full row-rank equal to N. The maximum SIR value is

$\gamma_1 = E_1 (\lambda_{\min} + \sigma^2)^{-1}$, and, with such a signal allocation, the linear MMSE detector degenerates to the matched-filter detector. That $\boldsymbol{\phi}_1$, the direction of least interference, maximizes SIR was also recognized in [9].

This approach can be extended to joint signal design. We formulate the problem as solving for the signal matrix that maximizes the minimum capacity, given the (strict) bandwidth (specified by N), the number of users K (with $K > N$), and the power constraints. The problem is a constrained optimization that is solved, for $\mathbf{E} = \mathbf{I}_K$, by the following proposition.

Proposition 1 *Given N, K, and $\mathbf{E} = \mathbf{I}_K$, Welch Bound Equality (WBE) signals (see e.g. [3]) maximize the minimum capacity (or equivalently SIR), i. e.,*

$$\mathbf{S}_{\mathrm{WBE}} \in \arg \max_{\mathbf{S} \in \mathbb{C}^{N \times K}} \min_{k=1,\ldots,K} \mathrm{SIR}_k(\mathbf{S}). \tag{2}$$

This result is a consequence of the following fact: generalized WBE signals are such that each column is the left singular vector corresponding to the minimum singular value of the other weighted columns. With such an allocation, the linear MMSE and matched-filter detectors are equivalent. Moreover, also under linear MMSE detection, generalized WBE signals solve a joint signal design and power control problem for SIR-based QoS requirements [6], and issues of their practical implementation have been recently addressed in [9–12]. Finally, they also maximize sum-capacity [4] (and symmetric-capacity in the equal-power case [3]), but this capacity can be achieved only by using joint ML decoding, rate-splitting [13], or MMSE decision feedback detection [5]. Receivers based on linear detectors, on the other hand, are not suited to achieve the high rates promised by the capacity region, and in fact perform poorly in terms of error rate for generalized WBE signals [14].

IV. Shaping the Capacity Region under MMSE Decision Feedback Detection

In this section, we assume that the base station employs the vertex-achieving, single-user code-based MMSE-DFD [5], and consider the problem of designing signals to minimize bandwidth while ensuring that each user meets a QoS requirement [1]. The requirements are specified in terms of desired rates (in bits/sec.) for arbitrary reliability, which translates into SIR requirements. The underlying assumption is that the weakest user achieves its QoS, and, hence, that other users have extra power relative to that needed to achieve their respective QoS. Next, we formulate the problem analytically and, because it is not tractable, propose instead a recursive, greedy algorithm for joint signal design.

A. Problem formulation The MMSE-DFD, which decodes users successively, is described e.g. in [15]. Assuming users are decoded in the increasing order of their indices, it forms the estimate Z_k at stage k, given by $Z_k = X_k \mathbf{f}_{k-\mathrm{M}}^{\mathsf{H}} \mathbf{s}_k + \sum_{j=k+1}^{K} X_j \mathbf{f}_{k-\mathrm{M}}^{\mathsf{H}} \mathbf{s}_j + \mathbf{f}_{k-\mathrm{M}}^{\mathsf{H}} \underline{W}$, where $\mathbf{f}_{k-\mathrm{M}}$ is the MMSE filter in a *user-expurgated channel*, i. e., assuming that only users $k+1, \ldots, K$ (future users) are active, and where past users' symbols have been perfectly estimated and canceled (no error propagation). This can be achieved if capacity-achieving codes are used to make the error probabilities arbitrarily small.

Let $\widehat{\mathbf{SIR}}$ be the K-dimensional vector that denotes the target SIRs. The problem can then be formulated analytically as

$$\hat{\mathbf{S}} \in \arg \min_{\mathbf{SIR}(\mathbf{S}) \geq \widehat{\mathbf{SIR}}} \mathrm{rank}(\mathbf{S}), \tag{3}$$

where $\mathbf{SIR}(\mathbf{S})$ is the K-dimensional vector of achieved SIRs (with perfect feedback) by $\mathbf{S}$, and the vector inequality is component-wise.

In general, the problem in (3) is not analytically tractable. Instead, we propose a recursive, greedy algorithm that, at each stage, meets the QoS requirement and tries to preserve bandwidth for the next stage. The driving idea relies on the solution to (1) that, for a power- and bandwidth-constrained user, the direction of least interference maximizes SIR.

A trivial upper bound on the minimum rank is K, and a generally unachievable lower bound was derived in [1, Prop. 3]. It is equal to the minimum bandwidth of an IWMA system that contains the target rate-tuple, and it can be achieved by variable signaling-interval combined with joint ML decoding or rate-splitting [13], neither of which are practical.

B. Joint signal design algorithm As in [1], the target SIRs are expressed, for each user, as a fraction of its SNR had it been the only active user: $\widehat{\mathrm{SIR}}_k = \eta_k E_k / \sigma^2$, $\eta_k \in (0, 1]$, and users are arranged in the decreasing order of $1 - \eta_k$. Hence, signals for users with higher excess power are designed first. The algorithm starts with the last user, user K, which sees a single-user Gaussian channel. At stage $K - k + 1$, the signal for user k is designed ($k = K - 1, \ldots, 1$), and signals for future users, which have already been designed, are grouped in $\hat{\mathbf{S}}_{k+1}$. Let $\gamma_k = E_k (\lambda_{\min}^{(k+1)} + \sigma^2)^{-1}$, where $\sqrt{\lambda_{\min}^{(k+1)}}$ is the minimum singular value of $\hat{\mathbf{S}}_{k+1} \hat{\mathbf{E}}_{k+1}^{1/2}$, denote the maximum SIR that user k can achieve if its signal lies in $\hat{\mathcal{S}}_{k+1}$.

At stage $K - k + 1$, compute γ_k and compare it with $\widehat{\mathrm{SIR}}_k$ to determine whether the QoS can be met with or without incrementing the signal space dimension. In either case, infinitely many signals will satisfy the QoS. The final selection should maximize the likelihood of maintaining the signal space dimension constant at the next stage (i. e., stage $K - k + 2$), which, in turn, depends only on γ_{k-1}. Note that γ_{k-1} decreases monotonically as a function of $\lambda_{\min}^{(k)}$, the minimum eigenvalue of $\mathbf{S}_k \mathbf{E}_k \mathbf{S}_k^{\mathsf{H}}$, which, in turn, depends on $\mathbf{s}_k$. Therefore, the strategy is to choose $\hat{\mathbf{s}}_k$ to maximize γ_{k-1}, or equivalently minimize $\lambda_{\min}^{(k)}$, while satisfying the QoS constraint $\mathrm{SIR}_k(\hat{\mathbf{s}_k}) \geq \widehat{\mathrm{SIR}}_k$. The algorithm is then as follows.

$\hat{s}_K \longleftarrow 1$

for $k = K - 1$ to 1, compute γ_k

$$\text{if } \gamma_k \geq \widehat{\mathrm{SIR}}_k, \qquad \hat{\mathbf{S}}_k \longleftarrow \left[\hat{\mathbf{s}}_k,\, \hat{\mathbf{S}}_{k+1}\right]$$
$$\text{where} \quad \hat{\mathbf{s}}_k \in \arg\min \lambda_{\min}^{(k)}$$
$$\text{subject to } \; ||\mathbf{s}_k|| = 1,\; \mathbf{s}_k \in \hat{\mathcal{S}}_{k+1},\; \mathrm{SIR}_k(\mathbf{s}_k) \geq \widehat{\mathrm{SIR}}_k$$

$$\text{if } \gamma_k < \widehat{\mathrm{SIR}}_k, \qquad \hat{\mathbf{S}}_k \longleftarrow \left[\hat{\mathbf{s}}_k,\, \begin{bmatrix} \mathbf{0}^{\mathsf{T}} \\ \hat{\mathbf{S}}_{k+1} \end{bmatrix}\right]$$
$$\text{where} \quad \hat{\mathbf{s}}_k \in \arg\min \lambda_{\min}^{(k)}$$
$$\text{subject to } \; ||\mathbf{s}_k|| = 1,\; s_{k1} \neq 0,\; \mathrm{SIR}_k(\mathbf{s}_k) \geq \widehat{\mathrm{SIR}}_k,$$

where $\mathbf{0}^{\mathsf{T}}$ and s_{k1} denote a $(K-k)-$dimensional row vector of zeros and the first component of $\mathbf{s}_k$, respectively.

The two constrained eigenvalue optimization problems above, referred to as C_1 and C_2, respectively, are solved in Proposition 2. Note that, at stage k, $\hat{\mathbf{S}}_{k+1}$ has full row-rank by construction (denoted by N_{k+1}), and dimensions $N_{k+1}\times(K-k)$. It follows that, for C_1, the rank is preserved ($N_k = N_{k+1}$), while it is incremented for C_2 ($N_k = N_{k+1}+1$), in which case $\hat{\mathbf{S}}_k$ is formed by zero-padding $\hat{\mathbf{S}}_{k+1}$ with $\mathbf{0}^\mathsf{T}$.

Proposition 2 *The solutions to the constrained optimization problems C_1 and C_2 are*

C_1 : *when* $\lambda_{\min}^{(k+1)}/\sigma^2 \le \eta_k^{-1}-1$, *let* j *be such that*

$$\lambda_j/\sigma^2 \le \eta_k^{-1}-1 \le \lambda_{j+1}/\sigma^2, \quad \textit{then}$$

$$\hat{\mathbf{s}}_k \quad \longleftarrow \quad \begin{cases} \hat{s}_1\,\boldsymbol{\phi}_j + \sqrt{1-\hat{s}_1^2}\,\boldsymbol{\phi}_{j+1}, & \textit{if} \;\; j < N_{k+1} \\ \boldsymbol{\phi}_{N_{k+1}}, \quad \textit{if} \quad \lambda_{N_{k+1}}/\sigma^2 \le \eta_k^{-1}-1 \end{cases}$$

C_2 : *when*$\lambda_{\min}^{(k+1)}/\sigma^2 > \eta_k^{-1}-1$,

$$\hat{\mathbf{s}}_k \quad \longleftarrow \quad \begin{bmatrix} \hat{s}_2 \\ \sqrt{1-\hat{s}_2^2}\,\boldsymbol{\phi}_1 \end{bmatrix},$$

where $\hat{s}_1$ and $\hat{s}_2$ are given by

$$\hat{s}_1^2 \;=\; \frac{\lambda_{j+1}+\sigma^2}{\lambda_{j+1}-\lambda_j}\left(1-\eta_k\left(\lambda_j/\sigma^2+1\right)\right),$$

$$\hat{s}_2^2 \;=\; \frac{\eta_k\left(\lambda_{\min}^{(k+1)}/\sigma^2+1\right)-1}{\lambda_{\min}^{(k+1)}/\sigma^2},$$

where $\hat{\mathbf{S}}_{k+1}\mathbf{E}_{k+1}\hat{\mathbf{S}}_{k+1}^\mathsf{H} = \boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^\mathsf{H}$ is its spectral decomposition, $\boldsymbol{\Lambda} = \mathrm{diag}\left(\lambda_{\min}^{(k+1)}, \lambda_2, \ldots, \lambda_{N_{k+1}}\right)$ the diagonal matrix of eigenvalues (all positive), arranged in nondecreasing order, and $\boldsymbol{\Phi} = \left[\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_{N_{k+1}}\right]$ the unitary matrix of eigenvectors.

C. Comments For C_1, the QoS is met without incrementing rank (or equivalently bandwidth). The solution consists of a linear combination of two consecutive eigenvectors whose indices, as well as the power sharing coefficient, are determined by the objective and constrained functions. Note that the case where $j=1$ and $\hat{s}_1=1$ corresponds to maximizing SIR as in (1). However, preserving rank comes at a price. Indeed, because in this case $\lambda_{\min}^{(k)} \ge \lambda_{\min}^{(k+1)}$, γ_{k-1} is potentially reduced, and so is the likelihood that $\hat{\mathrm{SIR}}_{k-1}$, the QoS at stage $K-k+2$, can be met without incrementing rank. If $j \ne 1$, then $\lambda_{\min}^{(k)} = \lambda_{\min}^{(k+1)}$ and there is no penalty. In the extreme case where $\lambda_{N_{k+1}}/\sigma^2 \le \eta_k^{-1}-1$, the QoS is so small that it can be met with all the power along the direction of *maximum* interference. This is typically very rare and, when $j=1$, the increase in the minimum eigenvalue is mitigated by appropriately distributing power (i. e., through $\hat{s}_1$).

On the other hand, for C_2, the QoS can only be met by incrementing rank: the first component is determined by the constraint and objective functions, and the vector of remaining components is proportional to the eigenvector $\boldsymbol{\phi}_1$. However, the benefit is a guaranteed reduction of $\lambda_{\min}^{(k)}$. Indeed, from the eigenvalue-interlacing theorem [16, Th. 4.3.4], $\lambda_{\min}^{(k)} \le \lambda_{\min}^{(k+1)}$ regardless of the choice for new signal. Consequently, incrementing rank at stage $K-k+1$ results in a potentially higher value of γ_{k-1} and a greater likelihood of meeting the QoS at stage $K-k+2$ without incrementing rank. Note that $\eta_k = 1$ (i. e., user k desires maximum SIR) implies case C_2, that $\hat{s}_2 = 1$, and hence that $\hat{\mathbf{s}}_k \perp \hat{\mathcal{S}}_{k+1}$.

In either case, the solution to the joint signal design is such that, at each stage, the QoS requirement is just met, while maximizing the likelihood of preserving bandwidth at the next stage. Note also that the algorithm shapes the capacity region so that the target rate-tuple lies at a vertex.

Moreover, the algorithm yields orthogonal signals, i. e., $\mathbf{S} = \mathbf{I}_K$, and IWMA, i. e., $\mathbf{S} = \mathbf{1}^\mathsf{T}$, if, and only if, $\eta_k = 1$ and $\eta_k \le (1 + 1/\sigma^2 \sum_{j=k+1}^{K} E_j)^{-1}$ for all k, respectively.

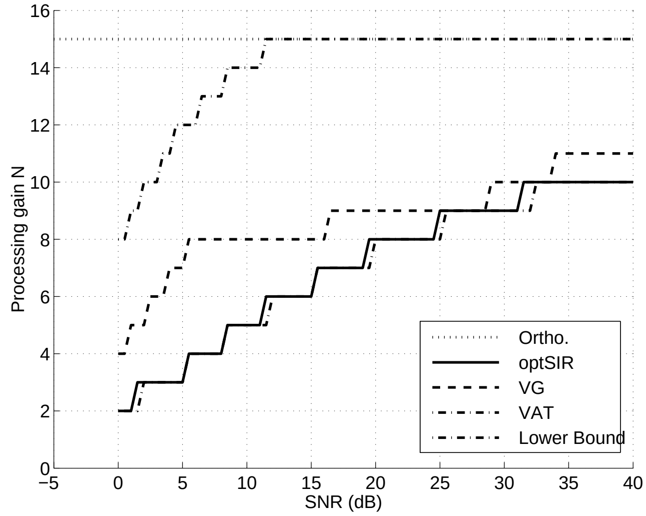

Figure 1: Processing gain vs. SNR for different signal designs and a quadratic power distributions ($E_k = (K-k+1)^2$).

V. Numerical examples

Consider a 15-user system with a quadratic power distribution ($E_k = (K-k+1)^2$), and symmetric SIR constraint equal to the SNR of the weakest user (if it were the only active user), i. e., equal to $\mathrm{SNR} = 10\log_{10}(1/\sigma^2)$. Figure 1 plots the processing gain versus SNR for the new design, the one in [1], and the linear-MMSE, power-control-based approach of [6], referred to as KV, VG, and VAT, respectively. Also included is the lower bound. The KV design successfully bridges the gap between the VG design and the lower bound, and, in fact, almost always achieves this bound. That the VAT design performs worst was already noted in [7].

Next, we plot the maximum number of users as a function of the processing gain for fixed SNR, power distribution, and symmetric SIR constraint. The results in Figure 2 are for the linear power distribution and $\mathrm{SNR} = 10$ dB. The proposed signal design almost always achieves the lower bound. Moreover, all curves are quasi-linear, a behavior that was observed for varied power distributions and SNR. Defining the load factor $\beta = K/N$, we summarize its values for several configurations in the following table. For all SNRs and power distributions, the KV design is the most efficient, and this efficiency increases with lower SNR and more disparate power. For instance, at $\mathrm{SNR} = 0$ dB and with a quadratic power distribution, 100 users can be accommodated with a processing gain equal to 9.

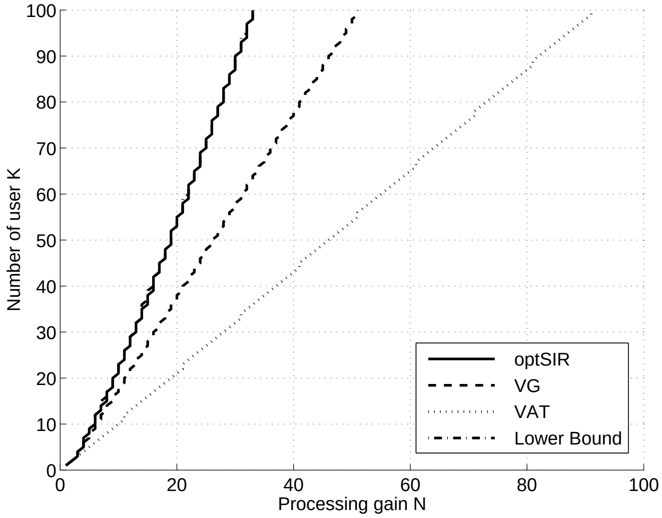

Figure 2: Processing gain vs. number of users for the KV, VG, and VAT signal designs, along with the lower bound, for a linear power distribution ($E_k = K-k+1$) and $\mathrm{SNR} = 10\mathrm{dB}$.

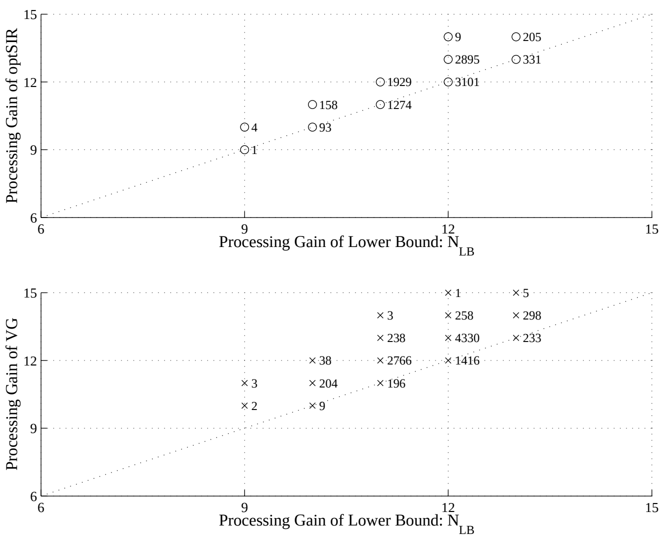

Figure 3: Histograms for asymmetric QoS with quadratic power distribution and $\mathrm{SNR} = 10\mathrm{dB}$.

SNR (dB)	VAT	linear power VG	linear power KV	quad. power VG	quad. power KV
0.0	1.94	2.92	8.2	4.95	12.2
5.0	1.3	1.94	4.0	2.4	6.65
10.0	1.09	1.92	2.75	1.95	4.3
20.0	1.0	1.68	1.82	1.91	2.66

Finally, consider a 15-user example with asymmetric, randomly generated, SIR constraints, such that the QoS for user k is uniformly distributed in $[0,\, E_k/\sigma^2]$. The processing gain for the KV and VG signal designs are determined and compared to the lower bound. Figure 3 plots the histogram of the processing gain of the two signal designs versus the processing gain of the lower bound, for a quadratic power distribution, 10 dB, and 10^4 trials. The processing gain for the KV tightly hugs the lower bound, whereas the VG processing gain is more scattered. A similar behavior was observed for other power distributions and SNRs.

VI. Conclusion

A new joint signal design algorithm is proposed to minimize strict bandwidth under rate-specified QoS requirements. It constructs signals recursively by carefully sharing power between different directions in the signal space to exactly meet the QoS requirements with as little bandwidth as possible. Consequently, it shapes the capacity region so that the desired rate-tuple exactly lies at a vertex. Numerical examples illustrate that its performance is very close to that of the generally unachievable lower bound. In fact, preliminary analysis suggest that it is optimal, i. e., that no other signal set can meet the QoS requirements with greater spectral efficiency. This optimality can, in turn, be exploited to maximize the symmetric capacity of CWMA systems. This issue is addressed in [17].

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