Achieving Vertices of the Capacity Region of the Synchronous Gaussian Correlated-Waveform Multiple-Access Channel with Decision-Feedback Receivers

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Abstract — **We consider users operating synchronously on a correlated-waveform multiple-access (CWMA) channel with additive Gaussian noise. CWMA is a multiple-access scheme where the users are assigned signature waveforms; it subsumes both orthogonal signaling (e.g., TDMA) and identical-waveform signaling (as in the conventional Gaussian multiple-access channel (C-GMAC)) as special cases. We obtain a decision-feedback receiver (DFR) for the CWMA channel which, in spite of its lower complexity than the optimal receiver, achieves the vertices of the CWMA channel's capacity region, implying in addition that the users can choose their single-user codes independently of each other. This result generalizes the well known successive decoder of the C-GMAC.**

I. Introduction

The discrete-time equivalent model of the M-user synchronous Gaussian CWMA (SG-CWMA) channel is a linear vector channel given by [1]

$$\underline{Y} = \mathbf{R}\underline{X} + \underline{N}, \qquad (1)$$

where the M-dimensional vector $\underline{X}$ contains the users' symbols, the $M \times M$ matrix $\mathbf{R}$ contains the correlations of the users' waveforms, and the Gaussian random vector, $\underline{N}$, represents noise with $\sigma^2\mathbf{R}$ as it covariance matrix. When every element of $\mathbf{R}$ is unity, we have the C-GMAC, for which successive decoding [2, Sec. 14.3.6] can be used to achieve the total capacity of the channel at a vertex point of the capacity region. The successive decoder sees an effective single-input/single-output (SISO) channel for each user so that the complexities of true multiuser coding and decoding are avoided.

When the waveforms of a SG-CWMA channel are energy-weighted root-mean-squared (EW-RMS) bandlimited, it has been shown [3] that the total capacity is maximized when the signature waveforms are neither orthogonal nor identical. This was the motivation in [4] for applying the ideas of successive decoding to the SG-CWMA channel by introducing the DFR structure. This structure combines low-complexity (linear in M) multiuser detection (with soft outputs) and sequential decoding of the users. The former operation is parameterized by a feed-forward matrix, $\mathbf{F}$, and the latter by a feedback matrix, $\mathbf{B}$. Because each user effectively sees a SISO Gaussian channel, a DFR allows the users to choose single-user codes independently of each other. In [4] it was shown that there exist an $\mathbf{F}$, $\mathbf{B}$ pair that simultaneously maximizes the capacity of each user when the order in which they are decoded is given; the result is the capacity-maximizing DFR (CM-DFR).

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II. Successive Decoding for CWMA

It can be shown that the CM-DFR allows the users of a SG-CWMA channel to achieve the total capacity at a vertex point of the capacity region. That is, the CM-DFR essentially generalizes successive decoding to the SG-CWMA channel. Of course, when all users have the same signature waveform, the channel reduces to the C-GMAC and the CM-DFR reduces to the usual successive decoder.

This property of the CM-DFR can be explained analytically by considering the chain rule for mutual information:

$$I(\underline{Y};\underline{X}) = I(X_1;\underline{Y}) + \sum_{i=2}^{M} I(X_i;\underline{Y}|X_1,\ldots,X_{i-1}). \qquad (2)$$

The CM-DFR essentially employs a Bayesian sufficient statistic, $T_1(\underline{Y})$, for the pair $(X_1,\underline{Y})$. It is known [2, Sec. 2.10] [5, Chap. 10] that $I(X_1,\underline{Y}) = I(X_1,T_1(\underline{Y}))$. When $\underline{X}$ is a zero-mean Gaussian vector, there exists a scalar sufficient statistic. For example, the Gauss-Markov theorem yields the following sufficient statistic for the first user[2],

$$T_1(\underline{Y}) = E\{X_1\underline{Y}^{\mathrm{T}}\}\left(E\{\underline{Y}\,\underline{Y}^{\mathrm{T}}\}\right)^{-1}\underline{Y}. \qquad (3)$$

Proceeding similarly for the other users we get that

$$I(\underline{X};\underline{Y}) = I(X_1;T_1) + \cdots + I(X_M;T_M), \qquad (4)$$

where, with $\underline{Y_i} = \underline{Y} - E\{\underline{Y}|X_1,\ldots,X_{i-1}\}$, each

$$T_i = E\{X_i\underline{Y_i}\}\left(E\{\underline{Y_i}\,\underline{Y_i}^{\mathrm{T}}\}\right)^{-1}\underline{Y_i} \qquad (5)$$

is a scalar, Gaussian random variable. Hence, the total capacity of the channel is achieved at a vertex point of the capacity region using this generalized form of successive decoding.

References

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[2]If $\underline{Y}$ is not full rank, it can be replaced by $\tilde{\underline{Y}}$, any full-rank subset of $\underline{Y}$ that has the same rank as $\underline{Y}$.