Noncoherent Decision Feedback Multiuser Detection : Optimality, Performance Bounds, and Rules for Ordering Users

Mahesh K. Varanasi and Deepak Das
Dept. of Electrical & Computer Engineering
University of Colorado, Boulder, CO 80309-0425 USA
Email: dasd@ucsu.colorado.edu varanasi@schof.colorado.edu

Abstract — **An analytical framework for noncoherent decision feedback detection is introduced while considering nonorthogonal binary modulation (NBM) over the synchronous Gaussian K-user channel. Following the key idea of noncoherent decorrelating decision feedback detection (NC-DDFD) proposed in [1], a K-parameter class of NC-DDFDs is defined. The symmetric energy measure is defined as the worst-case (over users) asymptotic effective energy for noncoherent detection. Without making any simplifying assumption about error propagation, the NC-DDFD that optimizes symmetric energy among the K-parameter class of detectors is derived. Since the NC-DDFD based on the generalized likelihood ratio test (GLRT) proposed in [1] belongs to the K-parameter class of NC-DDFDs, the optimum NC-DDFD outperforms the GLRT based NC-DDFD in symmetric energy. Like the latter detector, the optimum NC-DDFD does not require the knowledge of the energies or phases of any of the users' transmissions. Exact expressions for symmetric energy and upper and lower bounds for symbol error rate (SER) and asymptotic effective energies are obtained for the optimum NC-DDFD. Rules for ordering users are obtained that guarantee that the optimum NC-DDFD can user-wise outperform the parallel bank of post-decorrelative GLRT detectors of [3].**

I. System Model

We consider a binary signaling scheme where each of the K users of a multiuser channel transmits one of two nonorthogonal signals to send one bit of information. After passing through an additive white Gaussian noise (AWGN) channel, the superposition of the K signals arrive in symbol synchronism at the receiver. A bank of $2K$ matched filters at the receiver is used to extract the sufficient statistics. The psuedo-linear discrete-time model for the sampled outputs of the matched filters can be obtained as in [3] as $\mathbf{y} = \mathbf{RAb} + \mathbf{n}$, where $\mathbf{y}$ is $2K$-dimensional with the matched filter ouputs arranged user-wise $\mathbf{y} = \left(\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T\right)^T$ with the 2-dimensional $\mathbf{y}_k$ containing the k^{th} user's matched filter outputs. The signal correlation matrix $\mathbf{R}$ is $2K \times 2K$ and assumed to be positive definite. The $2K \times 2K$ complex amplitude matrix $\mathbf{A}$ is diagonal with the k^{th} 2×2 block $\mathbf{A}_{kk} = \mathrm{diag}\{\sqrt{E_k}e^{j\phi_{k1}}, \sqrt{E_k}e^{j\phi_{k2}}\}$ where E_k is the k^{th} user's received energy per bit, assumed equal for both signals of the same user, and ϕ_{ki} are carrier phases modeled as independent, uniformly distributed random variables in $[0, 2\pi]$. The $2K$-dimensional information bearing vector $\mathbf{b} = (\mathbf{b}_1^T, \cdots, \mathbf{b}_K^T)^T$ is such that $\mathbf{b}_k \in \{(0,1),(1,0)\}$ depending on whether the signal transmitted by user k is the first or the second signal, respectively. The noise vector $\mathbf{n}$ is zero-mean, complex, circularly symmetric Gaussian random vector with covariance matrix $\sigma^2\mathbf{R}$.

II. The K-Parameter Class of NC-DDFDs

Assume that the users are indexed in the order in which they are detected. The k^{th} user's decision is made after the decisions of past users $1, \cdots, k-1$ are made. Following the idea of noncoherent decorrelation [3] we define $\mathbf{z} = \mathbf{R}^{-1}\mathbf{y}$. Let user 1 be detected according to the post-decorrelative rule that decides on the first or second signal (denoted as $\hat{i}_1 = 1$ or 2) transmitted according to whether $|z_1(1)|^2$ is greater or less than $\beta_1|z_1(2)|^2$. $z_1(1)$ and $z_1(2)$ are the first two elements of $\mathbf{z}$, β_1 is some arbitrary real-valued and positive parameter. The k^{th} user's decision is made as follows: suppose that the past user decisions are $\hat{\mathbf{i}}_1^{k-1} = (\hat{i}_1, \cdots, \hat{i}_{k-1})^T$, with $\hat{i}_j \in \{1,2\}$. Following the idea of noncoherent decision feedback introduced in [1], consider for user k, an 'expurgated' version of the MF outputs $\mathbf{y}_{ex}^{(k)} = \left(\mathbf{y}_1(\hat{i}_1), \cdots, \mathbf{y}_{k-1}(\hat{i}_{k-1}), \mathbf{y}_k^T, \cdots, \mathbf{y}_K^T\right)^T$. Consider also the matrix $\mathbf{R}_{ex}^{(k)}$ which is a $2K - k + 1$ principal submatrix of $\mathbf{R}$ obtained by retaining its rows and columns corresponding to the same indices that are used to retain the elements of $\mathbf{y}$ to form the sub-vector $\mathbf{y}_{ex}^{(k)}$. Let $\mathbf{z}_k^{df}(1)$ and $\mathbf{z}_k^{df}(2)$ denote the k^{th} and the $(k+1)^{st}$ elements of $(\mathbf{R}_{ex}^{(k)})^{-1}\mathbf{y}_{ex}^{(k)}$, respectively. The decision for user k is chosen to be the first or the second signal depending on whether $|z_k^{df}(1)|^2$ is greater or less than $\beta_k|z_k^{df}(2)|^2$, respectively. β_k is a real-valued and positive parameter corresponding to user k and is allowed to be a function of past decisions. This completes the description of the K-parameter class of NC-DDFDs. We will refer to an arbitrary member of this class as ϕ^{β}.

The NC-DDFD based on the generalized likelihood ratio test (GLRT) proposed in [1] is a member in the class of NC-DDFDs defined above. In particular, the parameters $\{\beta_k\}$ take on values that depend on past decisions and have the nice feature of not requiring the knowledge of signal energies for their determination. While retaining this desirable feature, a key objective of this paper is to seek an NC-DDFD that is optimum in symmetric energy among the class of NC-DDFDs.

III. Symmetric Energy

If the bit error probability in the multiuser channel for user k of a NC-DDFD $\phi(\beta)$ (denoted as $P_k(\sigma, \phi(\beta))$ is expressed in the form of the minimum error probability formula for binary orthogonal signalling over a single-user channel, so that we let $P_k(\sigma,\phi(\beta))$= $2^{-1}\exp(-e_k(\sigma,\phi(\beta))/2\sigma^2)$, then $e_k(\sigma,\phi(\beta))$ is the effective energy whose limit as $\sigma \to 0$ is defined as *asymptotic effective energy* and denoted as $E_k(\phi(\beta))$. The *symmetric energy* $E(\phi(\beta)) = \min_{1\le k\le K} E_k(\phi(\beta))$ is defined as the worst-case asymptotic effective energy.

The results mentioned in the abstract will be presented.

References

[1] M. K. Varanasi and D. Das, "Noncoherent Decision Feedback Multiuser Detection for Nonorthogonal Multipulse Modulation", presented at *CISS 1998* , Princeton, NJ, March 1998.

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