Time Limited and Bandwidth Constrained Signal Design for the Fast Fading Noncoherent MIMO Channel

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***Abstract*— We address the problem of bandwidth constrained space-time signal design for the noncoherent Rayleigh block-fading channel. Existing design techniques for this channel subdivide the coherence interval into smaller time blocks and use repetitions of a basic waveform to signal in each sub-block. When the coherence time of the channel is short this access technique becomes questionable, due to the inverse relationship between bandwidth and time support. In particular, there may not be sufficient time support to allow matched filtered reception with finite (or nearly finite) Shannon bandwidth waveforms. To address this problem, we consider other notions of bandwidth, such as the root-mean square (RMS) bandwidth and fractional out of band energy (FOBE), which are appropriate for signals with finite time support. We extend our previous work on unconstrained signal designs for the block fading channel to incorporate such bandwidth constraints. The resulting signal constellations can be used 1) as a comparison point for any signal design procedure and 2) to conclude that there is a performance advantage to be had when signals are properly matched to the finite time support of the channel.**

I. Introduction

Previous work on signal designs for the noncoherent communication channel have not focused attention on the issue of bandwidth utilization when the coherence time is short. By enforcing a strict dimensionality constraint on the signal set, it is assumed that it is possible to employ band-limited signalling on the block-fading channel. However, when the channel is constant only over finite blocks of time we can not use basis functions with a time support larger than this coherence interval to build signals. Consequently, finite bandwidth signalling schemes are not feasible. This situation is the most pronounced when the coherence time is short, which is also the situation in which noncoherent techniques are most likely to be employed as reliable channel estimation is difficult.

To avoid the contradiction that arises when trying to use signals with finite time and frequency support, we must use other notions of bandwidth in designing signals, such as the root mean square (RMS) bandwidth, fourth order bandwidth, or fractional out of band energy (FOBE). Each of these bandwidth definitions is well defined for finite duration signals.

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We first derive the power spectrum of the space-time signal. We then extend our previous work on finite dimensional unitary space-time signal designs[1] to the bandwidth constrained problem through the use of the asymptotic union bound (AUB)[2].

A. Power spectrum of the Space-Time Signal

We begin by deining the general continuous time model for the received signal at a spatial location x when M transmit antennas are employed:

$$r_x(t) = \sum_{k=-\infty}^{\infty} \sum_{i=1}^{S} c_i(k) \sum_{m=1}^{M} h_{m,x}(k) s_m^i(t-kT), \quad (1)$$

where $s_m^i(t)$ is the signal transmitted through the m^{th} antenna to convey the information symbol i; $c_i(k)$ is non-zero for only one value of i, selecting the appropriate signal; and $h_{m,x}(k)$ is the value of the fading coefficient describing the wireless channel linking antenna m to location x. The signalling period T is assumed to be no bigger than the coherence time of the channel. We may collect the signal into the vector $\mathbf{s}^i(t) = [s_1^i(t) \ \cdots \ s_M^i(t)]^{\mathsf{T}}$ to form the compact model

$$r_x(t) = \sum_{k=-\infty}^{\infty} \sum_{i=1}^{S} c_i(k) \mathbf{h}_x^{\mathsf{T}}(k) \mathbf{s}^i(t-kT). \quad (2)$$

Here $\mathbf{h}_x(k) = [h_{1,x}(k) \cdots h_{M,x}(k)]^{\mathsf{T}}$ is modelled as a zero-mean complex normal random vector with correlation matrix

$$E\mathbf{h}_x(k)\mathbf{h}_x^{*}(k) = \mathbf{\Sigma}_x \quad (3)$$

The power spectrum depends on the cross-correlation of the $c_i(k)'s$ and of the signals. Without loss of generality, we will assign a random phase to each $c_i(k)$ in order to force their cross-correlation into the form $E[c_i(k)c_{i'}^{*}(k')] = 1/S\delta(k-k')\delta(i-i')$, assuming independent signaling.

The periodically varying autocorrelation function of $r_x(t)$ is given by

$$\begin{aligned} R_{r_x}(t,\tau) &= E[r_x^{*}(t) r_x(t+\tau)] \quad (4) \\ &= \frac{1}{S} \sum_k \sum_i \mathbf{s}^i(t-kT)^{*} \overline{\mathbf{\Sigma}_x} \mathbf{s}^i(t+\tau-kT). \end{aligned}$$

We have used the symbol $\overline{\mathbf{h}_x}(k)$ to denote the element-wise complex conjugate of $\mathbf{h}_x(k)$, $\overline{\mathbf{h}_x}(k) = (\mathbf{h}_x^*)^\mathsf{T}$, similarly $\overline{\mathbf{\Sigma}_x} = (\mathbf{\Sigma}_x^*)^\mathsf{T}$.

We define the wide sense stationary correlation function $R(\tau)$ by averaging the cyclo-stationary function, $R_{r_x}(t, \tau)$ over one period (see e.g. [3]):

$$\begin{aligned} R_{r_x}(\tau) &= \frac{1}{ST}\int_0^T R_{r_x}(t,\tau)dt \\ &= \frac{1}{S}\sum_i \operatorname{tr}\left\{\overline{\mathbf{\Sigma}_x}\mathbf{R}_{\mathbf{s}(i)}(\tau)\right\}, \end{aligned} \tag{5}$$

where the dependency on k is dropped because each signal $s_m^i(t)$ has time support $[0, T]$ and we have defined the correlation matrix

$$\mathbf{R}_{\mathbf{s}^i}(\tau) = \int_0^T \mathbf{s}^i(t)^*\mathbf{s}^i(t-\tau)dt. \tag{6}$$

The element-wise Fourier transform of $R_{r_x}(\tau)$ yields the matrix valued power spectrum $P_{r_x}(f)$:

$$P_{r_x}(f) = \frac{1}{ST}\sum_i \operatorname{tr}\left\{\overline{\mathbf{\Sigma}_x}\mathbf{P}_{s^i}(f)\right\}, \tag{7}$$

where $\mathbf{P}_{s^i}(f)$ is the element-wise Fourier transform of $\mathbf{R}_{s^i}(\tau)$. In the special case of independent identical fading[1], $\mathbf{\Sigma}_x = 1/M\mathbf{I}$, we drop the dependency on x and the power spectral density has the simple form

$$P_r(f) = \frac{1}{MST}\sum_i \operatorname{tr}\left\{\mathbf{P}_{s^i}(f)\right\} = \frac{1}{MST}\sum_{i,m}|\widehat{s_m^i}(f)|^2. \tag{8}$$

This is simply the average of the magnitude-squared Fourier transforms of the signals from each antenna.

B. Bandwidth Constrained Signals

Having derived the power spectral density associated with our modulation scheme, we now present a technique to build signals under bandwidth constraints. Since we are restricting our attention to time limited signals, there is some freedom in the definition of bandwidth. Two common notations which are applicable to this channel are the fractional out of band energy (FOBE) [4] and the root mean squared (RMS) bandwidth [5] (see also [6]). We will also consider the fourth order moment bandwidth measure introduced in [7]. The three measures of bandwidth are defined by

$$\begin{aligned} bw_{FOBE}(P_{r_x}(f)) &= \frac{\int_{-B}^{B} P_{r_x}(f)df}{\int_{-\infty}^{\infty} P_{r_x}(f)df}, \\ bw_{RMS}(P_{r_x}(f)) &= \left[\frac{\int_{-\infty}^{\infty} f^2 P_{r_x}(f)df}{\int_{-\infty}^{\infty} P_{r_x}(f)df}\right]^{1/2} \\ bw_{4th}(P(f)) &= \left[\frac{\int_{-\infty}^{\infty} f^4 P_{r_x}(f)df}{\int_{-\infty}^{\infty} P_{r_x}(f)df}\right]^{1/4}, \end{aligned} \tag{9}$$

[1]For a rich scattering environment, this is achieved when the transmit antennas produce uncorrelated waveforms. I.e. that the antennas are sufficiently well separated and isolated.

where the FOBE measure is defined with respect to some cut-off frequency, B.

We may associate an orthonormal set of basis functions, $\{u_d(t)\}$, to each definition of bandwidth through an eigen-decomposition of the corresponding time and bandwidth limiting operator (see e.g. [4]- [7]). When designing signals which minimize the bandwidth under consideration, we can restrict our attention to signals which are formed as linear combinations of these basis functions without loss of generality, i.e. each signal has the form

$$s_m^i(t) = \sum_{d=1}^{D} \mathbf{s}_m^i(d)u_d(t), \tag{10}$$

where generally D should equal S, the cardinality of the signal set. In practice, we generally choose $D \ll S$ since it is typically only the first several eigenvectors which matter (the corresponding eigenvalues typically decay quickly). This matter is discussed further in Section III-B.

For the FOBE measure, the appropriate basis functions are the prolate spheroidal wave functions, defined in [4]. For the RMS bandwidth measure, the basis functions are the truncated sinusoids $u_d(t) = \sin(d\pi t/T)$ on $[0, T]$ [6]. The basis functions for the fourth order bandwidth measure are derived in [7].

The chief advantage that the RMS and fourth order moment measures of bandwidth have over the FOBE measure is an improved spectral roll-off, which may be appropriate for designing signals to meet practical bandwidth constraints (such as FCC masks). In Figure 1 we plot the spectrum of the first basis function for each measure. Notice that the fourth order moment has the best spectral roll-off of the three.

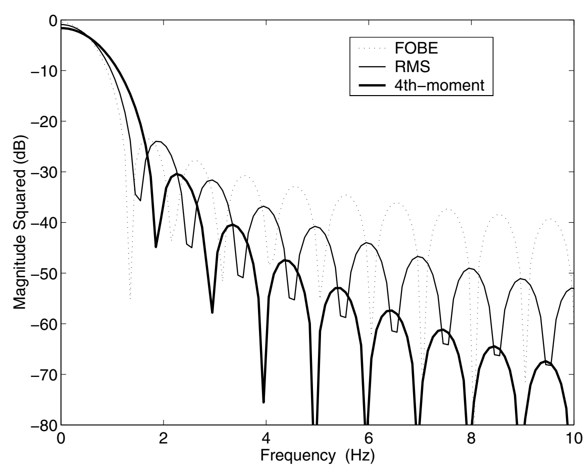

Fig. 1. Magnitude spectrum of $u_1(t)$ for the FOBE, RMS, and 4th-moment bandwidth measures (each normalized to $T = 1$s).

Once the basis functions have been determined, the bandwidth of the signal set can generally be defined in terms of the expansion coefficients, $\mathbf{s}_i^m(d)$. Let us form the space-time representation of the i^{th} possible transmit signal, $\mathbf{S}_i \in \mathbb{C}^{D\times M}$, by $\{\mathbf{S}_i\}_{d,m} = \mathbf{s}_i^m(d)$. For the case of uncorrelated fading we have

$$bw(P_r(f))^\alpha = \frac{\sum_{i=1}^{S} \text{tr}\left\{\mathbf{S}_i^* \Gamma \mathbf{S}_i\right\}}{\sum_{i=1}^{S} \text{tr}\left\{\mathbf{S}_i{}^* \mathbf{S}_i\right\}}, \tag{11}$$

where the diagonal scaling matrix Γ is formed from eigenvalues of a certain continuous time operator (see [7]) which is related to the bandwidth measure employed and $\alpha = \{1, 2, 4\}$ corresponding to the FOBE, RMS, and 4th order bandwidth measures, respectively. For the special case of RMS bandwidth, for example, we have $\Gamma_{k,k} = k^2$. Notice also that we have repressed the dependency on the spatial location x in equation (11) since the bandwidth is independent of the location in i.i.d. fading.

II. Noncoherent Space-Time Detection

We now review space-time detection signals on the noncoherent block fading channel, a more detailed derivation can be found in [2]. After projecting the received waveform onto the orthonormal basis $\{u_d(t)\}$, the discrete time received signal at the n^{th} antenna (there are N receive antennas in total) is of the form

$$\mathbf{y}_n = \sqrt{\overline{\gamma}} \mathbf{S}_i \mathbf{h}_n + \mathbf{n}_n. \tag{12}$$

Here $\mathbf{S}_i \in \mathbb{C}^{D\times M}$ is the i^{th} signal from a set of cardinality S, $\mathbf{h}_n = [h_{1,n} \cdots h_{M,n}]^\mathsf{T} \in \mathbb{C}^{M\times 1}$ is the vector of fading coefficients linking the transmit antennas to sensor n, and $\mathbf{n}_n$ is additive noise.

Stacking the D observations per receiver antenna yields the DN sufficient statistics

$$\mathbf{y} = [\mathbf{y}_1^\mathsf{T}, \ldots,\ \mathbf{y}_N^\mathsf{T}]^\mathsf{T} = \sqrt{\overline{\gamma}} \boldsymbol{\mathcal{S}}_i \boldsymbol{h} + \boldsymbol{n}, \tag{13}$$

where $\boldsymbol{\mathcal{S}}_i = \mathbf{I}_N \otimes \mathbf{S}_i$ ($\otimes$ is the Kronecker product [8] and $\mathbf{I}_N$ is the $N \times N$ identity matrix), $\boldsymbol{h} = [\mathbf{h}_1^\mathsf{T}, \ldots,\ \mathbf{h}_N^\mathsf{T}]^\mathsf{T}$, and $\boldsymbol{n} = [\mathbf{n}_1^\mathsf{T}, \ldots,\ \mathbf{n}_N^\mathsf{T}]^\mathsf{T}$. We assume that the additive noise is spatially white, but the fading need not be, i.e., $\boldsymbol{\Sigma} = E[\boldsymbol{h}\boldsymbol{h}^*]$ need not be diagonal[2]. We normalize by the additive noise power so that $E[\boldsymbol{n}\boldsymbol{n}^*] = \mathbf{I}$, the signals are normalized so that $\text{tr}\left\{\mathbf{S}_i^* \mathbf{S}_i\right\} = M$, and the fading process is normalized so that $E[\mathbf{h}_n^* \mathbf{h}_n] = 1$. We further assume that "unitary" signals are used so that $\boldsymbol{\mathcal{S}}_i^* \boldsymbol{\mathcal{S}}_i = \mathbf{I}$. Consequently, the scalar $\overline{\gamma}$ denotes the average SNR at each receiver (i.e. $\overline{\gamma}$ is the normalization factor).

Conditioned on the transmission of signal $\boldsymbol{\mathcal{S}}_i$, and denoting the determinant of a matrix $\mathbf{X}$ as $|\mathbf{X}|$, the measurement has likelihood function

$$\begin{aligned} f_i(\mathbf{y}) \quad &= \quad \frac{1}{\pi^{ND}\left|\mathbf{I}_{DN} + \overline{\gamma}\boldsymbol{\mathcal{S}}_i \boldsymbol{\Sigma} \boldsymbol{\mathcal{S}}_i^*\right|} \times \\ &\quad \exp\left\{-\mathbf{y}^* \left(\mathbf{I} + \overline{\gamma}\boldsymbol{\mathcal{S}}_i \boldsymbol{\Sigma} \boldsymbol{\mathcal{S}}_i^*\right)^{-1} \mathbf{y}\right\}. \end{aligned} \tag{14}$$

[2]In a rich scattering environment, $\mathbf{\Sigma}$ will be diagonal when the receive antennas are sufficiently well separated to guarantee independent fading paths and the antenna elements are sufficiently well isolated. If $\boldsymbol{\Sigma}$ is block diagonal and the sub-blocks $\mathbf{\Sigma}_k$ are additionally diagonal, then we say we have uncorrelated transmission and reception.

The optimal detection strategy is the maximum likelihood rule[3]

$$\begin{aligned} \phi^{ML} : \widehat{i} \quad &= \quad \arg\max_i f_i(\mathbf{y}) \\ &= \quad \arg\max_i \mathbf{y}^* \boldsymbol{\mathcal{S}}_i \left(\mathbf{I} + \overline{\gamma}^{-1} \boldsymbol{\Sigma}^{-1}\right)^{-1} \boldsymbol{\mathcal{S}}_i^* \mathbf{y} \end{aligned} \tag{15}$$

where we have applied a matrix inversion identity [8] to the quadratic form in the likelihood function.

To evaluate the probability of error for the maximum likelihood test of (15) we employ a union upper bound on the probability of error:

$$P \leq \frac{1}{S} \sum_{i=1}^{S} \sum_{\substack{k=1 \\ i \neq k}}^{S} P_{i,k}, \tag{16}$$

where P is the probability of a symbol error and $P_{i,k}$ is the probability that signal $\boldsymbol{\mathcal{S}}_k$ is preferred by the detector to signal $\boldsymbol{\mathcal{S}}_i$, when $\boldsymbol{\mathcal{S}}_i$ was transmitted.

When $D \geq 2M$ we find that as the SNR grows large, the pairwise error probabilities approach [2]

$$P_{ik} \sim \binom{2MN-1}{MN} \frac{\overline{\gamma}^{-MN}}{|\boldsymbol{\Sigma}||\mathbf{I} - \mathbf{S}_i^* \mathbf{S}_k \mathbf{S}_k^* \mathbf{S}_i|^N}. \tag{17}$$

The corresponding union bound, denoted as the *asymptotic union bound*, is given by

$$\begin{aligned} P^{AUB} \quad &= \quad \frac{1}{S} \sum_{i=1}^{S} \sum_{k \neq i} P_{ik} \\ &= \quad \frac{1}{S} \binom{2MN-1}{MN} \frac{\overline{\gamma}^{-MN}}{|\boldsymbol{\Sigma}|} \times \\ &\quad \sum_{i=1}^{S} \sum_{k \neq i} \frac{1}{|\mathbf{I} - \mathbf{S}_i^* \mathbf{S}_k \mathbf{S}_k^* \mathbf{S}_i|^N}. \end{aligned} \tag{18}$$

III. Bandwidth Constrained Signal designs over G(M,D)

We are now ready to consider the general problem of designing signals for use on a noncoherent space-time channel with strict bandwidth constraints. We will first proceed as in [1] and consider signal designs based on optimization over the Grassmanian manifold $G(M, D)$, the space of all M-dimensional subsets of $\mathbb{C}^D$. A motivation for this formulation comes from the fact that unitary signals are generally preferable over the noncoherent channel since the unknown channel matrix effectively destroys all information except the left singular vectors of the space-time signal. These singular vectors are in turn only unique up to the subspace they span and hence $G(M, D)$ emerges as a natural space on which to pose the problem.

A design technique for the noncoherent space-time channel which optimizes the signal constellation over $G(M, D)$ with

[3]We shall assume that the fading correlation is full rank. If $\boldsymbol{\Sigma}$ is singular with rank r, we may form the eigen-decomposition-decomposition $\boldsymbol{\Sigma} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^*$ with $\mathbf{U} \in \mathbb{C}^{NM\times r}$. We can then replace the signals by $\{\boldsymbol{\mathcal{S}}_i \mathbf{U}\}$ and treat the fading as full rank with diagonal correlation matrix $\boldsymbol{\Lambda}$.

respect to the AUB was presented in [1]. The algorithm recursively adds signals to an existing set in such a way as to minimize the additional contribution to the AUB. We will only treat the case of i.i.d. fading in this paper.

The optimization problem can be stated as follows. Assuming that we have designed K signals, $\{\mathbf{S}_k\}_{k=1}^{K}$, we seek to add J new signals $\{\mathbf{S}_{k'}\}_{k'=K+1}^{K+J}$ such that we minimize the AUB:

$$d(\{\mathbf{S}_k\}_{k=1}^{K+J}) = d(\{\mathbf{S}_k\}_{k=1}^{K}) + 2\sum_{k=1}^{K+J}\sum_{\substack{k'=K+1\\k'\neq k}}^{K+J}\frac{1}{|\mathbf{I}-\mathbf{S}_{k'}^{*}\mathbf{S}_k\mathbf{S}_k^{*}\mathbf{S}_{k'}|^{N}}. \quad (19)$$

Notice that only the second term in this expression depends on the new signals. The bandwidth constraint is enforced by ensuring that the new signals obey

$$\sum_{k'=K+1}^{K+J}\mathrm{tr}\left\{\mathbf{S}_{k'}^{*}\Gamma\mathbf{S}_{k'}\right\} \leq MJ\mathrm{B}^{\alpha}, \quad (20)$$

where α is defined in equation 11. It is clear that blocks of signals designed to meet this constraint will also meet the joint bandwidth constraint. Notice that for $J = S$ and $K = 0$, this is a joint optimization over all of the signals while for $J < S$ this is a recursive update procedure. A modified Fletcher-Powell algorithm was used to perform this constrained optimization over $G(M, D)$. For details on the optimization implementation please see [1].

Notice that the above algorithm incrementally designs signal sets, J signals at a time. For the special case of $J = S$, the technique yields the full optimization problem. It is also possible to iteratively improve the signal set by first designing with an incremental strategy ($J < S$) and then using the resulting design as an initial point in the full optimization.

A. Examples

The first examples we consider compare the performance of a rank constrained signal design of the type considered in [1] with a strict dimensionality constraint of 4 channel uses to that of a bandwidth constrained design which uses the same RMS bandwidth. To calculate the RMS bandwidth of the rank constrained design we assume that the signals are used in conjunction with the first 4 RMS basis functions. Figure 2 shows the asymptotic union bound for $S = 128$ signals with $M = 2$ transmit and $N = 2$ receive antennas. The RMS bandwidth was $B = 136/T$Hz, where T is the signalling period. The asymptotic bound on the symbol error rate is shown in each figure. Notice that we achieve an improvement in the operating SNR of over 2dB through the use of the RMS constraints in the optimizations.

The next example we consider employs the fourth order bandwidth measure with $M = 2$ transmit and $N = 2$ receive antennas. The fourth order bandwidth was fixed at $8/T$Hz. In Figure 3 we plot the asymptotic union bound for the fourth order bandwidth constrained signal constellation and the rank constrained signal set employing the same bandwidth in $D =$ 4 dimensions. We see that the bandwidth constrained design achieves just under a 2dB improvement in the operating SNR over the rank constrained design.

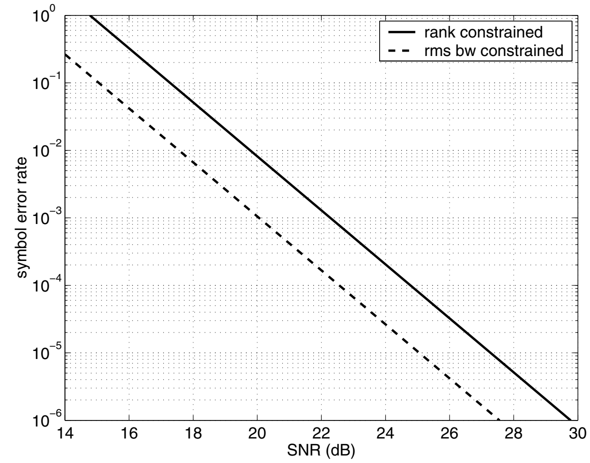

Fig. 2. Symbol error rate versus SNR for $S = 128$ signals, $M = 2$ transmit antennas, and $N = 2$ receive antennas for the rank and the RMS bandwidth constrained designs.

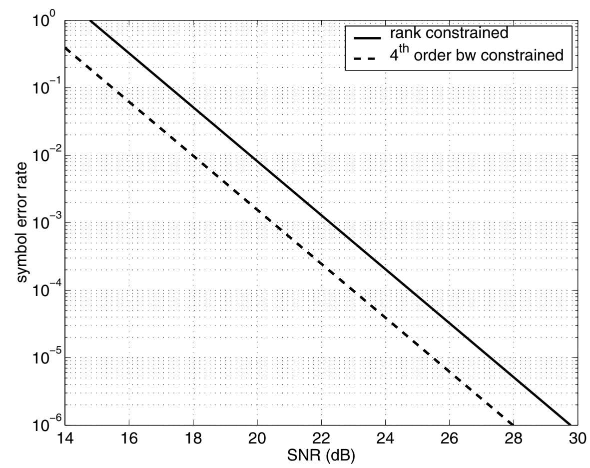

Fig. 3. Symbol error rate versus SNR for $S = 128$ signals, $M = 2$ transmit antennas, and $N = 2$ receive antennas for the rank and the fourth order bandwidth constrained designs.

B. How big should D be made?

One drawback to the bandwidth constrained signal designs is that the optimization must be performed in a high dimensional space (the same dimension as the signal cardinality). However, the penalty for the higher dimensional modes increases quickly for the bandwidth measures that we have consider, suggesting that the designs could be performed in a lower dimensional space. In Figure 4 we consider the effect of varying the dimensionality, D, for a fixed RMS bandwidth of $25/T$Hz with $M = 2$ transmit antennas, $N = 1$ receive antennas, and $S = 64$ symbols. We notice that there is no noticeable improvement in the symbol error rate (we plot the asymptotic bound) when D is increased above 5. The full

RMS optimization would be performed with $D = S = 64$, for which the optimization is prohibitively slow due to the relatively large dimension of the parameter space. Luckily we find that we can get by with a much smaller size problem, which may be solved in a reasonable amount of time.

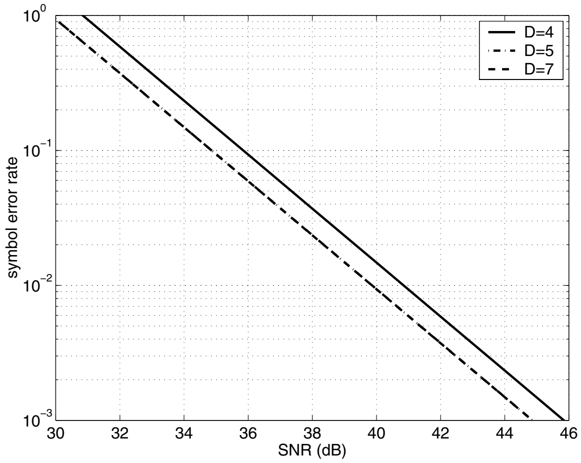

Fig. 4. Symbol error rate versus SNR for $S = 64$ signals, $M = 2$ transmit antennas, $N = 1$ receive antennas, and several values of the dimensionality, D.

IV. Summary and Future Work

In this paper we have addressed the issue of bandwidth efficient signal designs for the noncoherent space-time Rayleigh block fading channel. The key issue in this work is that the absolute (or Shannon) bandwidth is not an appropriate measure on such channels when the fading coefficients are quickly time varying, as is usually true when noncoherent reception is employed. In order to address this problem we have considered more general definitions of bandwidth, including the RMS and fourth order bandwidth measures, and extended our previous work on noncoherent signal designs (the Successive Updates algorithm) to incorporate bandwidth constraints. Future work will consider the coupling of such constraints with more structured designs (such as those of [9]-[11]) which have a lower complexity receiver. Additionally, the optimality (or lack thereof) of unitary signalling under bandwidth constraints should be established.

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