Outage Capacities of Space-Time Architectures

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Abstract — **This paper considers non-ergodic multi-input multi-output (MIMO) fading channels. We compare the outage capacities yielded by the optimal unconstrained system and some recently proposed space-time architectures for i.i.d. Gaussian inputs. All the systems considered here are known to yield outage capacities having the same (maximum) rate of growth with the signal-to-noise ratio (SNR). For each system, an asymptotically tight lower bound on the outage capacity is derived. The analysis reveals that the diagonal BLAST architecture with the zero-forcing front end is asymptotically optimal with respect to the outage capacity. Another space-time architecture is shown to be optimal at all SNRs with respect to the outage capacity.**

I. Introduction

For narrowband, point-to-point communication systems, the use of multiple transmit and receive antennas is an attractive option to significantly increase the spectral efficiency. Our focus here is on delay-constrained systems where a transmitted codeword experiences only one fading realization and the channel state information (CSI) is known perfectly to the receiver but unknown at the transmitter end (perfect CSI-R and no CSI-T). For such systems, outage capacity is a relevant figure of merit. Outage capacity can be defined as the maximum rate that can be transmitted reliably over all channel realizations except a subset whose probability is less than a pre-specified fraction. From a practical point of view, the outage capacities give good indication of the rates that are achievable with acceptably small probability of error over channels with sufficiently large coherence time. Outage capacity and outage probability of the unconstrained MIMO system [1] over Rayleigh fading channel has received much recent attention. However, since at any rate R and SNR ρ, the optimal input covariance matrix is unknown [1], a uniform power assumption is made in that the input covariance matrix is taken to be the scaled identity matrix. In other words, independent, identically distributed (i.i.d.) Gaussian inputs are assumed. Under this assumption, asymptotic distributions of the mutual information (in the limit as the number of receive and/or transmit antennas go to infinity) were obtained in [2,3]. [4,5] and [6] showed that for a finite N receive, K transmit antenna MIMO system, the outage capacity, denoted here by $C_\rho^{ML}(\epsilon)$, has a growth rate characterized by $\lim_{\rho\to\infty} \frac{C_\rho^{ML}(\epsilon)}{\log(\rho)} = \min(N,K)$. [5] in fact shows that the growth rate factor $\min(N,K)$ is achieved for a more general set of fading channels. This factor corresponds to the maximum multiplexing gain point in the tradeoff curve obtained in [7]. However, the capacity order (or the maximum multiplexing gain) is a *coarse measure* of performance of a space-time architecture and it is often not sufficiently informative. For instance, it is not enough to distinguish between the outage capacities of the unconstrained architecture and several sub-optimum architectures such as BLAST because they all have the same growth rate of outage capacity with SNR.

In this paper we give a sharp characterization of the outage capacity — for finite systems, with only mild assumptions on the fading distributions and the uniform power assumption — in the form of *asymptotically tight* (in the limit of high SNR) lower bounds on the outage capacity of the optimum unconstrained system as well as several sub-optimum architectures all of which have the highest capacity order of $\min(N,K)$. By asymptotically tight bound we mean that the high SNR limit of the difference between the actual outage capacity and the bound is identically equal to zero.

Several space-time architectures have been proposed to garner some of the advantages of the unconstrained MIMO system but at lower decoding complexities [8–10]. Here we consider several versions of the vertical and diagonal BLAST architectures [8,10,11], all of which employ single-input single-output (SISO) codes and sub-optimal decoding. Nevertheless, these architectures achieve the maximum possible capacity order (or multiplexing gain) as evidenced by the (high SNR) diversity-multiplexing trade-off curves for these architectures obtained in [7] that show that indeed the highest diversity order achievable is non-zero for all multiplexing gains upto $\min(N,K)$.

Our interest in this work is therefore to develop a sharper analysis of outage capacities and provide comparative statements about the respective outage capacities of these architectures as well as the unconstrained system for both finite SNR as well as in the limit of high SNR. We develop such an analysis and thereby characterize the asymptotic loss in the outage capacity of each architecture relative to the optimum unconstrained system. Interestingly, for example, our analysis reveals the result that the D-BLAST architecture employing the zero-forcing front-end [12] is asymptotically optimal with respect to (w.r.t.) the outage capacity. Further, the threaded space-time architecture [9] which involves spatial formatting of *independent* codewords is shown to be optimal at all SNRs w.r.t. the outage capacity when the codewords are jointly decoded using the optimum decoder.

II. System Model

We consider a N receive and K transmit antenna MIMO channel governed by a discrete-time baseband model

$$\mathbf{y}_j = \mathbf{H}\mathbf{x}_j + \mathbf{v}_j\,, \tag{1}$$

where $\mathbf{x}_j$ is the $K \times 1$ channel input at time j and $\mathbf{y}_j$ is the corresponding $N \times 1$ channel output. The noise vector $\mathbf{v}_j$ is spatially and temporally white and is taken to have i.i.d. circularly symmetric complex normal elements with zero mean and unit variance ($\mathcal{CN}(0,1)$). The random channel matrix $\mathbf{H}$ has dimensions $N \times K$ and is perfectly known to the receiver (perfect CSI-R) but is unknown to the transmitter (no CSI-T). The transmitter however is assumed to know the distribution of $\mathbf{H}$. Further we assume the channel to be non-ergodic, i.e., as in [1], we assume that a channel realization is drawn randomly at the beginning of time and is held fixed for all uses of the channel. To obtain the outage capacities, we view $\mathbf{x}_j$ as a zero-mean random vector and impose the uniform power restriction $E[\mathbf{x}_j\mathbf{x}_j^\dagger] = \frac{\rho}{K}\mathbf{I}$ so that ρ represents the average transmit power. At a given ρ, any probability of error greater than some specified ϵ can be achieved for any rate less than the corresponding ϵ−outage capacity, if the coherence time of the channel is large enough. How large the coherence time must be can be inferred from the error exponents computed for the channel and the space-time architecture employed.

The results obtained here are valid for any fading distribution satisfying the twin conditions of $\mathbf{H}$ being finite as well as full rank with

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[1] A system with no restriction on coding and which employs the optimum decoder.

probability one. Then we have that

$$\lim_{y\to\infty} \Pr\left(\lambda_{max}(\mathbf{H}^\dagger\mathbf{H}) < y\right) = 1, \quad (2)$$

$$\begin{cases} \lim_{y\to 0^+} \Pr\left(\lambda_{min}(\mathbf{H}^\dagger\mathbf{H}) < y\right) = 0, & N \geq K \\ \lim_{y\to 0^+} \Pr\left(\lambda_{min}(\mathbf{H}\mathbf{H}^\dagger) < y\right) = 0, & N < K, \end{cases}$$

where $\lambda_{max}(.)$ and $\lambda_{min}(.)$ represent the maximum and the minimum eigenvalues of their matrix arguments, respectively. Also, outage capacities of the space-time architectures which employ sub-optimum decoders are computed under the assumption $N \geq K$ since many sub-optimum decoders considered here require that condition. Finally, to illustrate, we also specialize our results to the Rayleigh fading channel, where the elements of $\mathbf{H}$ are i.i.d. $\mathcal{CN}(0,1)$.

III. Unconstrained Architecture

We now consider the outage capacity of an unconstrained system with coding across transmit antennas and which employs the optimum decoder. With no CSI-T and under the uniform power restriction the outage probability $\Pr(\mathcal{O})$ at rate R and a given power ρ, is given by $\Pr(\mathcal{O}) = \Pr\left(\log\left(|\mathbf{I} + \frac{\rho}{K}\mathbf{H}^\dagger\mathbf{H}|\right) < R\right)$. For any $\epsilon \in (0,1)$, the resulting outage capacity, denoted as $\mathcal{C}_\rho^{ML}(\epsilon)$, is

$$\mathcal{C}_\rho^{ML}(\epsilon) = \sup\left\{R : \Pr\left(\log\left(\left|\mathbf{I} + \frac{\rho}{K}\mathbf{H}^\dagger\mathbf{H}\right|\right) < R\right) \leq \epsilon\right\}. \quad (3)$$

For $N \geq K$, we next expand $\mathcal{C}_\rho^{ML}(\epsilon)$ as $\mathcal{C}_\rho^{ML}(\epsilon) = K\log(\rho) + \log(\hat{\mathcal{C}}_\rho^{ML}(\epsilon))$ so that

$$\hat{\mathcal{C}}_\rho^{ML}(\epsilon) = \sup\left\{y : \Pr\left(\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < y\right) \leq \epsilon\right\}. \quad (4)$$

Note that for any $\rho > 0$, using the first condition in (2), we can conclude that $\hat{\mathcal{C}}_\rho^{ML}(\epsilon) < \infty$. Furthermore, for any $y \geq 0$, $\Pr(|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}| < y)$ is non-decreasing in ρ, so that $\hat{\mathcal{C}}_\rho^{ML}(\epsilon)$ is non-increasing in ρ. Further, we define

$$\hat{\mathcal{C}}_\infty^{ML}(\epsilon) = \sup\left\{y : \Pr\left(\left|\frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < y\right) \leq \epsilon\right\}, \quad (5)$$

and using the second condition in (2) we note that $\hat{\mathcal{C}}_\infty^{ML}(\epsilon) > 0$. Thus we have $\hat{\mathcal{C}}_\rho^{ML}(\epsilon) \geq \hat{\mathcal{C}}_\infty^{ML}(\epsilon) > 0$, and that $\lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{ML}(\epsilon)$ exists. Next, for any given $y > 0$, let $\mathcal{X}_\rho(\mathbf{H})$ and $\mathcal{X}_\infty(\mathbf{H})$ denote the indicator functions of the events $\{\mathbf{H} : |\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}| < y\}$ and $\{\mathbf{H} : |\frac{1}{K}\mathbf{H}^\dagger\mathbf{H}| < y\}$, respectively. It is readily seen that for any $\mathbf{H}$, $\lim_{\rho\to\infty}\mathcal{X}_\rho(\mathbf{H}) = \mathcal{X}_\infty(\mathbf{H})$. Then since $E[\mathcal{X}_\rho(\mathbf{H})] = \Pr\left(\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < y\right)$, using the Dominated Convergence Theorem (DCT) [13], we can show that

$$\lim_{\rho\to\infty}\Pr\left(\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < y\right) = \Pr\left(\left|\frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < y\right). \quad (6)$$

Using (6) and (5), we have that for arbitrary $\delta > 0$,

$$\lim_{\rho\to\infty}\Pr\left(\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < \hat{\mathcal{C}}_\infty^{ML}(\epsilon) + \delta\right) = \quad (7)$$
$$\Pr\left(\left|\frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < \hat{\mathcal{C}}_\infty^{ML}(\epsilon) + \delta\right) = \epsilon + \alpha,$$

for some $\alpha > 0$. From (7) we can guarantee the existence of ρ_0 such that $\forall\rho > \rho_0$,

$$\Pr\left(\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| < \hat{\mathcal{C}}_\infty^{ML}(\epsilon) + \delta\right) \geq \epsilon + \alpha/2 \quad (8)$$

so that $\lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{ML}(\epsilon) < \hat{\mathcal{C}}_\infty^{ML}(\epsilon) + \delta$. Since the above observation holds for arbitrary $\delta > 0$ we have that, $\lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{ML}(\epsilon) \leq \hat{\mathcal{C}}_\infty^{ML}(\epsilon)$ and hence $\lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{ML}(\epsilon) = \hat{\mathcal{C}}_\infty^{ML}(\epsilon)$. Similarly for $N < K$, expanding $\mathcal{C}_\rho^{ML}(\epsilon)$ as $\mathcal{C}_\rho^{ML}(\epsilon) = N\log(\rho) + \log(\hat{\mathcal{C}}_\rho^{ML}(\epsilon))$, we can show that

$$\lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{ML}(\epsilon) = \hat{\mathcal{C}}_\infty^{ML}(\epsilon) = \sup\left\{y : \Pr\left(\left|\frac{1}{K}\mathbf{H}\mathbf{H}^\dagger\right| < y\right) \leq \epsilon\right\}.$$

We summarize the results in the following:

Theorem 1: A lower bound to $\mathcal{C}_\rho^{ML}(\epsilon)$ is given by

$$\mathcal{C}_\rho^{ML}(\epsilon) \geq \tilde{\mathcal{C}}_\rho^{ML}(\epsilon) \triangleq \min(N,K)\log(\rho) + \log(\hat{\mathcal{C}}_\infty^{ML}(\epsilon)). \quad (9)$$

$\tilde{\mathcal{C}}_\rho^{ML}(\epsilon)$ is asymptotically tight in that $\lim_{\rho\to\infty}(\mathcal{C}_\rho^{ML}(\epsilon) - \tilde{\mathcal{C}}_\rho^{ML}(\epsilon)) = 0$.

A. The Rayleigh Fading MIMO Channel

For the special case of the Rayleigh fading channel, we will explicitly evaluate $\hat{\mathcal{C}}_\infty^{ML}(\epsilon)$. To this end, for $N \geq K$ we let $\mathbf{H}^\dagger\mathbf{H} = \mathbf{L}^\dagger\mathbf{L}$ denote the Cholesky decomposition with $\mathbf{L}$ being a $K \times K$ lower triangular matrix with positive diagonal elements. We then need to compute $\Pr(|\mathbf{H}^\dagger\mathbf{H}| < y) = \Pr\left(\prod_{i=1}^K L_{i,i}^2 < y\right)$. It is well known that that $\{L_{i,i}^2\}_{i=1}^K$ are independent chi-square random variables with $\left\{2\mathcal{D}_i \triangleq 2(N-K+i)\right\}_{i=1}^K$ degrees of freedom, respectively [14]. Using the distributions of $\{L_{i,i}^2\}$, after some work it can be verified that $\Pr\left(\prod_{i=1}^K L_{i,i}^2 < y\right)$ is equal to

$$c\int_0^\infty \cdots \int_0^\infty \left\{\exp(-x_{K-1} - \frac{x_{K-2}}{x_{K-1}} - \cdots - \frac{x_1}{x_2}) \times \right. \quad (10)$$
$$\left. x_1^{N-K+1} F_{L_{1,1}^2}(y/x_1)\, dx_1 \cdots dx_{K-1}\right\},$$

where $c = \frac{1}{\prod_{i=1}^{K-1}(N-K+i)!}$ and $F_{L_{1,1}^2}(u) = 1 - \exp(-u)(\sum_{m=0}^{N-K}\frac{u^m}{m!})$ for $u \geq 0$. (10) can be numerically evaluated, thereby yielding a simple algorithm to explicitly evaluate the fundamental constant $\hat{\mathcal{C}}_\infty^{ML}(\epsilon)$, and hence the asymptotic lower bound $\tilde{\mathcal{C}}_\rho^{ML}(\epsilon)$ for $\mathcal{C}_\rho^{ML}(\epsilon)$. Similarly when $N < K$ we can obtain the Cholesky decomposition of $\mathbf{H}\mathbf{H}^\dagger$ and proceed as before.

IV. V-BLAST Architecture

We now consider the V-BLAST architecture proposed in [11]. Here the input data stream is de-multiplexed into K sub-streams which are then *independently* coded using K single-input single-output (SISO) encoders. The K independent codewords of equal length are transmitted by the K transmitters, with the k^{th} codeword being transmitted by the k^{th} transmitter over J symbol intervals (the duration of a frame). Note that there is no spatial formatting in this scheme. This restriction of using independent SISO codes is imposed for allowing low complexity decoding, typically through the decorrelating decision feedback decoder (D-DFD) or its minimum mean square error counterpart the MMSE-DFD. There are three versions of V-BLAST. The first one, referred to here as simply V-BLAST, uses K SISO encoders of equal rates and employs the D-DFD or the MMSE-DFD with a fixed order of decoding. The second version, which we refer to as Ordered V-BLAST, also uses K SISO encoders of equal rates but uses the D-DFD or the MMSE-DFD with the SISO codes decoded in an order that is channel dependent. The third version, proposed as rate-tailored BLAST (RT-BLAST) in [10], employs the D-DFD in a fixed order of decoding but for K SISO encoders of possibly different rates (see also [15]).

A. V-BLAST

We start by considering the first version [8]. A brief description of V-BLAST [2] employing the D-DFD with a fixed order of decoding is as follows. The receiver obtains the QR decomposition $\mathbf{H} = \mathbf{UL}$, where the $K \times K$ lower triangular matrix $\mathbf{L}$ has positive diagonal elements and $\mathbf{U}^\dagger\mathbf{U} = \mathbf{I}$. It then transforms the received vectors $\{\mathbf{y}_j\}_{j=1}^J$ as $\mathbf{z}_j = \mathbf{U}^\dagger\mathbf{y}_j = \mathbf{L}\mathbf{x}_j + \tilde{\mathbf{v}}_j$. The SISO decoder for transmitter 1 makes a decision $\hat{\mathbf{x}}_1$, based on the soft statistics $\{z_{1,j}\}$. The soft statistics for the k^{th} SISO decoder obtained after feedback are denoted by $\{\tilde{z}_{k,j}\}$, where,

$$\begin{aligned}\tilde{z}_{k,j} &= z_{k,j} - \sum_{m=1}^{k-1} L_{k,m}\hat{x}_{m,j} \\ &= L_{k,k}x_{k,j} + \sum_{m=1}^{k-1} L_{k,m}(x_{m,j} - \hat{x}_{m,j}) + \tilde{v}_{k,j}.\end{aligned}$$

The k^{th} decoder decodes the codeword transmitted by the k^{th} transmit antenna assuming perfect feedback. For any sum rate $R > 0$, the corresponding outage probability is given by [8, 16],

$$\begin{aligned}\Pr(\mathcal{O}) = \Pr\left(\cup_{i=1}^K \left\{\log\left(1 + \frac{\rho}{K}L_{i,i}^2\right) < \frac{R}{K}\right\}\right) = \\ \Pr\left(\log\left(1 + \frac{\rho}{K}\min_{1\le i\le K}\{L_{i,i}^2\}\right) < \frac{R}{K}\right).\end{aligned}$$

Thus the corresponding outage capacity, denoted by $\mathcal{C}_\rho^{VB}(\epsilon)$, can be defined as

$$\mathcal{C}_\rho^{VB}(\epsilon) \triangleq \sup\left\{R : \Pr\left(\log\left(1 + \frac{\rho}{K}\min_{1\le i\le K}\{L_{i,i}^2\}\right) < \frac{R}{K}\right) \le \epsilon\right\}.$$

To obtain an asymptotically tight lower bound to $\mathcal{C}_\rho^{VB}(\epsilon)$, we set $\mathcal{C}_\rho^{VB}(\epsilon) = K\log(\rho) + K\log(\hat{\mathcal{C}}_\rho^{VB}(\epsilon))$, so that

$$\hat{\mathcal{C}}_\rho^{VB}(\epsilon) = \sup\left\{y : \Pr\left(\frac{1}{\rho} + \frac{1}{K}\min_{1\le i\le K}\{L_{i,i}^2\} < y\right) \le \epsilon\right\}. \quad (11)$$

At any $\rho > 0$, using the inequality $\left|\frac{1}{\rho}\mathbf{I} + \frac{1}{K}\mathbf{H}^\dagger\mathbf{H}\right| \ge \prod_{i=1}^K \left(\frac{1}{\rho} + \frac{1}{K}L_{i,i}^2\right)$, we have

$$\begin{aligned}(\hat{\mathcal{C}}_\rho^{VB}(\epsilon))^K &\le \sup\left\{y^K : \Pr\left(\prod_{i=1}^K\left(\frac{1}{\rho} + \frac{1}{K}L_{i,i}^2\right) < y^K\right) \le \epsilon\right\} \\ &\le \hat{\mathcal{C}}_\rho^{ML}(\epsilon) < \infty.\end{aligned}$$

Let us define

$$\hat{\mathcal{C}}_\infty^{VB}(\epsilon) \triangleq \sup\left\{y : \Pr\left(\frac{1}{K}\min_{1\le i\le K}\{L_{i,i}^2\} < y\right) \le \epsilon\right\}.$$

Then, noting that $\hat{\mathcal{C}}_\rho^{VB}(\epsilon)$ is non-increasing in ρ, and using the fact that $\min_{1\le i\le K}\{L_{i,i}^2\} \ge \lambda_{min}(\mathbf{H}^\dagger\mathbf{H})$ along with the second condition in (2), we can prove that

$$\hat{\mathcal{C}}_\rho^{VB}(\epsilon) \ge \hat{\mathcal{C}}_\infty^{VB}(\epsilon) > 0 \qquad \& \qquad \lim_{\rho\to\infty}\hat{\mathcal{C}}_\rho^{VB}(\epsilon) = \hat{\mathcal{C}}_\infty^{VB}(\epsilon).$$

Theorem 2: An asymptotically tight lower bound to the outage capacity of V-BLAST is given by $\tilde{\mathcal{C}}_\rho^{VB}(\epsilon) = K\log(\rho) + K\log(\hat{\mathcal{C}}_\infty^{VB}(\epsilon))$.

Next, we consider V-BLAST with the MMSE-DFD with fixed order of decoding. Obtaining the outage capacity in this case even under the uniform power restriction is an open problem, since the optimal input distribution is not known. However, i.i.d. Gaussian inputs are commonly assumed and the resulting outage capacity is $\mathcal{C}_\rho^{VBM}(\epsilon) \triangleq \sup\{R : \Pr(\mathcal{O}) \le \epsilon\}$, where $\Pr(\mathcal{O})$ now equals

$$\Pr\left(\log\left(1 + \frac{\rho}{K}\min_{1\le i\le K}\left\{\mathbf{h}_i^\dagger\left(\mathbf{I} - \mathbf{G}_{(i),\rho}\right)\mathbf{h}_i\right\}\right) < \frac{R}{K}\right)$$

where $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_K]$, $\mathbf{H}_{(i)} = [\mathbf{h}_{i+1}, \cdots, \mathbf{h}_K]$ and

$$\mathbf{G}_{(i),\rho} = \mathbf{H}_{(i)}\left(\frac{K}{\rho}\mathbf{I} + \mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)}\right)^{-1}\mathbf{H}_{(i)}^\dagger,$$

respectively. Next we note that with $\mathbf{H}^\dagger\mathbf{H} = \mathbf{L}^\dagger\mathbf{L}$,

$$\begin{aligned}L_{i,i}^2 &= \mathbf{h}_i^\dagger(\mathbf{I} - \mathbf{H}_{(i)}(\mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)})^{-1}\mathbf{H}_{(i)}^\dagger)\mathbf{h}_i \quad (12) \\ &= ||\mathbf{h}_i||^2 - \mathbf{h}_i^\dagger\mathbf{H}_{(i)}(\mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)})^{-1}\mathbf{H}_{(i)}^\dagger\mathbf{h}_i\end{aligned}$$

but since for any $\rho > 0$, $(\mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)})^{-1} > (\frac{K}{\rho}\mathbf{I} + \mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)})^{-1}$, we have that

$$L_{i,i}^2 < \mathbf{h}_i^\dagger(\mathbf{I} - \mathbf{G}_{(i),\rho})\mathbf{h}_i. \quad (13)$$

Further, for any $y > 0$, using the DCT with (13) and (12) we have

$$\begin{aligned}\lim_{\rho\to\infty}\Pr\left(\log\left(\frac{1}{\rho} + \frac{1}{K}\min_{1\le i\le K}\left\{\mathbf{h}_i^\dagger(\mathbf{I} - \mathbf{G}_{(i),\rho})\mathbf{h}_i\right\}\right) < y\right) \quad (14) \\ = \Pr\left(\log(\frac{1}{K}\min_{1\le i\le K}\{L_{i,i}^2\}) < y\right).\end{aligned}$$

Then using (14) we have the following

Theorem 3: The outage capacity of V-BLAST with the MMSE-DFD is higher than that of V-BLAST with D-DFD for any SNR, and moreover, the high SNR limit of the outage capacity of the former coincides with that of the latter, i.e.,

$$\mathcal{C}_\rho^{VBM}(\epsilon) \ge \mathcal{C}_\rho^{VB}(\epsilon)$$

and

$$\lim_{\rho\to\infty}(\mathcal{C}_\rho^{VBM}(\epsilon) - \mathcal{C}_\rho^{VB}(\epsilon)) = 0.$$

B. Ordered V-BLAST

We next introduce V-BLAST employing the D-DFD along with the optimal channel dependent ordering [16]. We let $\mathcal{P}$ denote the set of all $K!$ permutations of $\{1, \cdots, K\}$ and let $\mathbf{H}^\pi = \mathbf{U}^\pi\mathbf{L}^\pi$ denote the QR decomposition corresponding to any permutation $\pi \in \mathcal{P}$, where $\mathbf{H}^\pi$ is obtained by permuting the columns of $\mathbf{H}$ according to π. Then at rate R the outage event corresponding to the permutation π is

$$\mathcal{O}_\pi = \left\{\mathbf{H} : \log\left(1 + \frac{\rho}{K}\min_{1\le i\le K}\{(L_{i,i}^\pi)^2\}\right) < \frac{R}{K}\right\}.$$

The optimal channel dependent ordering selects the permutation which maximizes $\min_{1\le i\le K}\left\{(L_{i,i}^\pi)^2\right\}$ among all permutations in $\mathcal{P}$ [8, 16]. Then, the resulting outage probability is given by

$$\Pr(\mathcal{O}) = \Pr\left(\log\left(1 + \frac{\rho}{K}\max_{\pi\in\mathcal{P}}\min_{1\le i\le K}\left\{(L_{i,i}^\pi)^2\right\}\right) < \frac{R}{K}\right),$$

and the outage capacity, $\mathcal{C}_\rho^{VB-ord}(\epsilon)$, equals

$$\sup\left\{R : \Pr\left(\log\left(1 + \frac{\rho}{K}\max_{\pi\in\mathcal{P}}\min_{1\le i\le K}\left\{(L_{i,i}^\pi)^2\right\}\right) < \frac{R}{K}\right) \le \epsilon\right\},$$

respectively. Regarding the outage capacity of Ordered V-BLAST and its comparison with V-BLAST (unordered), we simply state the following theorem without proof.

[2] Coded V-BLAST is referred to as H-BLAST in [8].

Theorem 4: The Ordered V-BLAST architecture has a higher outage capacity than V-BLAST, its unordered counterpart for any $\rho > 0$, i.e.,

$$\mathcal{C}_\rho^{VB-ord} \geq \mathcal{C}_\rho^{VB}(\epsilon)\,.$$

Moreover, an asymptotically tight lower bound to the outage capacity of Ordered V-BLAST $\mathcal{C}_\rho^{VB-ord}(\epsilon)$ is given by $\tilde{\mathcal{C}}_\rho^{VB-ord}(\epsilon) = K\log(\rho) + K\log(\hat{\mathcal{C}}_\infty^{VB-ord}(\epsilon))$, where

$$\hat{\mathcal{C}}_\infty^{VB-ord}(\epsilon) = \sup\left\{y : \Pr\left(\frac{1}{K}\max_{\pi\in\mathcal{P}}\min_{1\leq i\leq K}\{(L_{i,i}^\pi)^2\} < y\right) \leq \epsilon\right\}.$$

Next, we turn to Ordered V-BLAST with the MMSE-DF decoder. Assuming i.i.d Gaussian inputs, we have that $\mathcal{C}_\rho^{VBM-ord}(\epsilon)$ is given by,

$$\sup\{R : \Pr\left(\log(1 + \frac{\rho}{K}\max_{\pi\in\mathcal{P}}\min_{1\leq i\leq K}\left\{(\mathbf{h}_i^\pi)^\dagger(\mathbf{I} - \mathbf{G}_{(i),\rho}^\pi)\mathbf{h}_i^\pi\right\}) < \frac{R}{K}\right) \leq \epsilon\right\}$$

We state next our theorem that summarizes our results for Ordered V-BLAST with the MMSE-DF decoder and its performance relative to the previously considered V-BLAST architectures.

Theorem 5: At any $\rho > 0$, the outage capacity of Ordered V-BLAST with MMSE-DF decoder is higher than that of its unordered counterpart as well as that of the Ordered V-BLAST with D-DF decoder. i.e.,

$$\mathcal{C}_\rho^{VBM-ord}(\epsilon) \geq \mathcal{C}_\rho^{VBM}(\epsilon) \;\;\&\;\; \mathcal{C}_\rho^{VBM-ord}(\epsilon) \geq \mathcal{C}_\rho^{VB-ord}(\epsilon)\,.$$

Moreover, an asymptotically tight lower bound to $\mathcal{C}_\rho^{VBM-ord}(\epsilon)$ is given by $\tilde{\mathcal{C}}_\rho^{VB-ord}(\epsilon)$, i.e., in Ordered V-BLAST, the MMSE-DF decoder yields an outage capacity that is asymptotically the same as that yielded by the D-DF decoder, i.e.,

$$\lim_{\rho\to\infty}(\mathcal{C}_\rho^{VBM-ord}(\epsilon) - \mathcal{C}_\rho^{VB-ord}(\epsilon)) = 0.$$

C. Rate-Tailored BLAST

We next consider the rate-tailored BLAST (RT-BLAST) architecture proposed in [10] which is identical to V-BLAST employing the D-DFD (fixed order) except that the equal rate restriction is removed. For the Rayleigh fading channel, significant improvements in the performance (joint error probability) of uncoded V-BLAST were obtained earlier through optimal rate and power allocations by the authors in [15]. The works of [10, 16] independently suggest improving the performance of coded V-BLAST by minimizing the outage probability over rate allocations. The outage capacity of RT-BLAST under the uniform power restriction, denoted by $\mathcal{C}_\rho^{RT}(\epsilon)$, was obtained in [10] and an asymptotic (high SNR) analysis was conducted in [6]. The work in [6] obtained a lower bound on $\mathcal{C}_\rho^{RT}(\epsilon)$ by upper bounding the outage probability via a Chernoff bound. This lower bound sufficed to show that RT-BLAST has the maximum capacity order, i.e., that $\lim_{\rho\to\infty}\frac{\mathcal{C}_\rho^{RT}(\epsilon)}{\log(\rho)} = K$. However, the bound in [6] is not tight.

Here, we do not restrict ourselves to Rayleigh fading channels. Moreover, through a much simpler argument than in [6], we obtain a lower bound — denoted by $\tilde{\mathcal{C}}_\rho^{RT}(\epsilon)$ — which also grows with ρ as $K\log(\rho)$ but is *asymptotically tight.*

For RT-BLAST, $\mathcal{C}_\rho^{RT}(\epsilon)$ is equal to

$$\sup\left\{\sum_{i=1}^K R_i : \Pr\left(\cup_{i=1}^K\left\{\log\left(1 + \frac{\rho}{K}L_{i,i}^2\right) < R_i\right\}\right) \leq \epsilon\right\}.$$

We set $\mathcal{C}_\rho^{RT}(\epsilon) = K\log(\rho) + \log(\hat{\mathcal{C}}_\rho^{RT}(\epsilon))$, so that $\hat{\mathcal{C}}_\rho^{RT}(\epsilon)$ equals

$$\sup\left\{\prod_{i=1}^K y_i : \Pr\left(\cup_{i=1}^K\left\{\frac{1}{\rho} + \frac{1}{K}L_{i,i}^2 < y_i\right\}\right) \leq \epsilon\right\}. \qquad (15)$$

At any $\rho > 0$, comparing (15) with (11), we have that $\hat{\mathcal{C}}_\rho^{RT}(\epsilon) \geq (\hat{\mathcal{C}}_\rho^{VB}(\epsilon))^K$, so that $\mathcal{C}_\rho^{RT}(\epsilon) \geq \mathcal{C}_\rho^{VB}(\epsilon)$. Also it can be shown that

$$\hat{\mathcal{C}}_\rho^{RT}(\epsilon) \leq \sup\left\{\prod_{i=1}^K y_i : \Pr\left(\prod_{i=1}^K\left(\frac{1}{\rho} + \frac{1}{K}L_{i,i}^2\right) < \prod_{i=1}^K y_i\right) \leq \epsilon\right\} \leq \hat{\mathcal{C}}_\rho^{ML}(\epsilon) < \infty\,.$$

Regarding the outage capacity of RT-BLAST, we have the following

Theorem 6: At any $\rho > 0$, we have that the outage capacity of RT-BLAST is bounded below and above by the outage capacities of V-BLAST and the optimum unconstrained system, i.e.,

$$\mathcal{C}_\rho^{VB}(\epsilon) \leq \mathcal{C}_\rho^{RT}(\epsilon) \leq \mathcal{C}_\rho^{ML}(\epsilon)\,.$$

Furthermore, an asymptotically tight lower bound to the outage capacity of RT-BLAST is given by

$$\mathcal{C}_\rho^{RT}(\epsilon) \geq \tilde{\mathcal{C}}_\rho^{RT}(\epsilon) = K\log(\rho) + \log(\hat{\mathcal{C}}_\infty^{RT}(\epsilon))\,,$$

where $\hat{\mathcal{C}}_\infty^{RT}(\epsilon)$ is defined as

$$\hat{\mathcal{C}}_\infty^{RT}(\epsilon) \triangleq \sup\left\{\prod_{i=1}^K y_i : \Pr\left(\cup_{i=1}^K\left\{\frac{1}{K}L_{i,i}^2 < y_i\right\}\right) \leq \epsilon\right\},$$

D. Outage Capacity of V-BLAST and RT-BLAST for the Rayleigh Fading Channel

Over the Rayleigh fading channel we can further refine the results obtained for V-BLAST and RT-BLAST both of which employ the D-DFD with a fixed order of decoding. Note that the cdf of $L_{i,i}^2$ is given by $F_i(x) = 1 - \exp(-x)(\sum_{m=0}^{\mathcal{D}_i-1}\frac{x^m}{m!})$ so that letting $F_{min}(.)$ denote the cdf of $\min_{1\leq i\leq K}\{L_{i,i}^2\}$, we have that

$$F_{min}(x) = 1 - \prod_{i=1}^K\left(\exp(-x)\left(\sum_{m=0}^{\mathcal{D}_i-1}\frac{x^m}{m!}\right)\right).$$

Therefore, we have $\hat{\mathcal{C}}_\infty^{VB}(\epsilon) = F_{min}^{-1}(\epsilon)/K$, whence the asymptotically tight lower bound on the outage capacity of V-BLAST can be explicitly computed as $\tilde{\mathcal{C}}_\rho^{VB}(\epsilon) = K\log\rho + K\log(\frac{F_{min}^{-1}(\epsilon)}{K})$.

Next, it can be shown that

$$\log(\hat{\mathcal{C}}_\infty^{RT}(\epsilon)) = \max_{\substack{y_i>0,\ 1\leq i\leq K\\ \sum_{i=1}^K \ln(1-F_i(y_i))\geq\ln(1-\epsilon)}}\left\{\sum_{k=1}^K\log(y_k)\right\}$$

is a *concave maximization problem*, so that the asymptotically tight lower bound for RT-BLAST can also be computed using standard numerical optimization algorithms for the Rayleigh fading channel.

V. Diagonal BLAST (D-BLAST) Architecture

In this section we consider the D-BLAST architecture where the transmitted matrix comprises of diagonally layered SISO codewords. The decoder consists of a zero-forcing or MMSE filter front end followed by SISO decoding and decision feed-back of "previous" diagonals.

A. D-BLAST with Zero-Forcing Filtering

Over Rayleigh fading channels, the outage capacity of D-BLAST with the zero-forcing front end was computed in [12] through simulations. The rate loss caused due to frame set up and termination was neglected without a proper justification. Also, [12] observed that for large ρ, the resulting outage capacity was very close to $\mathcal{C}_\rho^{ML}(\epsilon)$. In the following, we show that for large framelength and a suitable choice of *diagonal width* the rate loss can indeed be neglected. We then prove that D-BLAST with the zero-forcing front end is asymptotically optimal with respect to the outage capacity.

With zero-forcing filtering and error-free decision feedback, we obtain a parallel channel model given by [7, 12],

$$z_i^g = L_{i,i}x_i + \tilde{v}_i, \ \ 1 \le i \le K. \tag{16}$$

Each SISO codeword of length nK, where $n \ge 1$ is referred to as the *diagonal width*, is picked from a code of rate $\frac{R}{K}$. Assuming the framelength to be $J = lnK$ for some $l \ge 1$, we have that there are $M = K(l-1)+1$ independent codewords in each frame. For $l >> 1$, the loss due to set up and termination is negligible in the sense that the rate (in bits per channel use) achieved is $\frac{RnKM}{KJ} = R\frac{lK-(K-1)}{lK} \approx R$. However the number of codewords, M, also scales linearly with l. Under perfect feedback each codeword cycles through the K parallel channels in (16) and the outage event under the uniform power restriction can be defined as, $\mathcal{O} = \{\mathbf{H}; \frac{1}{K}\sum_{i=1}^{K}\log\left(1+\frac{\rho}{K}L_{i,i}^2\right) < \frac{R}{K}\}$. Let $\Pr(\mathcal{O})$ denote the resulting outage probability. Further, let $\mathcal{E}_i$ denote the error event for the i^{th} codeword so that the frame error event is $\mathcal{E} \triangleq \cup_{i=1}^{M}\mathcal{E}_i$. To show that any rate $R-\alpha$, for arbitrary $\alpha > 0$ can be achieved, we need to take $l >> 1$ and prove the existence of a SISO code of length nK and rate $\frac{R}{K} - \delta$, for arbitrary $\delta > 0$, which guarantees a frame error probability $\Pr(\mathcal{E})$ arbitrarily close to $\Pr(\mathcal{O})$. This can be done as follows. Letting $\mathcal{E}_i^g$ denote the error event for the i^{th} codeword under perfect feedback and $\mathcal{O}^c$ denote the complement of the outage event, we upper bound $\Pr(\mathcal{E})$ as,

$$\Pr(\mathcal{E}) = \Pr(\cup_{i=1}^{M}\mathcal{E}_i) = \Pr(\cup_{i=1}^{M}\mathcal{E}_i^g) \le \Pr(\mathcal{O}) + \Pr(\cup_{i=1}^{M}\mathcal{E}_i^g, \mathcal{O}^c)$$
$$\le \Pr(\mathcal{O}) + \sum_{i=1}^{M}\Pr(\mathcal{E}_i^g, \mathcal{O}^c)$$

Using results on compound channels, we can guarantee the existence of a code of rate $\frac{R}{K} - \delta$ and length nK whose error probability decays *exponentially* with n under the event $\mathcal{O}^c$. Thus since $\sum_{i=1}^{M}\Pr(\mathcal{E}_i^g, \mathcal{O}^c)$ increases linearly with l and decays exponentially with n, we can choose l and n so that the rate loss is negligible and *simultaneously* $\Pr(\mathcal{E})$ is arbitrarily close to $\Pr(\mathcal{O})$.

Consequently, with rigorous justification, we have shown that the outage capacity of the D-BLAST architecture, denoted as $\mathcal{C}_\rho^{DB}(\epsilon)$, is equal to

$$\mathcal{C}_\rho^{DB}(\epsilon) = \sup\left\{R : \Pr\left(\sum_{i=1}^{K}\log\left(1+\frac{\rho}{K}L_{i,i}^2\right) < R\right) \le \epsilon\right\}.$$

Next, we state the following theorem without proof which says that the outage capacity of D-BLAST is sandwiched between that of RT-BLAST and the optimum unconstrained architecture.

Theorem 7: At any $\rho > 0$, we have that

$$\mathcal{C}_\rho^{RT}(\epsilon) \le \mathcal{C}_\rho^{DB}(\epsilon) \le \mathcal{C}_\rho^{ML}(\epsilon) < \infty.$$

Quite interestingly, the D-BLAST architecture achieves asymptotically the same performance as the optimum unconstrained architecture. Using the fact that $|\mathbf{H}^\dagger\mathbf{H}| = \prod_{i=1}^{K}L_{i,i}^2$, we have

Theorem 8: In the limit of high SNR, we have that

$$\lim_{\rho\to\infty}(\mathcal{C}_\rho^{ML}(\epsilon) - \mathcal{C}_\rho^{DB}(\epsilon)) = 0$$

Thus with respect to outage capacity, D-BLAST with the zero-forcing front end is asymptotically optimal.

The next theorem states that the outage capacity of D-BLAST is always higher than Ordered V-BLAST.

Theorem 9: We have that at any finite $\rho > 0$,

$$\mathcal{C}_\rho^{VB}(\epsilon) \le \mathcal{C}_\rho^{VB-ord}(\epsilon) \le \mathcal{C}_\rho^{DB}(\epsilon) \le \mathcal{C}_\rho^{ML}(\epsilon).$$

Proof: Of the three inequalities, the only one that remains to be proved is that $\mathcal{C}_\rho^{VB-ord}(\epsilon) \le \mathcal{C}_\rho^{DB}(\epsilon)$. To accomplish this we will equivalently prove that

$$\sum_{i=1}^{K}\log\left(1+\frac{\rho}{K}L_{i,i}^2\right) \ge K\max_{\pi\in\mathcal{P}}\log\left(1+\frac{\rho}{K}\min_{1\le i\le K}\{(L_{i,i}^\pi)^2\}\right). \tag{17}$$

Using the fact that for any $c > 0$, $\log(1+c\exp(x))$ is convex in $x > 0$, along with Jensen's inequality [17], we have that

$$\sum_{i=1}^{K}\log\left(1+\frac{\rho}{K}L_{i,i}^2\right) \ge K\log\left(1+\frac{\rho}{K}(\prod_{i=1}^{K}L_{i,i}^2)^{1/K}\right). \tag{18}$$

For any permutation π, we have

$$|\mathbf{H}^\dagger\mathbf{H})| = \prod_{i=1}^{K}L_{i,i}^2 = |(\mathbf{H}^\pi)^\dagger\mathbf{H}^\pi| = \prod_{i=1}^{K}(L_{i,i}^\pi)^2. \tag{19}$$

Using (19) in (18) with the fact that $(\prod_{i=1}^{K}(L_{i,i}^\pi)^2)^{1/K} \ge \min_{1\le i\le K}\{(L_{i,i}^\pi)^2\}$, we have that for any $\pi\in\mathcal{P}$,

$$\sum_{i=1}^{K}\log\left(1+\frac{\rho}{K}L_{i,i}^2\right) \ge K\log\left(1+\frac{\rho}{K}\min_{1\le i\le K}\{(L_{i,i}^\pi)^2\}\right). \tag{20}$$

The result in (17) now follows from (20). ■

D-BLAST with the MMSE front end was shown to be optimal in [7, 8] using a fact proved earlier in [18] and by ignoring the rate loss. The latter step can be justified by arguments similar to the ones given here for the zero-forcing front-end.

VI. Optimal Spatial Formatting

We focus on the TST architecture proposed in [9]. In this architecture a K layer TST frame consists of K, spatially formatted (independent) codewords of equal lengths, the j^{th} codeword corresponds to the j^{th} layer for $1 \le j \le K$. The codewords are generated by K independent SISO encoders of equal rate. The framelength is assumed to be a multiple of K. Letting $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_J]$ denote the transmitted frame and $\lfloor a \rfloor_b = a \ mod \ b$, we have that, $\{X_{\lfloor t+m-1\rfloor_K+1,t}\}_{t=1}^{J}$ are the J consecutive symbols of the m^{th} codeword (or m^{th} layer). Then collecting K consecutive received vectors in $\tilde{\mathbf{y}} = [\mathbf{y}_1^T \cdots, \mathbf{y}_K^T]^T$, the equivalent linear model can be written as,

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{z}} + \tilde{\mathbf{v}} \tag{21}$$

where $\tilde{\mathbf{z}} = [\mathbf{z}_1^T \cdots, \mathbf{z}_K^T]^T$ with $\mathbf{z}_m = [z_{1,m}, \cdots, z_{K,m}]^T$ where $z_{j,m} = X_{\lfloor j+m-1\rfloor_K+1,j}$, $1 \le j \le K$ and $\tilde{\mathbf{H}}$ is equal to

$$[\mathbf{e}_1\otimes\mathbf{h}_2, \mathbf{e}_2\otimes\mathbf{h}_3, \cdots, \mathbf{e}_K\otimes\mathbf{h}_1, \mathbf{e}_1\otimes\mathbf{h}_3, \cdots, \mathbf{e}_K\otimes\mathbf{h}_2, \cdots,$$
$$\mathbf{e}_1\otimes\mathbf{h}_1, \cdots, \mathbf{e}_K\otimes\mathbf{h}_K],$$

where $\mathbf{e}_k$ is the k^{th} unit vector of length K. Using the model in (21) along with the fact that the codewords across layers are *independent*, the outage event for the optimum decoder, $\mathcal{O}^{TST-opt}$, can be written as

$$\mathcal{O}^{TST-opt} = \left\{\mathbf{H} : \bigcup_{\substack{\mathcal{L}\subseteq\{1,\cdots,K\}\\ \mathcal{L}\ne\phi}}\left\{\frac{1}{K}\sum_{t=1}^{K}\right.\right.$$
$$\left.\left.\log\left|\mathbf{I}+\frac{\rho}{K}\sum_{m\in\mathcal{L}}\mathbf{h}_{\lfloor t+m-1\rfloor_K+1}\mathbf{h}_{\lfloor t+m-1\rfloor_K+1}^\dagger\right| < \frac{|\mathcal{L}|R}{K}\right\}\right\}$$

and the resulting outage capacity is $\mathcal{C}_\rho^{TST-opt}(\epsilon) = \sup\{R : \Pr(\mathcal{O}^{TST-opt}) \le \epsilon\}$. Before proving our optimality result for the TST, we need the following lemma stated without proof.

Lemma 1: For any $N \times K$ matrix $\mathbf{H}$ and any non-empty subset $\mathcal{L} \subseteq \{1, \cdots, K\}$ we have,

$$\sum_{t=1}^{K} \log \left| \mathbf{I} + \frac{\rho}{K} \sum_{m \in \mathcal{L}} \mathbf{h}_{\lfloor t+m-1 \rfloor_K + 1} \mathbf{h}^{\dagger}_{\lfloor t+m-1 \rfloor_K + 1} \right| \geq |\mathcal{L}| \log \left| \mathbf{I} + \frac{\rho}{K} \mathbf{H}\mathbf{H}^{\dagger} \right|.$$

Thus by lemma (1) we have that $\mathcal{O}^{TST-opt}$ in fact can be simplified as,

$$\mathcal{O}^{TST-opt} = \left\{ \mathbf{H} : \log \left| \mathbf{I} + \frac{\rho}{K} \mathbf{H}\mathbf{H}^{\dagger} \right| < R \right\} \quad (22)$$

Consequently we have the following theorem

Theorem 10: The TST architecture employing optimum decoding is optimal w.r.t the outage capacity, i.e. $\mathcal{C}^{TST-opt}_{\rho}(\epsilon) = \mathcal{C}^{ML}_{\rho}(\epsilon)$ at any SNR ρ.

VII. Simulation Results

We now demonstrate the effectiveness of the analytical lower bounds over Rayleigh fading channels. In the following examples we plot the outage capacities in bits per channel use (PCU) against SNR (per receive antenna per symbol interval).

In Fig. 1 we consider two systems with $(6,6)$ and $(4,4)$ transmit and receive antenna pairs, respectively. For each system, for $\epsilon = .1,\ .01$, we have plotted $\mathcal{C}^{ML}_{\rho}(\epsilon)$, $\mathcal{C}^{DB}_{\rho}(\epsilon)$, which were obtained through simulations and the analytical lower bound $\tilde{\mathcal{C}}^{ML}_{\rho}(\epsilon)$ computed using (10). Note that at each ϵ and sufficiently high SNR, $\mathcal{C}^{ML}_{\rho}(\epsilon)$ and $\mathcal{C}^{DB}_{\rho}(\epsilon)$ are practically indistinguishable from their asymptotically tight lower bound, $\tilde{\mathcal{C}}^{ML}_{\rho}(\epsilon)$. Also, the convergence to the lower bound is faster for larger ϵ.

In Fig. 2 we consider a $(4,4)$ system and for $\epsilon = .05$, plot $\mathcal{C}^{RT}_{\rho}(.05)$, $\mathcal{C}^{VB-ord}_{\rho}(.05)$, $\mathcal{C}^{VB}_{\rho}(.05)$ along with their respective asymptotically tight lower bounds. Again the computed lower bounds are seen to be tight for sufficiently high SNR. Also RT-BLAST performs slightly better than Ordered V-BLAST and provides an asymptotic gain of about 6.4 dB over V-BLAST which uses the D-DFD with a fixed order.

VIII. Conclusions

We compared the outage capacities yielded by the unconstrained system and some recently proposed space-time architectures. Through asymptotically tight lower bounds, the asymptotic (high SNR) loss in the outage capacity of each architecture relative to the optimum unconstrained system was determined. Further, the D-BLAST architecture using the zero-forcing front-end was shown to be asymptotically optimal w.r.t the outage capacity whereas the TST architecture with optimum decoding was shown to be optimal at all SNRs.

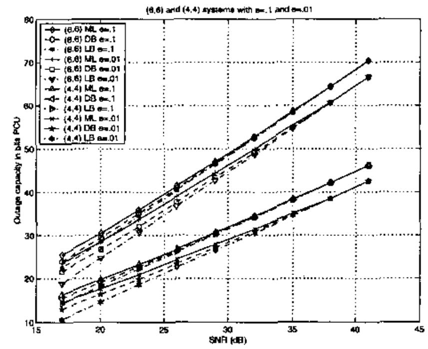

Fig. 1. Outage Capacity Comparison

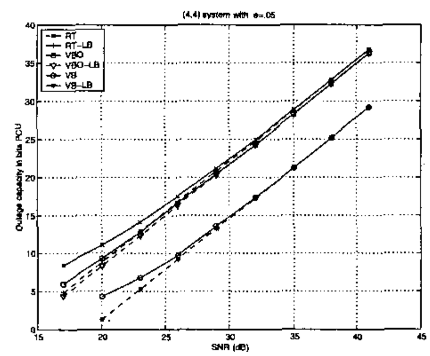

Fig. 2. Outage Capacity Comparison

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